The Evaluation of Sensors' Reliability and their Tuning for Multisensor Data Fusion within the Transferable Belief Model

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Abstract. We develop a method to evaluate the reliability of a sensor in a classification task when the uncertainty is represented by belief functions as understood in the transferable belief model.
This reliability is represented by a discounting factor that minimizes the distance between the pignistic probabilities computed from the discounted beliefs and the actual values of the data in a learning set.
We then describe a method to tune the discounting factors of several sensors when their reports are merged in order to reach an aggregated report. They are computed so that together they minimize the distance between the pignistic probabilities computed from the combined discounted belief functions and the actual values of the data in a learning set.
The first method produces the reliability of a sensor considered alone. The second method considers a set of sensors, and weights each of them so that together they produce the best predictor.

1 Introduction

The belief function theory, in particular the Transferable Belief Model (TBM), is more and more used to represent and deal with uncertainty. It can be seen as a generalization of subjective probability theory. The TBM allows to handle data collected from partially reliable sensors. It can represent full, partial and even total ignorance. The conjunctive rule of combination provides the tool to aggregate the reports produced by several sensors in order to get their merged report. It seems to be perfectly adapted for multisensor data fusion [14].

Sensors use different approaches and types of measurements and work in different environments, and their reliability can vary from one to the other. One way to take in consideration the reliability and applicability of a sensor consists in weighing / discounting their reports.

In the TBM, a sensor reports about the actual value of a variable is represented by a belief function. The reliability of the sensor is represented by a discounting factor, i.e., a coefficient that 'weights' the belief function produced by the sensor. Reliability and discounting are linked by the idea that if a sensor is felt as unreliable by the user, he/she will discount what the sensor states. Discount can be understood as 'partially disregard'. The smaller the reliability, the larger the discounting. The discounting factor is a well defined concept in belief function theory (see section 2.3), whereas reliability will be used unformally hereafter.

This paper addresses the problem of assessing the discounting factor to be applied to the beliefs generated by the sensors. We develop two methods applicable in two contexts. The first consists in assessing the discounting factor to be applied to one sensor by comparing its report (represented by a belief function) with the actual values. The second consists in assessing the values of the discounting factor to be given to each of several sensors when their reports must be merged. It is obtained by comparing the merged discounted belief function with the actual values.

The first method concerns one sensor, the second concerns a group of sensors who jointly must produce an aggregated decision.

It may seem odd that we speak of 'beliefs held by a sensor', but the term belief is to be taken in a neutral way. No philosophical or psychological connotation is to be introduced. It is just a tradition that the functions that represent the sensor report are called 'belief function', hence the 'belief' term. Classically, sensors produce likelihoods. Here we just replace the term likelihood by beliefs, what enhances that we use belief functions and not probability functions.

This paper is composed as follows. We start by giving an overview of the basics of the TBM. Next, we present the multisensor data fusion within the belief function formalism. We then describe the two methods for assessing sensor reliabilities and for tuning them. Each method is illustrated by an example explaining its unfolding.

In this paper, we speak of sensors, but all we present here can be applied directly to other problems, like expert opinion pooling. An expert is just a sensor, and his/her opinion is equivalent to a sensor report. Data fusion and opinion pooling are analogous.

Experts differ in level of expertise, some of them are more reliable than others due to their better knowledge, training, experience, intelligence ... To express their opinions, experts may use different background, methodology and even knowledge. Hence, the necessity to consider the expert's reliability when receiving their opinions, and consequently these judgments must be appropriately 'discounted'.

Thus, the concepts of expert, opinion and expert opinion pooling are equivalent to those of sensor, report and data fusion. The methods presented in this paper can be applied directly to this other domain. Note that other researchers have proposed to assess experts' discounting factors within the belief function theory, we basically mention the one developed by Zouhal and Denoeux [15].

2 Belief Function Theory

In this section, we briefly review the basics of the belief function theory as interpreted in the Transferable Belief Model (TBM). For a more detailed explanation and other basics see [6, 10, 11].

2.1 Definitions

Let Θ be a finite set of elementary and mutually exclusive hypotheses related to a given problem domain. It is called the frame of discernment. One value of Θ, denoted θ_0, corresponds to the actual value of Θ. This actual value is not known by the belief holder (the sensor).

A basic belief assignment (bba) is a function m from the power set of Θ, denoted 2^{Θ}, to $[0, 1]$ verifying:

$$\sum_{A \subseteq \Theta} m(A) = 1 \tag{1}$$

The basic belief mass (bbm) $m(A)$ given to $A \subseteq \Theta$ is the amount of belief specifically assigned to the event $\theta_0 \in A$ and that cannot support any subset of Θ more specific than A.

The belief function (bel) represents the belief assigned to an event $A \subseteq \Theta$. It is equal to the sum of bbm committed to the subsets of A. For each bba m, there corresponds a belief function bel such that: $bel : 2^{\Theta} \to [0, 1]$, and defined by:

$$bel(A) = \sum_{\emptyset \neq B \subseteq A} m(B), \ \forall A \subseteq \Theta. \tag{2}$$

A vacuous belief function is such that $m(\Theta) = 1$ and $m(A) = 0, \ \forall A \subseteq \Theta, A \neq \Theta$. It represents a state of the total ignorance.

2.2 Combination

Consider two pieces of evidence on the same frame Θ represented by the two bbas m_1 and m_2, the joint bba quantifying the combined impact of these two pieces of evidence is obtained through the conjunctive combination rule as follows [8]:

$$(m_1 \circledcirc m_2)(A) = \sum_{B,C \subseteq \Theta : B \cap C = A} m_1(B).m_2(C) \tag{3}$$

where $\circledcirc$ denotes the operator of conjunction. The classical Dempster's rule of combination is the conjunctive combination rule where the result is normalized by dividing each term by $(1 - (m_1 \circledcirc m_2)(\emptyset))$. It is defined as:

$$(m_1 \oplus m_2)(A) = K. \sum_{B,C \subseteq \Theta : B \cap C = A} m_1(B).m_2(C) \tag{4}$$

where

$$K^{-1} = 1 - \sum_{B,C \subseteq \Theta : B \cap C = \emptyset} m_1(B).m_2(C) \tag{5}$$

and

$$(m_1 \oplus m_2)(\emptyset) = 0. \tag{6}$$

K is called the normalization factor.

The conjunctive combination rule and Dempster's rule of combination are commutative and associative, so we can combine several belief functions iteratively and in any order.

2.3 Discounting

Reliability, i.e. our opinion about the 'value' of a sensor, varies from sensor to sensor. The idea is to weight more heavily the reports produced by the 'best' sensors and conversely for the 'bad' ones. For $\alpha \in [0, 1]$, let $(1 - \alpha)$ be the degree of 'confidence' we assign to the sensor. It can be encoded into a bba defined on the set {reliable, not reliable} such that [9]:

$$m(\textit{reliable}) = 1 - \alpha \qquad \text{and} \qquad m(\textit{not reliable}) = \alpha \tag{7}$$

Suppose the bba m on Θ represents the sensor report about the actual value of Θ. The result of combining the sensor report with the bba given in (7) is a new bba, denoted m^α, defined as:

$$m^\alpha(A) = (1 - \alpha).m(A) \text{ for } A \subset \Theta \tag{8}$$

$$m^\alpha(\Theta) = \alpha + (1 - \alpha).m(\Theta) \tag{9}$$

This operation is called a discounting by Shafer [6] and the coefficient α is called the discounting factor. The larger α, the closer m^α is from the vacuous belief function.

2.4 Pignistic Transformation

To make decisions in the TBM, we build a probability function $BetP$ on Θ, called the pignistic probability function, by applying the pignistic transformation [10]. It is defined by:

$$BetP(A) = \sum_{B \subseteq \Theta} \frac{|A \cap B|}{|B|} \frac{m(B)}{1 - m(\emptyset)}, \ \forall A \subseteq \Theta \tag{10}$$

3 Multisensor Data Fusion with the TBM

Multisensor systems can be used for the detection, localization and recognition of objects in a given area [1]. Handling information collected by different sensors requires an evidence gathering process, called a multisensor data fusion process, in order to get, hopefully, a 'better' information. The TBM offers a formal way to combine sensor data what is achieved by the conjunctive combination rule.

As mentioned before, sensors do not usually have the same level of reliability, so before pooling sensor reports (hence combining their belief functions), each belief function should be discounted to take into account the sensor reliability represented by the discounting factor. When these discounting factors are not known, they must be assessed. We propose two methods for such an assessment which correspond to the two different contexts mentioned in the introduction.

4 Evaluating Sensor's Reliability

4.1 Introduction

Finding an 'automatic' method to assess the sensor's reliability relative to a given problem requires information regarding the judgments given previously by the sensor concerning 'past' events (related to the same problem) for which the truth is known by us and not by the sensor. Then, a comparison between the truth and the sensor's judgments allows to derive the reliability of the sensor.

In practice, one domain where we can get this kind of information is represented by classification problems[1]. In such problems, we can get the sensor's reports on the classes to which an object belongs, a class otherwise well known by us. In the following subsections, we focus on classification problems. The method can easily be adapted to other domains, the underlying schema being quite general.

4.2 The Framework

Let T be a set composed of n objects denoted by o_j $(j = 1, 2, ..., n)$. Each object has to belong to one of the possible classes relative to the given problem. The set of classes is defined by $\Theta = \{\theta_1, \theta_2, ..., \theta_p\}$. For each object o_j, we know its class, denoted c_j with $c_j \in \Theta$, and the sensor produces a bba, denoted $m^\Theta\{o_j\}$ on Θ, that represents its opinion on the actual value of c_j.

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5.3 Assessing the Discounting Factor

The first method considers one sensor for which we want to assess its reliability, thus its discounting factor. This is done by comparing the bba produced by the sensor about the class of each of the n objects with their actual classes.

If we knew the discounting factor α applicable to a sensor we would discount the bba it generates by the discounting factor. So we would compute the bba $m^{\Theta,\alpha}\{o_j\}$ using relations (8) and (9).

If we had to decide which class objects o_j belongs to, we would then compute the pignistic probability from $m^{\Theta,\alpha}\{o_j\}$. Let the result be denoted by $BetP^{\Theta,\alpha}\{o_j\}$. This probability function is then to be compared with the actual value c_j of object o_j. Let the indicator function δ be defined as $\delta_{j,i} = 1$ if $c_j = \theta_i$ and 0 otherwise.

The distance between the pignistic probability computed from the discounted sensor's report and the indicator function δ is used as a measure of the reliability of the sensor for what concerns object o_j, and their sum over the n objects is used as a measure of the overall reliability of the sensor. It is denoted $TotalDist$ and defined as:

$$TotalDist = \sum_{j=1}^{n} \sum_{i=1}^{p} (BetP^{\Theta,\alpha}\{o_j\}(\theta_i) - \delta_{j,i})^2$$

We then define the reliability of the sensor as $(1-\alpha \in [0,1])$ where α minimizes $TotalDist$, i.e., the α that makes the discounted opinions of the sensor as good as possible, thus that makes the values of $BetP^{\Theta,\alpha}\{o_j\}$ as close as possible to $\delta_{j,i}$.

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5.4 Explicit Computation with Normalized Belief Functions

In the special but common case where all bbas are normalized (thus $m(\emptyset) = 0$), it is possible to explicitate the value of α from the initial bba $m^{\Theta}\{o_j\}$. Let $BetP^{\Theta}\{o_j\}$ be the pignistic probability function computed from $m^{\Theta}\{o_j\}$, hence before discounting. The solution for α is given in the next theorem.

Theorem 1. *Let a set of normalized bbas $m^{\Theta}\{o_j\}$ defined on the set of classes $\Theta = \{\theta_1, \ldots, \theta_p\}$ for objects $o_j, j = 1, \ldots, n$. Let the indicator function $\delta_{j,i} = 1$ if the object o_j belongs to the class θ_i, and 0 otherwise. The discounting factor α that minimizes:*

$$TotalDist = \sum_{j=1}^{n} \sum_{i=1}^{p} (BetP^{\Theta,\alpha}\{o_j\}(\theta_i) - \delta_{j,i})^2$$

where $BetP^{\Theta,\alpha}\{o_j\}$ is the pignistic probability function computed from the discounted bba $m^{\Theta,\alpha}\{o_j\}$, is given by:

$$\alpha = \min(1, \max(0, \frac{\sum_{j=1}^{n} \sum_{i=1}^{p} (\delta_{j,i} - BetP^{\Theta}\{o_j\}(\theta_i)).BetP^{\Theta}\{o_j\}(\theta_i)}{n/p - \sum_{j=1}^{n} \sum_{i=1}^{p} BetP^{\Theta}\{o_j\}(\theta_i)^2}))$$

Proof. Given $m^{\Theta}\{o_j\}$, we have:

$$\begin{aligned} m^{\Theta,\alpha}\{o_j\}(\theta) &= (1-\alpha)m^{\Theta}\{o_j\}(\theta) && \text{if } \theta \subset \Theta \\ &= (1-\alpha)m^{\Theta}\{o_j\}(\Theta) + \alpha && \text{if } \theta = \Theta \end{aligned}$$

The pignistic probability $BetP^{\Theta,\alpha}\{o_j\}$ computed from $m^{\Theta,\alpha}\{o_j\}$ can be expressed as a function of the pignistic probability $BetP^{\Theta}\{o_j\}$ computed directly from $m^{\Theta}\{o_j\}$. For simplicity sake, we omit the $\{o_j\}$ index hereafter. One has:

$$\begin{aligned} BetP^{\Theta}(\theta_i) &= \sum_{\theta_i \in \theta} \frac{m^{\Theta}}{|\theta|} \\ BetP^{\Theta,\alpha}(\theta_i) &= \sum_{\theta_i \in \theta} \frac{m^{\Theta,\alpha}(\theta)}{|\theta|} \\ &= \sum_{\theta_i \in \theta} \frac{(1-\alpha)m^{\Theta}(\theta)}{|\theta|} + \alpha/p = (1-\alpha)BetP^{\Theta}(\theta) + \alpha/p \end{aligned}$$

For simplicity sake, we write $P_{ij} = BetP^{\Theta}\{o_j\}(\theta_i)$. The term to be minimized becomes:

$$TotalDist = \sum_{j=1}^{n} \sum_{i=1}^{p} (BetP^{\Theta,\alpha}\{o_j\}(\theta_i) - \delta_{ji})^2 = \sum_{j,i} ((1-\alpha)P_{ij} + \alpha/p - \delta_{ji})^2$$

Its extremum is reached when its derivative is null, hence when:

$$\begin{aligned} 0 &= \frac{d\, TotalDist}{d\alpha} \\ &= 2\sum_{j,i}((1-\alpha)P_{ij} + \alpha/p - \delta_{ji})(-P_{ij} + 1/p) && (11) \\ &\propto \sum_{j,i} -(1-\alpha)P_{ij}^2 - \alpha n/p + \sum_{j,i} \delta_{ji}P_{ij} + (1-\alpha)n/p + \alpha n/p - n/p && (12) \\ &= \sum_{j,i} -(1-\alpha)P_{ij}^2 - \alpha n/p + \sum_{j,i} \delta_{ji}P_{ij} && (13) \end{aligned}$$

$$\text{Thus, } \alpha = \frac{\sum_{j,i}(\delta_{ji} - P_{ij})P_{ij}}{n/p - \sum_{j,i} P_{ij}^2}$$

□

Once the discounting factors are computed for several sensors, the observed values can be used to order several sensors: the smaller the value the better the sensor. It could be used to select optimal sensors. It can also be used to discount the reports produced by the sensor in the future.

5.5 The Simplified Equivalent

Usually, probabilities are easier to understand than discounting factors. So one way to get a feeling of what represents the value α of a discounting factor, we consider a highly simplified case where there are just 2 objects that can be either a or b. Both are a's. You are sure that object 1 is a, but have a probability $\pi \leq 0.5$ that object 2 is a and $1-\pi$ that it is b. So you are right for object 1 and quite wrong with object 2 (with probability $1-\pi$). If π was known, we could compute its related discounting factor by applying the previous relations. When π is unknown but α is known, we can compute the value of π that underlies α in our simplified schema. We have: $\alpha \in [0,1]$, compute

$$\pi = \frac{3 - 2\alpha - \sqrt{1 + 4\alpha - 4\alpha^2}}{4 - 4\alpha}$$

This is the value π that would produce α in the simplified schema where we deal with only two objects and two classes and where the sensor is only uncertain about the class of one object. This π represents the probability that the sensor is correct and that would induced a discounting factor equals to α.

5.6 Example 1

Suppose there are two sensors S_1, S_2 applied to classify aerial targets. The possible classes are: $\Theta = \{Airplane, Helicopter, Rocket\}$. In order to find the degree of reliability of these two sensors, table 1 presents their reports on the classes of 4 objects where their classes are known by us (a part of a learning set), but not by the sensors S_1, S_2. At the first row of the table, we have the actual class of each object, then we present the two sensors' bbas on the classes of these objects (since they do not know the truth).

Table 1. The sensors' bbas and the truth

Truth	Airplane	Helicopter	Airplane	Rocket
S_1	o_1	o_2	o_3	o_4
$\emptyset$	0	0	0	0
Airplane	0	0	0	0
Helicopter	0	0.5	0.4	0
Rocket	0.5	0.2	0	0
Airplane ∪ Helicopter	0	0	0	0
Airplane ∪ Rocket	0	0	0.6	0.6
Helicopter ∪ Rocket	0.3	0	0	0.4
Airplane ∪ Helicopter ∪ Rocket	0.2	0.3	0	0
S_2	o_1	o_2	o_3	o_4
$\emptyset$	0	0	0	0
Airplane	0	0.3	0.2	0
Helicopter	0	0	0	0
Rocket	0	0	0	0
Airplane ∪ Helicopter	0.7	0.4	0	0
Airplane ∪ Rocket	0	0	0	0
Helicopter ∪ Rocket	0	0	0.6	1
Airplane ∪ Helicopter ∪ Rocket	0.3	0.3	0.2	0

Assume that the discounting factors assigned to the two sensors S_1 and S_2 are respectively α_1 and α_2.

Let's focus on the first sensor, we have to update the bbas relative to the objects o_1, o_2, o_3 and o_4 by taking into account α_1. We get:

$m_{S_1}^{\Theta,\alpha_1}\{o_1\}(Rocket) = 0.5(1-\alpha_1)$, $m_{S_1}^{\Theta,\alpha_1}\{o_1\}(Helicopter \cup Rocket) = 0.3(1-\alpha_1)$, $m_{S_1}^{\Theta,\alpha_1}\{o_1\}(\Theta) = 0.2 + 0.8\alpha_1$

$m_{S_1}^{\Theta,\alpha_1}\{o_2\}(Helicopter) = 0.5(1-\alpha_1)$, $m_{S_1}^{\Theta,\alpha_1}\{o_2\}(Rocket) = 0.2(1-\alpha_1)$, $m_{S_1}^{\Theta,\alpha_1}\{o_2\}(\Theta) = 0.3 + 0.7\alpha_1$

$m_{S_1}^{\Theta,\alpha_1}\{o_3\}(Helicopter) = 0.4(1-\alpha_1)$, $m_{S_1}^{\Theta,\alpha_1}\{o_3\}(Airplane \cup Rocket) = 0.6(1-\alpha_1)$, $m_{S_1}^{\Theta,\alpha_1}\{o_3\}(\Theta) = \alpha_1$

$m_{S_1}^{\Theta,\alpha_1}\{o_4\}(Airplane \cup Rocket) = 0.6(1-\alpha_1)$, $m_{S_1}^{\Theta,\alpha_1}\{o_4\}(Helicopter \cup Rocket) = 0.4(1-\alpha_1)$, $m_{S_1}^{\Theta,\alpha_1}\{o_4\}(\Theta) = \alpha_1$

The corresponding discounted $BetP$ relative to the first sensor is summarized in this following table:

Table 2. S_1's discounted BetPs

S_1	o_1	o_2	o_3	o_4
Airplane	$0.07 - 0.27\alpha_1$	$0.10 - 0.23\alpha_1$	$0.30 - 0.03\alpha_1$	$0.30 - 0.03\alpha_1$
Helicopter	$0.22 - 0.12\alpha_1$	$0.60 + 0.27\alpha_1$	$0.40 + 0.07\alpha_1$	$0.20 - 0.13\alpha_1$
Rocket	$0.72 + 0.38\alpha_1$	$0.30 - 0.03\alpha_1$	$0.30 - 0.03\alpha_1$	$0.50 + 0.17\alpha_1$

For example the computation of $BetP_{S_1}^{\Theta,\alpha_1}\{o_1\}(Heliocpter)$ is done as follows: $BetP_{S_1}^{\Theta,\alpha_1}\{o_1\}(Heliocpter) = \frac{0.3(1-\alpha_1)}{2} + \frac{0.2+0.8\alpha_1}{3} = 0.22 - 0.12\alpha_1$

Using the different values of $BetPs$, the whole distance relative to the sensor S_1 will be equal to:

$$TotalDist = \sum_{j=1}^{4} \sum_{i=1}^{3} (BetP^{\Theta,\alpha_1}\{o_j\}(\theta_i) - \delta_{j,i})^2$$

Hence

$$TotalDist = 0.41\alpha_1^2 - 0.56\alpha_1 + 2.81;$$

Minimizing $TotalDist$ under the constraint $0 \leq \alpha_1 \leq 1$ gives as a result $\alpha_1 = 0.68$. Hence, the discounting factor to be given to sensor S_1 by taking into account its opinions on the classes of the objects o_j, $j = 1, 2, 3, 4$, is equal to 0.68.

Applying the same procedure for the beliefs given by the second sensor (S_2), we get $\alpha_2 = 0.52$ as the discounting factor of this sensor. Thus sensor S_2 is (a little) better than sensor S_1.

Just to get an idea about what represents the two discounting factors (see section 5.5), their equivalent in the highly simplified schema of 2 objects produce π values of 0.21 and 0.28, respectively, what can be understood as 'the sensors are really not good, and the second is just a little better then the first'. This is indeed what the data also show.

6 Tuning Sensors' Reports

Evaluating sensors within this second framework is based on taking into account the sensors' bbas together and not independently as we have done in the previous section.

The idea is to build the best predictor from a set of available sensors. Bad ones should be discounted more than good ones. The present method is applicable when the main objective is to get the best aggregated report induced from those given by the sensors.

This requires assessing the 'best' values of the discounting factors to be allocated to each sensor knowing that their discounted 'beliefs' will be merged.

The 'best' discounting factors are those that will make the pignistic probabilities induced by the conjunctive combination of the discounted bba's as close as possible from the actual values, just as done in the previous section. Such process is named tuning sensors' reports.

In order to derive the optimal set of discounting factors, we apply the following steps. Suppose we knew the discounting factors, we would then:

- For each bba $m^{\Theta}_{S_k}\{o_j\}$, discount it by its discounting factor α_k given to the sensor S_k. We get $m^{\Theta,\alpha_k}_{S_k}\{o_j\}$. This process will be applied for the bba given by the sensor for each object.
- For each object o_j $(j = 1, ..., n)$, combine the different discounted bba's by applying the conjunctive rule. We get:

$$m^{\Theta}\{o_j\} = m^{\Theta,\alpha_1}_{S_1}\{o_j\} \circledcirc .... \circledcirc m^{\Theta,\alpha_k}_{S_k}\{o_j\} \tag{14}$$

  $m^{\Theta}\{o_j\}$ is a joint bba representing the induced belief on the class to which object o_j belongs computed by taking into account the data collected from all the sensors.
- Compute the corresponding $BetP^{\Theta}\{o_j\}$ (relative to the bba $m^{\Theta}\{o_j\}$) representing the pignistic probability on the class of object o_j.
- For each object o_j, compute the distance between $BetP^{\Theta}\{o_j\}$ and the real class of o_j. This distance is defined by:

$$Dist\{o_j\} = \sum_{i=1}^{p} (BetP^{\Theta,\alpha}\{o_j\}(\theta_i) - \delta_{j,i})^2$$

  where $\delta_{j,i} = 1$ if $c_j = \theta_i$ and 0 otherwise.
- Compute $TotalDist$ as follows:

$$TotalDist = \sum_{j=1}^{n} Dist\{o_j\} \tag{15}$$

  This variable depends on the discounting factors $\alpha_1, \alpha_2, \ldots, \alpha_k$.

- In order to find the optimal discounting factors, we have to minimize $TotalDist$ on the α's under the constraints $0 \leq \alpha_\nu \leq 1,\ \forall \nu \in \{1, .., k\}$

Example 2. Let's use the same data in the example 1 (see table 1). Let's apply our second method on the two sensors' reports by assuming that we want to get the merged report.

Once S_1'bbas and S_2'bbas are discounted, we get respectively $m^{\Theta,\alpha_1}_{S_1}\{o_j\}$ and $m^{\Theta,\alpha_2}_{S_2}\{o_j\}$ where $j = 1, 2, 3, 4$, which are linear functions of the discounting factors. For each object o_j,we compute the joint bba $m^{\Theta}\{o_j\}$:

$$m^{\Theta}\{o_j\} = m^{\Theta,\alpha_1}_{S_1}\{o_j\} \circledcirc m^{\Theta,\alpha_2}_{S_2}\{o_j\}$$

where the α terms are at most of the form $\prod_{i=1,\ldots,I} \alpha_i$ where I is the number of sensors ($I = 2$ in the present case).

The corresponding discounted $BetPs$ relative to the these bbas are also linear functions of the same product terms. The value of $Dist\{o_j\}$ relative to the objects, as wells as $TotalDist$ are quadratic functions of the previous product terms.

So its minimization on the α_i is very simple and can be achieved by any minimization program. Even when we work more than two sensors, any minimization program can give the different values of α.

In the present case, $\alpha_1 = 0.28$ and $\alpha_2 = 0.12$. It should be enhanced that the α coefficients computed in this second method should not be assimilated to those computed with the first one. Here we want the α so that the multisensor is 'optimal', whereas in the first method, we compute α in order to evaluate the individual sensor quality.

7 Conclusion

In the TBM, degrees of reliability to give to sensors are represented by discounting factors. In this paper, we have presented one method for assessing these discounting factors in a classification context where we to have at our disposal a learning set where the classes of the object are perfectly known and when each sensor is considered alone.

We have also present a tuning method by which each sensor in a group of sensors is partially discounted so that the overall set of sensors is optimal.

These methods are presented by studying a classification problem. They can easily be extended to other problems of prediction. All that is required is a learning set and a distance between the prediction and the actual values.

We have presented operational methods to assess the discounting factors in two contexts. It will be useful for any problem of multisensor data fusion [14].

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