

The nature of the unnormalized beliefs encountered in the transferable belief model.

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Summary: Within the transferable belief model, positive basic belief masses can be allocated to the empty set, leading to unnormalized belief functions. The nature of these unnormalized beliefs is analyzed.

Keywords. Transferable belief model, belief functions, Dempster-Shafer theory, Dempster's rule of conditioning, normalization factors.

1. Introduction.

In subjective probability theory, a probability space (Ω, \mathcal{A}, P) is characterized by a set of worlds Ω , a Boolean algebra \mathcal{A} of subsets of Ω and a probability measure P defined on \mathcal{A} . Let ω be the world that corresponds to the actual state of affairs. We ignore which world corresponds to ω . We only know the strength $P(A)$ of our belief that $\omega \in A$, for all $A \in \mathcal{A}$. The normalization of P is an axiom of the theory: $P(\Omega) = 1$.

In the transferable belief model (TBM), the model we developed to quantify beliefs and that covers the same domain of application as the subjective probability measures, we distinguish between an open-world and a close-world context (Smets 1988, Smets and Kennes 1990). In the open-world context beliefs are not necessarily normalized. The nature of this possible lack of normalization is analysed in this paper.

The TBM is characterized by an unitary mass of belief that is distributed among the subsets of a finite frame of discernment Ω . For $A \subseteq \Omega$, $m(A)$ is the portion of our total initial belief supporting A ¹ and that could support any subset of A if further information

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¹ To support A means to support $\omega \in A$.

justifies it, but given the available information $m(A)$ supports only A . The function $m:2^\Omega \rightarrow [0,1]$ is called a basic belief assignment (bba) and the m values are called the basic belief masses (bbm), with

$$\sum_{A \subseteq \Omega} m(A) = 1$$

Based on the bba m , the functions $\text{bel}(A)$ and $\text{pl}(A)$ are defined for $A \subseteq \Omega$ by:

$$\text{bel}(A) = \sum_{\emptyset \neq X \subseteq A} m(X) \quad (1)$$

$$\text{pl}(A) = \sum_{A \cap X \neq \emptyset} m(X) \quad (2)$$

$$\text{bel}(\emptyset) = \text{pl}(\emptyset) = 0.$$

Hence $\text{pl}(A) = \text{bel}(\Omega) - \text{bel}(\bar{A})$

$$\text{bel}(\Omega) = \text{pl}(\Omega) = 1 - m(\emptyset) \leq 1.$$

For all $A \subseteq \Omega$, $\text{bel}(A)$ (read the belief of A) represents the degree of *justified specific support* given to A and $\text{pl}(A)$ (read the plausibility of A) represents the degree of *potential specific support* that could be given to A .

We say *specific* because the bbm's $m(X)$ included in $\text{bel}(A)$ are those that support A without supporting \bar{A} . Hence X must be a subset of A without being a subset of \bar{A} , i.e. $X \neq \emptyset$. So $m(\emptyset)$ is not included in $\text{bel}(A)$. Identically it is not included in $\text{pl}(A)$ as $\text{pl}(A)$ includes the bbm given to sets X compatible with A , and \emptyset is not compatible with A .

We say *justified* because *only* the bbm's given to subsets of A are included in $\text{bel}(A)$. For instance, consider two distinct elements x and y of Ω . The bbm $m(\{x,y\})$ given to $\{x,y\}$ could support x if further information indicates this. However given the available information the bbm can only be given to $\{x,y\}$.

We say *potential* because the bbm included in $\text{pl}(A)$ could be transferred to non empty subsets of A if some new information could justify such a transfer. It would be the case if we learn that all the worlds in \bar{A} are impossible. The fact that, after such a conditioning, the updated belief given to A will be equal to $\text{pl}(A)$ reflects the fact that $\text{pl}(A)$ is the maximal support that might be given to A . In particular, subsets $X \subseteq \Omega$ for which it is accepted that $\bar{\omega} \notin X$ receive a null plausibility. If it was not the case, then appropriate updating could induce a positive support to X , thus making supported worlds that were accepted as not including $\bar{\omega}$.

An important element of the TBM is the acknowledgement that a positive bbm can be allocated to the empty set or equivalently that $\text{bel}(\Omega)$ and $\text{pl}(\Omega)$ can both be less than one. The bbm $m(\emptyset)$ can be viewed as a *missing mass* as it is equal to $1 - \text{pl}(\Omega)$. On the

contrary, Shafer (1976) defines a belief function such that $m(\emptyset)=0$ or equivalently $bel(\Omega)=pl(\Omega)=1$.

In order to analyse the nature of $m(\emptyset)>0$, we define the nature of the frame of discernment in section 2. In section 3, we explain the conditioning process used in the TBM. In section 4, we explain the origin of the positive bbm given to \emptyset . In section 5, we analyse the case where the frame of discernment is not exhaustive. We summarize the results in section 6.

2. The frame of discernment Ω_L .

In the TBM, bel quantifies the strength of the beliefs held by a given agent Y at a given time t that an actual state of affairs ω belongs to subsets of possible worlds. The domain of Y 's beliefs at time t is a set of distinct possible worlds, one of them, denoted ω , corresponds to the actual state of affairs (Carnap 1956, 1962, Ruspini 1986). This set, is denoted by Ω_L and called the frame of discernment. It is defined as follows.

2.1. The logical construction of Ω_L .

Let L be a finite propositional language. Let $\Omega_L = \{\omega_1, \omega_2, \dots, \omega_n\}$ be the set of worlds that correspond to the interpretations of L . Propositions identify the subsets of Ω_L . Let T be the tautology and \perp be the contradiction. For any proposition X , let $\llbracket X \rrbracket \subseteq \Omega_L$ be the set of worlds identified by X . Let A be a subset of Ω_L , then f_A is any proposition that identifies A . So $A = \llbracket f_A \rrbracket$, $\emptyset = \llbracket \perp \rrbracket$ and $\Omega_L = \llbracket T \rrbracket$. The domain of bel and pl are sets of worlds in Ω_L . By definition the actual world ω is an element of Ω_L .

Given a frame of discernment Ω_L , the complement \bar{A} of a set of worlds $A \subseteq \Omega_L$ is the complement of A relative to Ω_L . It corresponds to $\llbracket \neg f_A \rrbracket$.

2.2. The evidential corpus.

The beliefs held by Y on Ω_L at time t are based on the set of all the pieces of evidence held by Y at time t . This set is called the evidential corpus and denoted EC_t^Y . Updating of EC_t^Y can be obtained by adding a new piece of evidence to EC_t^Y . Let $t' > t$ and let Ev be the piece of evidence added to EC_t^Y . Then $EC_{t'}^Y = EC_t^Y \cup \{Ev\}$. Ev induces a conditioning of Y 's beliefs on Ω_L if that piece of evidence is relevant to Y 's beliefs on Ω_L . The evidential corpus EC_t^Y can also be updated by the removing of a piece of evidence from EC_t^Y . Then $Ev \in EC_t^Y$ and $Ev \notin EC_{t'}^Y$. It results in a deconditionalization of Y 's beliefs on Ω_L if that piece of evidence was relevant to Y 's beliefs on Ω_L at time t

(Klawonn and Smets 1992). These two kinds of changes correspond to the expansion and contraction processes (Levi , 1980, pg. 25)

3. Conditioning in the TBM.

In this section, we describe the impact of a new pieces of evidence within the TBM. Let Ω_L be a frame of discernment. A piece of evidence I_A is added to EC_t^Y and I_A is such that Y takes for grant that $\bar{\omega} \in A$.

In the TBM, the bba given to $X \subseteq \Omega_L$ is transferred to $X \cap A$. When $X \cap A \neq \emptyset$, this transfer reflects the nature of the TBM and its masses: a bba given to a set X is a part of belief that supports X and might support any subset of X if further information justifies such a support to a more specific subset. The information that the worlds in \bar{A} are impossible is the kind of information that justifies such a transfer. Y updates his beliefs by conditioning m (or equivalently bel and pl) on A. The resulting bba m_A is:

$$m_A(B) = \begin{cases} \sum_{X \subseteq \bar{A}} m(B \cup X) & \forall B \subseteq A, \\ 0 & \text{otherwise} \end{cases}$$

Thus $m_A(\emptyset) = m(\emptyset) + \text{bel}(\bar{A})$.

$\text{bel}_A, \text{pl}_A$ are related to m_A by relations (1) and (2). The results are equivalent to:

$$\text{bel}_A(B) = \text{bel}(B \cup \bar{A}) - \text{bel}(\bar{A}) \quad \forall B \subseteq \Omega_L$$

$$\text{and} \quad \text{pl}_A(B) = \text{pl}(A \cap B) \quad \forall B \subseteq \Omega_L.$$

This updating corresponds to the unnormalized Dempster's rule of conditioning. Justifications of this rule can be found in Nguyen and Smets (1991) and in Klawonn and Smets (1992).

4. Updating beliefs.

Let Ω_L be the frame of discernment on which Y builds his/her beliefs at time t. Let m be the bba that quantifies these beliefs that result from EC_t^Y . We want to explain the meaning of $m(\emptyset) > 0$ as encountered in the TBM updating.

4.1. Let us suppose that Y learns the piece of evidence I_A that the worlds in \bar{A} happen to be impossible, with $A \subseteq \Omega_L, A \neq \emptyset$. Let bel' be the result of the updating of bel by this pieces of evidence, whereas bel_A is the updating obtained in the TBM. bel' is the belief function that results from $EC_t^Y \cup \{I_A\}$. We give our reason why bel' should be bel_A .

The transfer of the bbm $m(X)$, $X \subseteq \Theta$ to $X \cap A$ is accepted when $X \cap A \neq \emptyset$, as this transfer is at the core of the TBM. What has to be justified is the transfer of $bel(\bar{A})$ to $m_A(\emptyset)$ and the non-normalization.

$bel(\bar{A})$ is the sum of the bbm given to the non-empty subset of \bar{A} . The updating information I_A says that the worlds in \bar{A} are impossible. Therefore $bel(\bar{A})$ was a support given by Y at time t to a set of worlds that turns out to be impossible. It results from a conflict between the beliefs on Ω_L induced by the EC_t^Y and responsible for Y 's allocation of positive bbm to some non-empty subsets of \bar{A} and the beliefs on Ω_L induced by the new piece of evidence I_A that says that no positive bbm should be allocated to the non-empty subsets of \bar{A} . So $bel(\bar{A})$ quantifies the conflict between Y 's beliefs at time t and the beliefs induced by the updating piece of evidence. It must be eliminated, either by a transfer to some kind of 'absorbing world' that represents a 'contradictory' state, denoted as ϕ , or by being redistributed among the still possible worlds by some normalization process.

Two types of arguments can be used to show that the redistribution of $bel(\bar{A})$ among the worlds included in A is inappropriate. The first is based on the definition of the plausibility, the second on a homomorphisme requirement.

For any subset X of the frame of discernment, we defined $pl(X)$ as the degree of potential specific support that could be given to X . By potential we mean that $pl(X)$ represents the maximal degree of justified support that might be allocated to X . Being maximal, $pl(X)$ can not increase after conditioning. This explains why we require that $pl_A(A) = pl(A)$, in which case the redistribution of $bel(\bar{A})$ among the subsets of A is not allowed. This argument can be questioned as it is based on a particular definition of the plausibility .

For the homomorphisme requirement (Gärdenfors, 1988), consider two belief functions bel' and bel'' defined on a frame of discernment Ω_L and a random device which outcome indicates which belief function is selected. Let p be the probability of bel' being selected and let $q=1-p$ be the probability that bel'' is selected. Let bel be the belief function on Ω_L resulting from the overall schema, so $bel(X) = p bel'(X) + q bel''(X) \forall X \subseteq \Omega_L$ (proved in Smets 1990b). The homomorphisme requirement states that the same relation should hold whatever the conditioning subset:

Homomorphisme requirement :

If $bel(X) = p bel'(X) + (1-p) bel''(X)$, $\forall X \subseteq \Omega_L, p \in [0,1]$,

then $\text{bel}_A(X) = p \text{bel}'_A(X) + (1-p) \text{bel}''_A(X) \quad \forall X \subseteq \Omega_L, \forall A \subseteq \Omega_L$

The homomorphisme is satisfied iff $\text{bel}(\bar{A})$ is not redistributed among the subsets of A .

Proof: In general $\text{bel}_A(X) = \text{bel}(X \cup \bar{A}) - \text{bel}(\bar{A}) + g(\text{bel}(\bar{A}))$. The first and second terms corresponds to the transfer of the bbm given to the subsets of Ω_L compatible with A . The third is the fraction of $\text{bel}(\bar{A})$ due to the hypothetical redistribution of $\text{bel}(\bar{A})$ among the subsets of A . The homomorphisme requirement becomes: for $p \in [0,1]$,

$$\forall X \subseteq \Omega_L, \quad \text{bel}_A(X) = p \text{bel}'_A(X) + (1-p) \text{bel}''_A(X)$$

or

$$\text{bel}(X \cup \bar{A}) - \text{bel}(\bar{A}) + g(\text{bel}(\bar{A})) = p [\text{bel}'(X \cup \bar{A}) - \text{bel}'(\bar{A}) + g'(\text{bel}'(\bar{A}))] + (1-p) [\text{bel}''(X \cup \bar{A}) - \text{bel}''(\bar{A}) + g''(\text{bel}''(\bar{A}))]$$

where the g functions can depend on the belief function considered.

Given $\text{bel}(X) = p \text{bel}'(X) + (1-p) \text{bel}''(X)$, the last relation becomes:

$$g(p \text{bel}'(\bar{A}) + (1-p) \text{bel}''(\bar{A})) = p g'(\text{bel}'(\bar{A})) + (1-p) g''(\text{bel}''(\bar{A}))$$

$$\text{or} \quad g(px + qy) = p g'(x) + q g''(y)$$

which solutions for g, g' and g'' are linear functions (Aczel, 1966, pg. 144).

So $g(x) = a x + b$, with a and b constant,

$$\text{and} \quad \text{bel}_A(X) = \text{bel}(X \cup \bar{A}) - \text{bel}(\bar{A}) + a \text{bel}(\bar{A}) + b.$$

The requirement that $\text{bel}_A(\bar{A}) = 0$ implies that: $a \text{bel}(\bar{A}) + b = 0$ whatever $\text{bel}(\bar{A})$,

hence $a = b = 0$.

QED

The satisfaction of the homomorphisme requirement or the definition of the plausibility function imply that $\text{bel}(\bar{A})$ can not be redistributed among the subsets of A . So $\text{bel}(\bar{A})$ can only be transferred to some contradictory state ϕ or to \emptyset .

We show now that there is at most one contradictory state ϕ . We accept that the updating process is the same when applied to a belief function defined on Ω_L or on the coarsening of Ω_L obtained by keeping the elements in A and regrouping the elements in \bar{A} into a single new element. We accept that the updated beliefs obtained in both frames are the same. It means that the 'contradictory' states absorbing the bbm given to the subsets of \bar{A} in the initial frame of discernment is identical to the 'contradictory' state absorbing the bbm given to the single element representing \bar{A} in the coarsening of the initial frame of discernment. This is true whatever $A \subseteq \Omega_L$. Therefore there is an unique contradictory state ϕ .

We show finally that $\phi = \emptyset$. By definition, $\text{bel}(X)$ and $\text{pl}(X)$, $X \subseteq \Omega_L$, include only those bbm given to non-empty subsets of X or non-empty subsets compatible with X ,

respectively. Even if $\phi \neq \emptyset$, ϕ may not be a subset of any subset of A , otherwise the $\text{bbm}(\bar{A})$ that was transferred to ϕ would be added into the beliefs given to these subsets of A , contrary to the fact that $\text{bel}(\bar{A})$ may not be redistributed among the subsets of A . So $\phi \not\subseteq A$. It may neither be a subset of Ω_L compatible with \bar{A} as otherwise $\text{pl}_A(\bar{A})$ would become positive in contradiction to the fact that all worlds in \bar{A} are assumed to be impossible. So $\phi \not\subseteq \Omega_L$, what is impossible if $\phi \neq \emptyset$. Therefore the only remaining solution is $\phi = \emptyset$, in which $\text{bel}' = \text{bel}_A$, and $\text{bel}(\bar{A})$ is thus transferred to $m_A(\emptyset)$.

4.2. We still have to consider the case $A = \emptyset$, i.e. the case where all the worlds in Ω_L are impossible. Y would be in a state of complete contradiction. It is a situation analogous to the one encountered in probability theory when conditioning on an event of zero probability. No natural solution exists and therefore could be imposed, hence there are no criteria to test the solution. In the TBM, such state of complete contradiction is translated by $m(\emptyset) = 1$, a bba that naturally translates the state of complete contradiction to be characterized.

We have argued that conditioning on the information $I_A = \text{'the worlds in } \bar{A} \text{ are impossible'}$ can result in the transfer of $\text{bel}(\bar{A})$ to $m_A(\emptyset)$. The information I_A was added to EC_t^Y , Y 's evidential corpus at time t , and m_A was the resulting bba induced by $\text{EC}_t^Y \cup \{I_A\}$. Before I_A was added to EC_t^Y , how to justify that the $\text{bbm}(\emptyset)$ induced by EC_t^Y could already be positive? It is due to the fact that the bba induced by EC_t^Y might already result from a conditioning on the pieces of evidence that were present in EC_t^Y . Contradiction could already be present in EC_t^Y ². Of course, contradiction within EC_t^Y is not necessarily present, in which case $\text{bel}(\Omega_L) = 1$. Once pieces of evidence accumulate in the evidential corpus, some contradiction will soon appear. Y can get rid of this contradiction by a deconditionalization process, the process by which the pieces of evidence in EC_t^Y that were responsible for the contradiction are excluded from EC_t^Y . The concept of deconditionalization is defined in Klawonn and Smets (1992).

In conclusion, the $\text{bbm}(\emptyset)$ quantifies the amount of conflict present in Y 's beliefs induced by Y 's evidential corpus EC_t^Y at time t . Renormalization would remove this conflict. This solution permits to hide conflicts but might be misleading as shown in the example described by Zadeh (1984).

² This is not the only possible origin for a positive bbm given to \emptyset . It may also result from the application of Dempster's rule of combination, what is equivalent to a conditioning on a dubious piece of evidence.

5. The epistemic construct of the frame of discernment.

We already studied the belief induced on a frame of discernment Ω_L derived from a propositional language L. In practice, the frame of discernment can also be built by listing the possible worlds, without referring explicitly to an underlying propositional language L. The frame of discernment is then an epistemic construct.

For this epistemic construct, one acknowledges the “limited understanding” of the agent Y that holds the beliefs (Walley, 1991). Y establishes a list Ω_t^Y of possible worlds, like the list encountered in a database. Because of Y’s “limited understanding”, Y is not able to imagine *all* the possible worlds. Ω_t^Y contains only those worlds that Y can conceive of at time t. The indices in Ω_t^Y emphasize the time dependency and the personal nature of the frame of discernment. Let Δ_t^Y denote the set of worlds not conceived by Y at t, but that could have been conceived if a logical approach had been taken, i.e. if one had started with the language L that underlies the worlds in Ω_t^Y . Thus $\Omega_t^Y \cup \Delta_t^Y = \Omega_L$ where Ω_L is the set of worlds that correspond to the interpretations of L.

Y’s beliefs can be expressed only for the worlds that he can conceived of, hence those in Ω_t^Y . By definition, Y do not conceive the worlds in Δ_t^Y , and therefore Y surely cannot express his beliefs that the actual state of affairs ω corresponds to one of the worlds in Δ_t^Y . Y’s beliefs at time t are only expressed for the subsets of worlds in Ω_t^Y .

By building Ω_t^Y , Y acknowledges implicitly - and maybe erroneously - that ω belongs to Ω_t^Y , therefore implicitly conditioning on Ω_t^Y a belief function that could have been defined on Ω_L . So all the bbm are allocated to subsets of Ω_t^Y . As contradiction could already result from this conditioning, some $m(\emptyset) > 0$ might be encountered. It corresponds to the belief that would have been allocated to Δ_t^Y .

6. Conclusions.

In summary, we have shown the nature of $m(\emptyset) > 0$. The bbm given to \emptyset at time t quantifies the amount of contradiction present at time t in the belief function that quantifies Y’s beliefs about the set of worlds that are conceivable for him at time t.

A merit of the TBM is that it keeps track of the contradiction present in the EC_t^Y . Renormalization as done in the Shafer’s approach leads to unnatural results like the one

described by Zadeh (1984). Counterexamples like Zadeh's explain why we gave up the normalization requirement when we start developing the TBM³.

Two questions about the meaning of $m(\emptyset) > 0$ merit consideration.

6.1. Suppose Y's initial beliefs are such that $\text{bel}(\omega \in \Omega_L) = 1$, and Y learns that $\omega \notin \bar{A}$ by adding the evidence I_A to EC_t^Y at $t' > t$: so $EC_{t'}^Y = EC_t^Y \cup \{I_A\}$. How come that $\text{bel}(\omega \in A)$ is not necessarily 1? The answer is to be found in the definition of bel : bel is the degree of justified support, and $m(\emptyset)$ is the amount of inherent contradiction present among the beliefs induced on Ω by Y's evidential corpus EC_t^Y . bel does not quantify unwarranted beliefs. For instance, whims are not included in bel . bel quantifies warranted supports and the evidential corpus might be such that it supports $\omega \in A$ somehow, but not fully, because it happens that EC_t^Y and I_A induce some contradictory supports on Ω .

The simple example can be encountered if $Ev_1 = 'X_1 \text{ says } \omega \in A'$ and $Ev_2 = 'X_2 \text{ says } \omega \in \bar{A}'$, where X_1 and X_2 are two different individuals. Let $EC_t^Y = \{Ev_1\} \cup \{Ev_2\}$. Ev_1 justifies Y's beliefs that $\omega \in A$, and Ev_2 justifies Y's beliefs that $\omega \in \bar{A}$. Both beliefs/supports are contradictory, and $m(\emptyset) = 1$ translates the fact that EC_t^Y actually does not justify any support for A or \bar{A} .

Historically, Shafer (1976) already qualified the bbm we allocate to \emptyset as the 'weight of conflict'. A high conflict would justify reconsidering how the evidential corpus influence beliefs (Laskey and Lehner, 1989). In the last example, where $m(\emptyset) = 1$, it is obvious that Y should reconsider the impact of the two pieces of evidence Ev_1 and Ev_2 , trying to discount or eliminate the 'most unreliable' one. When $1 > m(\emptyset) > 0$, the largest $m(\emptyset)$ the more seriously Y should reconsider how his evidential corpus influence his beliefs, but there is of course no real crisp limit between values of $m(\emptyset)$ that would correspond to acceptable conflict, and those that would not. It is a matter of degree. The way to update the impact of the evidential corpus in order to reduce the conflicts on Ω require information external to the model. It is not considered in this paper.

6.2. The second question is concerned with beliefs induced by randomness. Suppose a random device generates the mutually exclusive and exhaustive events ω_i , $i=1, 2, \dots, n$ with probabilities p_i . Should we quantify our belief that event ω_i will occur as $\text{bel}(\omega_i) =$

³ Historically this occurs during a discussion I had with L. Zadeh and R. Yager in a beautiful exotic restaurant in Acapulco in 1980.

p_i ? Does randomness warrants support? These questions are left open. It seems that the p_i 's are the pignistic probabilities induced on Ω by an underlying belief on Ω (Smets 1990b). If the p_i 's are not equal, it seems to reflect that the evidential corpus justifies some beliefs on some subset of Ω .

The aim of this paper was not to solve all the open questions in relation to $m(\emptyset) > 0$, but to provide a meaning for it. There are still pending questions that will be studied in future papers.

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