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Independence and Non-Interactivity in the Transferable Belief Model.

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Abstract

A survey of the use of belief functions to quantify the beliefs held by an agent, and in particular of their interpretation in the transferable belief model.

1 Introduction.

The study of independence seems to be an important topic for uncertainty management systems. Reasoning systems should take into account independence (resp. dependence) considerations in order to get an efficient performance and to simplify many reasoning tasks. The concept of independence, that could also be called irrelevance or informational irrelevance, is regarded as a relationship stating the conditions under which the knowledge of the value taken by one variable is irrelevant to the uncertainty about the value taken by another variable.

The notion of informational irrelevance has been extensively studied in probability theory, e.g. (Dawid, 1979,1998), (Pearl, 1988), (Lauritzen et al., 1990), where it is identified with independence or more specifically, conditional independence. It can be represented axiomatically or graphically. The concept of independence has also been studied in other non-probabilistic frameworks such that Spohn's theory of ordinal conditional functions (Spohn, 1988), Zadeh's possibility theory (Zadeh, 1978), (Hisdal, 1978), (Fonck, 1994), (Campos et al., 1995), (Vejnarová, 1999)..., and in an abstract framework that unifies different calculi called valuation-based system (Shenoy, 1994). However, the concept of (conditional) independence for variables has not been widely treated in belief-functions theory.

The aim of this work is to investigate some ways to define independence relationships between variables when uncertainty is expressed under the form of belief functions. Some other researches studying this topic are (Campos and Huete, 1993), (Shafer, 1976), and (Studeny, 1993).

In this presentation, we are concerned essentially with one interpretation of the belief functions theory, called the transferable belief model (Smets and Kennes, 1994) and we point out the fundamental problems related to the concept of independence in this framework. Indeed, the main problem we want to solve may be stated as follows: let X and Y be two (logically independent) variables taking their values on the sets $X = \{x_1, x_2, ...x_n\}$ and $Y = \{y_1, y_2, ...y_m\}$, respectively. Consider a belief function, representing a global piece of knowledge about the variables X and Y, on the Cartesian product $W = X \times Y$. Then, we want to define the concept of non-interactivity and of independence between X and Y. The definition of these concepts are not so universal.

Definition. Belief-function Non-Interactivity.

Given a bi-dimensional belief functions *bel* on W associated to its basic belief assignment m, the variables X and Y are said to be non-interactive with respect to m_W iff:

$$m_W(w) = m_X(A)m_Y(B) \ if \ w = A \times B, A \subseteq X, B \subseteq Y,$$

$$= 0 otherwise$$

where m_X and m_Y are the basic belief assignments obtained by the marginalization of m_W on X and Y, respectively.

This definition corresponds to the concept of non-interactivity as defined by Zadeh (1979). By analogy to stochastic independence for the probabilistic case, we can define belief-function independence - that we call "Doxastic Independence" - as follows:

Definition. Doxastic Independence.

Given a bi-dimensional belief functions bel_W on $W = X \times Y$ associated to its bba m_W, the variables X and Y are said to be doxastically independent with respect to m_W iff :

i.$m_Y(B|A) = m_Y(B) \ \forall A \subseteq X, \forall B \subseteq Y$ where $m_Y(B|A)$ is the conditional belief function over Y obtained by the Dempster's rule of conditioning, and

ii. if X and Y are doxastically independent with respect to both m_1 and m_2, then X and Y are doxastically independent with respect to $m_1 \oplus m_2$ obtained by applying Dempster's rule of combination on m_1 and m_2.

In this presentation, we show the links between the concepts of belief-function non-interactivity and doxastic independence.

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