The Transferable Belief Model for Expert Judgments and Reliability Problems.

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Abstract.
We show how the transferable belief model (TBM) can be used to assess and to combine expert opinions. The transferable belief model has the advantage that it can handle weighted opinions and their aggregation without the introduction of any ad hoc methods. We also study a paradigm in reliability analysis. It shows that the TBM analysis leads to a solution diametrically opposed to the Bayesian solution.

Keywords: transferable belief model, belief functions, Dempster Shafer theory, expert judgments, reliability, subjective probability.

1. Introduction.

Expert judgments are useful when no objective data are available. They are often used to assess an unknown probability π. Bayesians postulate that the expert opinion about the value of π can be summarized by a meta-probability distribution on [0,1]. They assess then some percentiles of that distribution of π in order to estimate the whole distribution. When several experts are involved, the individual meta-probability distributions are aggregated (often by averaging) into a pooled distribution, and the needed point estimates are derived from this pooled distribution.

We study the same process within the transferable belief model (TBM) framework. The TBM is a normative model to quantify someone's degree of belief (Smets and Kennes 1990). It claims that degrees of belief are quantified by belief functions. It is our interpretation of the Demspetr-Shafer theory. The TBM is developped outside of the scope of probability theory. It avoids any upper and lower probability or random set interpretations, escaping thus from the classical criticisms like those of Levi (1983), or Pearl (1990).

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The use of belief functions in reliability problems has been introduced in Shafer (1979) and studied recently in Ling and Rudd (1989), Wu et al (1990) and Dubois and Prade (1990).

The paper presents the needed information about the transferable blelief model and the belief functions. We show how to build the least committed belief function given the percentiles obtained as in the Bayesian anlysis. We show how to derive the pignistic probability function from that belief function in order to make decisions. The impact of the reliability of each expert is described. All experts' opinions are pooled according to Dempster's rule of combination. Point estimates of the unknown probability are derived from the pignistic probabilities induced by the pooled beliefs.

To show the importance of deciding which of the two concurrent models, the TBM or the Bayesian, is the best, we present a paradigm in reliability analysis, the breakable sensor. The two analysis lead to diametrically opposed conclusions. The choice between the two models is thus not purely academic.

2. The transferable belief model.

The transferable belief model is a model to quantify someone's degree of belief. It covers the same domain as the subjective probabilities. It is based on the following ideas:

1) our degree of belief that a proposition is true is quantified by a number between 0 and 1,
2) there exists a **two-level structure**:
   - a **credal** level where beliefs are entertained and
   - a **pignistic** level where beliefs are used for decisions making,
3) beliefs at the credal level are quantified by **belief** functions.
4) beliefs at the pignistic level are quantified by **probability** functions.
5) when a decision must be made, beliefs at the credal level induce beliefs at the pignistic level, i.e. there exists a transformation - called the pignistic transformation - from the set of belief functions to the set of probability functions that is applied whenever a decision must be made.

In the problem of expert judgment, the unknown variable is the value of the probability π that some well defined event will occur. The universe of discourse Ω on which beliefs are assessed is the interval [0,1]. Let $\Re$ be the set of all closed intervals of [0, 1].

The **transferable belief model** postulates that the impact of an evidence consists in allocating parts of an initial unitary amount of belief among the subsets of Ω. For $A \in \Re$, m(A) is a part of our belief that supports A (i.e. that π is in A), and that, due to lack of information, does not support any strict subinterval of A. The m's are called the **basic belief masses** (bbm) and the m function $m: \Re \rightarrow [0,1]$ is called a basic belief assignment (bba) with:

$$\sum_{A \in \Re} m(A) = 1$$

Note that we accept that some positive bbm might be given to the empty set Ø. The origin of such positive bbm is introduced in Smets (1988) where the concepts of open- and closed-world assumptions are defined.

We assume that our belief on [0,1] is so defined that positive bbm are given only to a finite number of closed intervals of [0,1].

The **degree of belief** bel(A) for $A \in \Re$ quantifies the total amount of belief supporting A without supporting $\bar{A}$. It is obtained by summing all the bbm given to non empty subintervals $X \in \Re$ with $X \subseteq A$, $X \neq \emptyset$.

$$\mathrm{bel}(A) = \sum_{\emptyset \neq X \subseteq A} m(X)$$

$$\mathrm{bel}(\emptyset) = 0$$

Suppose two belief functions bel_1 and bel_2 induced by two 'distinct' pieces of evidence. The question is to define a belief function $\mathrm{bel}_{12} = \mathrm{bel}_1 \oplus \mathrm{bel}_2$ resulting from the **combination** of the two belief functions, where the $\oplus$ symbolizes the combination operator. Shafer proposed to use Dempster's rule of combination in order to derive bel_{12}. The underlying intuitive idea is that the product of two bbm $m_1(X)$ and $m_2(Y)$ induced by the two distinct pieces of evidence on Ω supports $X \cap Y$, hence:

$$m_{12}(A) = \sum_{X \cap Y = A} m_1(X) \cdot m_2(Y)$$

Smets (1990b), Klawonn and Schwecke (1990), Klawonn ans Smets (1991) Hajek (1991) provide different justifications for the origin and the unicity of this rule. These justifications are obtained without introducing some underlying probability concepts. They are based essentially on the associativity and commutativity properties of the combination operator.

In Smets (1990a, 1990c), we show how to make **decisions** when the beliefs are quantified by belief functions. The satisfaction of some rationality requirements leads to the derivation of a unique transformation between the belief functions and the probability functions that must be used once decisions must be made. We call this transformation the pignistic transformation (from pignus = to bet in latin). Let BetP(A) be the pignistic probability (i.e. a probability measure) derived from the bba $m: \Re \rightarrow [0,1]$, one has

$$\mathrm{BetP}(A) = \sum_{B \in \Re} m(B) \frac{|A \cap B|}{|B|} \quad \forall A \in \Re$$

where |A| is the length of A. BetP is the appropriate probability function to be used to make decisions (using expected utilities theory).

Because of the lack of space, we can only present a summary of the other concepts and results relevant to this paper. Too often these concepts are considered as arbitrary, eventhoufh natural. Answers to these supposedly arbitrarinesses within the transferable beleif models are provided in the cited references. In fact, these concepts are perfectly justified, and many are even necessary.

1) **The principle of minimal commitment** is detailed in Delgado and Moral (1987), Dubois and Prade (1987), Hsia (1991), Kruse and Schwecke (1990), Moral (1985), Smets (1991), Yager (1986). When several belief functions are compatible with our knowledge, the application of the principle consists in selecting the one that gives the minimum support to every propositions (when possible). The selected belief function is called the least committed. The principle corresponds to: 'don't give more support than justify'.

2) **The conditioning rules.** It is the least committed admissible *specialization* of the initial belief function (Klawonn and Smets, 1991). It is also the least committed solution when the belief of A given B is understood as the belief in the conditional object A|B (Goodman et al. 1991, Nguyen and Smets 1991).

3) The problem on **how to combine non distinct pieces of evidence** has been first considered in Smets (1986). Ling and Rudd (1989) and Kennes (1991) introduce the concept of a **cautious rule of (conjunctive) combination**. It is based on the idea that each expert provides a belief function that results from his/her own expertise plus a common background. The rule permits to disentangle the underlying common background. It is idempotent. Ling and Rudd (1989) solved the case where the experts opinions are described by simple belief functions. Kennes (1991) presents the solution when the experts opinions are described by separable belief functions. The generalization to any pair of belief functions is under way. Its use for pooling expertise provided by experts who share a common background will be studied in a forthcomming paper. In the present paper we restrict ourselves to the idealized situation where the experts are 'independent' , i.e. the experts do not communicate together and do not use common evidence.

4) Suppose Your[2] belief on Ω is build from another expert's belief on Ω. Suppose you have some a priori belief about the reliability of the expert, characterized by a bba on the sapce reliable, not reliable} with m(reliable) = α and m(reliable or not reliable) = 1-α. Combining this belief with the one on Ω (through their vacuous extensions on the product space $\Omega\times${reliable, not reliable}) leads to the same belief as the one obtained by the application of the so called **discounting** principle introduced in Shafer (1976).

[2] You is the agent who held the beliefs or must integrate the other agents' beliefs into his/her own beliefs.

Finally we must insist that the transferable belief model is *not* a Bayesian model, even not a generalization of such a model. It is a completely different normative model that assumes only the existence of the bbm given to subsets and that can be transferred to more specific subsrets should further information becomes available. It is a model alternative to the Bayesian one. It covers the same domain of application, but use another modelization. It is based in part on the aknowledgement that the betting behavior justification of the Bayesians that leads to the use of additive measures, applies only once decisions are involved. This justification is accepted in the transferable belief model at the pignistic level, what explains why we require probabilities to quantify uncertainty at that level (Smets 1990a). But the betting behavior justification does not apply at the credal level (where beliefs are just entertained). The diachronic Dutch books (Jeffrey 1988) does not justify either that belief at the credal level must be quantified by probability functions, except if one postulates the equality between contingent and updated bets[3] (Walley 1991, page 287), an eaquality which justification can be questioned.

3. The belief function induced on [0,1] by the percentiles of π.

The data collected from the expert are some percentiles of their meta-probability distribution about the value of π. They are based on the expert's willingness to bet that π is in some intervals of [0,1]. Therefore these meta-probabilities are pignistic probabilities. The percentile x_p satisfy $\text{BetP}(\pi \le x_p) = p$ for $p \in P$ (where P is the set of collected percentiles - usually $P = \{0.05, 0.50, 0.95\}$). Let BetP_p denotes $\text{BetP}(\pi \le x_p)$.

The BetP_p observed for $p \in P$ are some of the values of BetP where BetP is the pignistic probability function that results from an unbderlying belief function held at the credal level. So the problem is to assess the underlying belief function on $\Omega = [0,1]$ which pignistic transformation BetP satisfies the known constraints BetP_p for $p \in P$. There are many such belief functions. The Minimal Commitment Principle can be evoked. Finding the least committed belief function which pignistic transformation satisfies the known constraints BetP_p is computationaly trivial.

Example 1: Suppose one tries to assess the probability π, and the collected percentiles are:

$$x_{.05} = .5 \qquad x_{.50} = .7 \qquad x_{.95} = .8.$$

The bbm given to the whole interval [0,1] must be such that BetP([0,.5]) = .05. The bbm given to [0,1] is spread equally on the interval [0,1] by the pignistic transformation. The value

[3] Both bets are held after a conditioning of Your initial beliefs on some event B . The **updated** bet is induced by Your present commitment to update your beliefs if You happen to observe just the event B. The **contingent** bet is induced by Your present dispositions to accept gambles which are called off if B fails to occur. (Walley 1991, page 284). Within the transferable belief model, the first conditioning is performed by Dempster's rule of conditioning applied to the belief funtcion encountered at the credal level, and the second conditioning is performed by the Bayesian conditioning applied to the pignistic probabilities encountered at the pignistic level (Smets and Kennes 1990).

m([0,1]) = .1 explains the .05 given to [0, .5] and is compatible with the other data.. The next constraint to be satisfied is the $x_{.95}$ = .8. The [.8, 1] interval received already a probability of m([0,1])∗(1-.8) = .1∗.2 = .02. The bbm that could justify the still unexplained pignistic probability 0.05 - 0.02 = 0.03 to be allocated to [.8, 1] is to be given to the largest left over interval, i.e. [.5, 1]. The portion of that bbm given to [.8, 1] - i.e. (1-.8)/(1-.5) = 2/5 - must be equal to 0.03. Hence m([.5, 1.]) = .03∗5/2 = .075. The next bbm are computed similarly.

The results are	m([.0, 1.]) = .100	m([.5, 1.]) = .075
	m([.5, .8]) = .600	m([.5, .7]) = .225

4. Discounting Expert Opinion.

Suppose You have an a priori opinion about the reliability of the experts. So the data provided by the experts (the least committed belief function which pignistic transformation is compatible with the observed percentiles) must be discounted as they are not fully reliable. Let bel be a belief function on Ω given by expert E, and let α quantifies the strength of the reliability You give to expert E's opinion. The discounted belief function bel^α induced in You by E's belief function bel is:

$$bel^\alpha(A) = \alpha\, bel(A) \qquad \text{for all } A\in \Re,\ A\neq[0,1]$$

and $$bel^\alpha([0,1]) = bel([0,1]),$$

All belief are reduced by a factor α and the amount of bbm lost by this process is reallocated to [0,1]. A value $\alpha = 1$ represents a full reliability, a value $\alpha = 0$ corresponds to total unreliability (and leads to bel^0 being the vacuous belief function, i.e. a belief function with $m(\Omega)=1$ that characterizes a state of total ignorance).

Example 2: Suppose the reliability that You give to the expert that gave the percentiles of example 1 is α = .8. Then the belief function bel^α induced in You by Expert E's opinion is described by the following bbm:

m^α([.0, 1.]) = .280	m^α([.5, 1.]) = .060
m^α([.5, .8]) = .480	m^α([.5, .7]) = .180

5. Combining Several Expert Opinions.

Suppose a set of n experts: E_1, E_2… E_n. Each expert provides his opinion on the unknown proposition π through his/her percentiles. For each expert E_i, one builds the least informative belief function bel_i that can justify the percentiles provided by expert E_i. Given Your a priori opinion α_i about the reliability of the expert E_i, the belief functions bel_i are discounted into $bel_i^{\alpha_i}$.

Suppose that the opinions provided by the experts are totally unrelated. They represent 'distinct' pieces of evidence. In that case, the various discounted belief functions $bel_i^{\alpha_i}$ are combined by Dempster's rule of combination.

$$bel = bel_1^{\alpha_1} \oplus bel_2^{\alpha_2} \oplus ... \oplus bel_n^{\alpha_n}$$

bel corresponds to the aggregation of the expert opinions, appropriately discounted. bel is Your belief about the value of π. From it, You compute Your pignistic probability about π. Decisions are made using this final pignistic probability function.

When the belief functions are not based on distinct pieces of evidence, Dempster's rule of combination does not apply. It must be replaced by the cautious Dempster's rule of combination.

Example 3: Suppose Expert E_1 discounted opinions are those derived in example 2.
Suppose Expert E_2 provides the following percentiles for π:

$x_{.05} = .4 \qquad x_{.50} = .6 \qquad x_{.95} = .9$

The corresponding bbm are:

$m_2([.0, 1.]) = .125 \quad m_2([.4, 1.]) = .225$
$m_2([.4, .9]) = .500 \quad m_2([.4, .6]) = .150$

Suppose $\alpha_2 = 1$. Then the bbm m obtained by the application of Dempster's rule of combination to $bel_1^{.8}$ and $bel_2^{1.}$ are:

$m([.0,1.]) = .035 \qquad m([.4,1.]) = .063 \qquad m([.4,.9]) = .140$
$m([.4,.6]) = .042 \qquad m([.5,1.]) = .021 \qquad m([.5,.9]) = .030$
$m([.5,.6]) = .108 \qquad m([.5,.8]) = .408 \qquad m([.5,.7]) = .153$

If one requires some point estimates on π (like the mean and the median), one builds the pignistic probability distribution based on the final bbm m. One could compute the mean or the median of this pignistic probability distribution. Table 1 presents the means and medians derived from $bel_1^{.8}$ and $bel_2^{1.}$ and from the combined belief function $bel_1^{.8} \oplus bel_2^{1.}$

	E1, $\alpha 1 = .8$	E2, $\alpha 2 = 1$	Combined
mean	.605	.620	.627
median	.624	.600	.610

Table 1: Means and medians derived from the pignistic probabilities obtained by the two expert and their combination.

6. The breakable sensors: disagreements between the Bayesian and TBM solutions.

To show that the choice between a Bayesian and a TBM approach is important, we analyse two cases of breakable sensors where the two approaches completely disagree in their conclusions. These examples might help the reader in deciding which approach is better.

Suppose I must check the temperature of a process. To do it I have a sensor that can check the temperature of the process. The temperature can only be hot or cold. If the temperature is hot (TH), the sensor light is red (R) and if the temperature is cold (TC), the sensor light is blue (B). The sensor is made of a thermometer and a device that turns the blue or the red light on according to the temperature reading. Unfortunately the thermometer can be broken.

The only known information is what is written on the box containing the sensor. "Warning: the thermometer included in this sensor can be broken. The probability that it is broken is 20%. When the thermometer is not broken, the sensor is a perfectly reliable detector of the temperature. When the thermometer is not broken, red light means the temperature is hot, blue light means that the temperature is cold. When the thermometer is broken, the sensor answer is unrelated to the temperature".

I am a new technician and I never saw this sensor before. I know nothing about it except the warning written on the box. I use it and the blue light gets on! How do I make up my mind about the temperature status? What is my opinion (belief) that the temperature status is hot or cold? Let us assume that the consequences (utilities) of the good and bad decisions are symmetrical: I am either right or wrong. We can thus avoid discussions about possible consequences.

The frame of discernment Ω on which beliefs will be allocated and updated is the finite boolean algebra of proposition or of sets (they are equivalent here) $\Omega = S x T x \Theta$, the cartesian product of spaces S, T, Θ where:

- S = {B, R}, the sensor status, Blue or Red,
- T = {TH, TC}, the temperature status, Hot or Cold
- Θ = {ThW, ThB}, the thermometer status, Working or Broken.

The eight elements of the space Ω are detailed in table 2 and are referred by the small letters.

	B		R	
	TH	TC	TH	TC
ThW	a	b	c	d
ThB	e	f	g	h

Table 2. The labels of the eight elements of Ω.

6.1. The probability analysis.

A probabilist build a probability distribution P on 2^{Ω} (with p the corresponding probability function). This probability distribution must satisfy several constraints:

1) The information on the box implies:

$$P(ThW) = P(\{a, b, c, d\}) = p(a) + p(b) + p(c) + p(d) = .8$$
$$P(ThB) = P(\{e, f, g, h\}) = p(e) + p(f) + p(g) + p(h) = .2$$

2) When the sensor is in working condition (ThW), the sensor is red (R) when the temperature is hot (TH) and blue (B) otherwise (TC). So

$$p(a) = p(d) = 0$$

3) When the sensor is broken (ThB), the sensor status (B or R) is unrelated to the temperature status. It translates into P(B|ThB, TH) = P(B|ThB, TC), i.e.

$$\frac{p(e)}{p(e) + p(g)} = \frac{p(f)}{p(f) + p(h)}.$$

4) Finally the status of the thermometer (ThB or ThW) is unrelated to the temperature (TH or TC). Hence P(ThW|TH) = P(ThW), i.e.

$$\frac{p(a) + p(c)}{p(a) + p(c) + p(e) + p(g)} = .8.$$

Let $x = P(B|ThB) = \frac{p(e) + p(f)}{p(e) + p(f) + p(g) + p(h)}$ denotes the probability that the sensor is blue when the thermometer is broken and $\alpha = P(TC) = p(b) + p(f) + p(h)$ denotes the a priori probability that the temperature is cold.

	B		R	
	TH	TC	TH	TC
ThW	0	$.8\ \alpha$	$.8\ (1-\alpha)$	0
ThB	$.2\ (1-\alpha)\ x$	$.2\ \alpha\ x$	$.2\ (1-\alpha)\ (1-x)$	$.2\ \alpha\ (1-x)$

Table 3: Probability distribution on $\Omega = S x T x \Theta$.

Table 3 presents the probability distribution p on Ω. The odd ratio for TC is:

$$\frac{P(TC|B)}{P(TH|B)} = \frac{.8 + .2x}{.2x} \frac{\alpha}{1-\alpha}$$

But the set of constraints is not sufficient to define uniquely α and x. Even if we knew α, what value should be given to x, the probability that the sensor status is blue when the thermometer is broken. Nothing in the available information tells us what value to give to x. A probabilist facing such a problem can follow several approaches.

- He can try to collect data about x … But in the present context, no further data can be collected about x.

- He can propose extraneous assumptions like:
  - the principle of insufficient reason: when the thermometer is broken, the sensor can only be blue or red. Knowing nothing more, I postulate that both options have the same probability, hence x = .5
  - a maximum entropy argument : it leads to the same result
  - a meta-probability that describes his belief about the possible values of x … but which one? Remember all what the technician knows is the warning on the box containing the sensor.

6.2. The transferable belief model analysis.

The basic belief masses m are:

$$m(\{b\}) = .8\ \alpha \quad m(\{c\}) = .8\ (1-\alpha) \quad m(\{e, g\}) = .2\ (1-\alpha) \quad m(\{f, h\}) = .2\ \alpha$$

Conditioning on B implies the transfer of all basic belief masses within the set {a, b, e, f}. The updated basic belief masses m_B are:

$$m_B(\{b\}) = .8\ \alpha \quad m_B(\{e\}) = .2\ (1-\alpha) \quad m_B(\{f\}) = .2\ \alpha$$

So $\quad bel(TC|B) \quad = bel_B(\{b, f\}) \quad = \dfrac{\alpha}{8\ \alpha\ +\ .2}$

and $\quad pl(TC|B) \quad = pl_B(\{b, f\}) \quad = \dfrac{\alpha}{8\ \alpha\ +\ .2}$

In fact, the solution is a probability measure (this results from the probabilistic nature of our a priori uncertainty on TC-TH).

Then $\quad \dfrac{bel(TC|B)}{bel(TH|B)} = \dfrac{P(TC|B)}{P(TH|B)} = \dfrac{1}{.2}\ \dfrac{\alpha}{1-\alpha}$

6.3. Several sensors.

Suppose several sensors are used simultaneously to check the tamperature and their outcomes (R or B) are independent given the temperature status. Let q_i be the probability that thermometer i is broken. Let $p_i=1-q_i$. Let x_i be the probability P(B|ThB) for sensor i. Suppose the data are B for sensors 1 to r and R for sensors r+1 to n. The odds are:

In the Bayesian approach:

$$\frac{P(TC|data)}{P(TH|data)} = \frac{\alpha}{1-\alpha} \prod_{i=1}^{r} \frac{p_i+q_i.x_i}{q_i.x_i} \prod_{i=r+1}^{n} \frac{q_i.(1-x_i)}{p_i+q_i.(1-x_i)}$$

In the transferable belief model approach:

$$\frac{P(TC|data)}{P(TH|data)} = \frac{\alpha}{1-\alpha} \prod_{i=1}^{r} \frac{1}{q_i} \prod_{i=r+1}^{n} \frac{q_i}{1}$$

As a practical example, suppose one uses three sensors with $q_1 = .03$, $q_2 = q_3 = .19$. Suppose one accepts for the Bayesian analysis that $x_i = .5$ (it is hard to imagine another choice in practice). Sensor 1 (the good one) answers B, sensors 2 and 3 (the bad ones) answer R. The odds become:

Bayes: $\frac{P(TC|data)}{P(TH|data)} = .78 \frac{\alpha}{1-\alpha}$

TBM: $\frac{P(TC|data)}{P(TH|data)} = 1.29 \frac{\alpha}{1-\alpha}$

So the odds decrease in the Bayesian analysis, and increase in the TBM analysis. Which fits common sense? The good sensor says TC, both bad sensors say TH. Baysians conclude that TH is supported by the data. The TBM concludes that TC is supported by the data.

Suppose five sensors with $q_1 = .01$, $q_2 = q_3 = q_4 = q_5 = .35$ and $x_i = .5$. Sensor 1 answers B, sensors 2 to 5 answer R. The odds become:

Bayes: $\frac{P(TC|data)}{P(TH|data)} = .40 \frac{\alpha}{1-\alpha}$

TBM: $\frac{P(TC|data)}{P(TH|data)} = 1.50 \frac{\alpha}{1-\alpha}$

The very good sensor say TC, the four very bad sensors say TH. Baysians conclude that TH is nevertheless supported by the data. The TBM concludes that TC is supported by the data. To decide which is more natural is left to the reader!

The lesson of these examples is that the choice of the model is important as it leads to diametrically opposed conclusions. We feel the TBM solution has the advantage that it does not require to assume any value for x_i and that it leads to more natural conclusions (according to us).

7. Conclusions.

For what concerns the expert opinions pooling, it is possible within the TBM to describe:
- the expert opinion on an unknown probability π,
- how to discount each expert opinion,
- how to combine (aggregate) the experts opinions
- how to derive combined densities and point estimates.

One major quality of this approach is the fact that no ad hoc methods or principle are introduced. All rules used are part of the transferable belief model. The TBM postulates 1) the existence of the bbm and 2) the bbm given to a set A is transferred to subsets of A when new information becomes available. All other properties of the TBM are derived from very general principle that we feel can hardly be criticized (and that are of course always more general than their probabilist counterparts).

Most justifications are detailed in Smets (1990a, 1990c). In particular, the discounting and the combination rules are not ad hoc. This contrasts with most of the probabilistic approaches where expert opinions are weighed and aggregated according to principles that are not part of the probability theory stricto sensu. The strict Bayesian with its meta-probabilities provides the alternative, but we feel the TBM is more general than the Bayesian model and do not require the introduction of the often artificial assumptions fed into a strict Baysian analysis.

In particular, the breakable sensors paradigm shows that the choice between the TBM and the Bayesian method is not an academic exercise. There are cases where their solutions completely conflict with each other. That the TBM solution is better or worse is up to now a matter of personal opinion. Indeed there is no way to prove which solution is the correct one. No simulation can achieved such a discrimination. Both approaches respond to different normative assumptions. Their quality can only be judged by the value of these assumptions, and the 'natturalness' of the derived solutions (especially when they diverge, as in the analysed breakable sensors paradigm).

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