The Origin of Physical Laws and Sensations

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Résumé : I will first present a non constructive argument showing that the mechanist hypothesis in cognitive science gives enough constraints to decide what a "physical reality" can possibly consist in. Then I will explain how computer science, together with logic, makes it possible to extract a constructive version of the argument by interviewing a Modest or Löbian Universal Machine. Reversing von Neumann probabilistic interpretation of quantum logic on those provided by the Löbian Machine gives a testable explanation of how both communicable physical laws and *incommunicable* physical knowledge, i.e. sensations, arise from number theoretical relations.

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Introduction

A lot of literature exists arguing for or against the mechanist hypothesis, roughly speaking the idea that we are machines, and this well before and after Descartes, Hobbes and their followers redeemed it in the so-called modern (recent) tradition.

It is *not* the purpose of this paper to argue for neither against mechanism. I propose instead to consider it as a working *hypothesis*, and to search its logical consequences. It will be easier to consider a stronger *digital* version of it, if only in order to get precise definitions crossing different scientific disciplines. Proceeding in this way will eventually leads us toward ‘‘pure’’ scientific questions, in the Popper testable sense, in particular under the form of mathematical and physical problems. It is not entirely unreasonable to expect a frank contradiction, in which case we would get a ‘‘refutation’’ of digital mechanism, but we must be careful not to confuse a contradiction with just some amount of weirdness[1].

Definition: Classical Digital mechanism, or Classical Computationalism, or just comp, is the conjunction of the following three sub-hypotheses:

[1] Especially when ‘‘nature’’ itself exhibits theoretical, practical and even exploitable quantum *weirdness*. See: A. Einstein, B. Podolski, and N. Rosen (1935): ‘‘Can quantum-mechanical description of physical reality be considered complete?’’ *Physical Review*, 47:777-780.. J. S. Bell (1964): ‘‘On the Einstein-Podolsky-Rosen paradox,’’ *Physics*, 1:195-200. D. Deutsch (1985): ‘‘Quantum theory, the Church-Turing principle and the universal quantum computer,’’ *Proc. R. Soc. Ac.*, 400:97-117.

1) The *yes doctor* hypothesis: It is the assumption, in cognitive science, that it exists a level of description of my parts (whatever I consider myself to be[2]) such that I would not be aware of any experiential change in the case where a functionally correct digital substitution is done of my parts at that level. We call that level the substitution level. More simply said it is the *act of faith* of those willing to say *yes* to their doctor for an artificial brain or an artificial body graft made from some description at some level. We will see such a level is unknowable. Note that some amount of folk or "grand-mother psychology" has been implicitly used under the granting of the notion of (self) awareness[3].

2) *Church Thesis*. A modern version is that all digital universal machines are equivalent with respect to the class of functions (from the natural numbers to the natural numbers) they can compute[4]. It can be shown that this entails such machines compute the same functions, but also they can compute them in similar ways, i.e. following similar algorithm. So, the thesis says, making abstraction of computation time, all digital universal machine can simulate each other exactly (I will say *emulate* each other).

3) *Arithmetical Realism* (AR). This is the assumption that arithmetical proposition, like "1+1=2," or Goldbach conjecture, or the inexistence of a bigger prime, or the statement that some digital machine will stop, or any Boolean formula bearing on numbers, are true independently of me, you, humanity, the physical universe (if that exists), etc. It is a version of Platonism limited at least to arithmetical truth. It should not be confused with the much stronger Pythagorean form of AR, AR+, which asserts that *only* natural numbers exist together with their nameable relations: all the rest being derivative from those relations.

To state the main results, it is helpful to give in advance the following definitions, although more precise formulations will be given naturally through the argumentation itself.

Definition *Fundamental Physics*: I define it by the correct-by-definition discourse about observable and verifiable anticipation of possible relatively evolving quantities and/or qualities.

We have tremendous empirical *evidences* that quantum mechanics is part of such a physics. [See Cabello quasi exhaustive and well ordered bibliography in the archive at Los Alamos http://arxiv.org/abs/quant-ph/0012089].

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[2] It could be the entire universe, but, in this case, this one, if it exists, must be supposed to be Turing emulable (perfectly simulable) for keeping comp. In the proof I will suppose the brain, or whatever is responsible for my awareness/consciousness, to be the one in the skull. Latter this supplementary assumption will be eliminated.

[3] It can be argued that such *grand-mother* use will be eliminated through the mathematical confirmation which follows, where the grand-mother is substituted by the Löbian Universal Machine. But as far as we can judge the mathematical confirmation, it should be seen (abductively) as a vindication of *grand-mother*.

[4] That thesis has been proposed independently by many authors. A shadow of the thesis exists in non published notes by Babbage concerning a system of functional notations that he used to describe its cogwheels computer. An explicit formulation has been given by Emil Post who derived "Gödel's theorem" from it in 1924 (about ten year before Gödel!). Turing and Markov did also propose the thesis. Gödel accepted it slowly after reading Turing 1936 paper. Church proposed it originally as a definition, but it is Kleene who created the vocable "Church's thesis" after having convinced himself that the "definition" cannot be refuted by diagonalisation, as we will illustrate below. The important papers are in M. Davis, editor (1965): *The Undecidable*, Raven Press, Hewlett, New York. See my 1994 text "Conscience & Mécanisme" for more information and references:

http://iridia.ulb.ac.be/~marchal/bxlthesis/consciencemecanisme.html

Definition *Fundamental Machine Psychology*: I define it by the correct-by-definition discourse that machines could have about themselves or about other machines. This will include in particular computer science, but also sets of propositions that some machine could correctly asserts about itself (named self-referentially correct discourses).

There is nothing normative in the use of the word "correct": if we ever *knew* that the reason the moon appears in the sky is that 667 little angels are pushing it there, then *that* would be the correct-by-definition explanation. We don't need to elaborate: eventually the word "correct" will just mean arithmetically true. This should be made clear through the reasoning which will follow.

The paper is divided into two parts:

Part 1 presents an informal but (hopefully) rigorous argument or proof, named the Universal Dovetailer Argument (UDA), in the form of a sequence of eight thought experiences[5], showing that it follows from comp that fundamental physics is necessarily reducible to fundamental psychology. Note that with comp, fundamental psychology is itself easily shown to be embeddable in number theory[6].

Part 2, thanks to the many discoveries of Gödel, Church, Turing, Post, Markov and many others (mainly the studies of Brouwer, Heyting, Löb, Grzegorczyk, Boolos, Goldblatt, Kuznetsov, Muravitski, Solovay, Visser) in the study of the self and in Arithmetical Self-Reference, will explain how to interview a Self-Referentially Correct Universal Machine (SRC Machine) on the UDA's conclusion to derive some comp logic of physical propositions. Then we will compare that logic with the empirical logic of physical propositions which have been inferred from observations. This will show that comp is testable and even that some test does already confirm it (and this does *not* mean that it *proves* it, of course). Before addressing the proof of the psychology/physics reversal and its mathematical confirmation, let me warn the sensible person that both can produce some amount of metaphysical vertigo[7].

I. The Universal Dovetailer Argument shows why comp necessarily forces a reversal between physics and machine psychology

Here is presented the argument showing that if we take seriously enough the computationalist hypothesis in the cognitive science then physics is reducible to machine psychology. The proof is divided into 8 steps. Each step is numbered and accompanied by a drawing featuring the principal idea of the step.

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[5] Such a sequence of thought experiences constitutes a giant "Platonic destructive thought experiment" in the nomenclature of James Brown. This means basically it is a proof, and this means that all the magic apparent in the conclusion was hidden in the hypotheses, or appeared by mistake. See J. R. Brown (1991): *The laboratory of the mind*, Routledge, London.

[6] A direct argument showing that Church Thesis rehabilitates a form of Pythagorism and makes plausible comp$^+$, that is comp with AR$^+$ instead of AR, is given in the appendix on Church thesis in my PhD thesis. Look at, or click on: http://iridia.ulb.ac.be/~marchal/lillethesis/these/node56.html#SECTION00104000000000000000

For those who accept COMP$^+$, the UDA is necessary only for explaining the reduction of physics to psychology, giving that comp+ makes the reduction of physics to number theory at once inescapable.

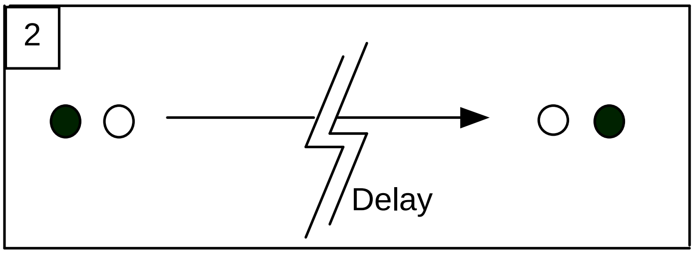
[7] This paper presents results obtained in my PhD thesis, "Calculabilité, Physique et Cognition" at the University of Lille (France). Available here: http://iridia.ulb.ac.be/~marchal/

1) Comp makes possible (only *in principle* but that is all we need), the use of classical[8] teleportation. You are read and cut, with the usual computer practice meaning, at Brussels. We assume the reading has been done, perhaps per chance, at a level equal or lower than the substitution level. The scanned (read) information is send by traditional means, by mails or radio waves for instance, at Helsinki, where you are correctly reconstituted with ambient organic material. "Correctly" by definition of comp. Note also we don't pretend the doctor know for sure the correct level of substitution. Actually comp will make such knowledge necessarily impossible. But comp says the level exists, and we will suppose the doctor has *bet* on the correct level.

In the figure the teleported individual is represented by a black box. Its annihilation is represented by a white box appearing at the left of the arrow. The reconstitution is represented by a white box at the right of the arrow. If we identify an individual with its (hopefully consistent) set of beliefs, the experience adds only a new belief (I did arrive in Helsinki) to the set, and the resulting set can be considered as a *consistent extension* of the set prior to the teleportation.

2) The step and figure 1 are just a restatement of the comp hypothesis. To proceed we need to introduce a key distinction between the notions of first person point of view and third person point of view. It will be enough, in the argument itself, to define them by the propositional content of personal diaries. The third person point of view is the content of a description of the experiment by an external observer which does not participate in the teleportation. The first person point of view is the content of the diary taken by the user of teleportation device. He is supposed to take it with him, so that the personal diary will be itself destroyed and reconstituted. To ease the reasoning, we neglect at first *reading* and *pasting* time, as we neglect the time travel of the descriptive information. In this simple teleportation experiment/experience there is no difference between the first and third person discourse, giving that both diaries will describe someone going from Brussels to Helsinki. Some pronouns can differ but they have similar references, and those are no more ambiguous than in their traditional grand-mother ordinary sense. At the second step, a difference between the 1-view and the 3-view will appear. It is a teleportation experiment where the reconstitution, or equivalently the travel, has been delayed for a period of one year (say).

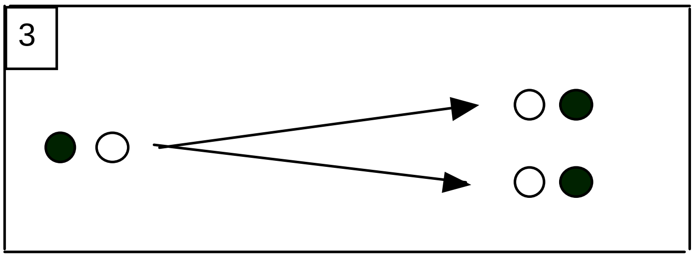
[8] Not to be confused with the Bennett & Al. quantum teleportation of quantum states. C. H. Bennett, G. Brassard, C. Crépeau, R. Jozsa, A. Peres, and W. Wooters: "Teleporting an unknown quantum state via dual classical and EPR channels," *Phys. Rev. Lett.*, 70: 1895-1899, 1993.



Giving that we assume comp, the "experiencer" has no ability, if deprived of any external clues (the reconstitution box has no window, etc.), to know anything about that delay. His diary will not and cannot mention it, so that the first person discourse is the same as in the preceding experiment. Contrariwise, the diary of the external observer will mention that very long delay. At step two the first and third person discourses are no more the same.

3) The third step is admittedly intriguing; its consequences are no less. The description encoded at Brussels after the reading-cutting process is just the description of a state of some Turing machine[9], giving that we assume comp. So its description can be duplicated, and the experiencer can be reconstituted simultaneously at two different places, for example Washington and Moscow. The reconstitution at Moscow is independent of the reconstitution at Washington, and comp makes the experiencer surviving this double teleportation experiment. At Brussels, before the experiment proceeds, the experimenter cannot give an argument for not surviving in Washington, so "to find oneself in Washington" gives a consistent extension. The same reasoning shows that "to find oneself in Moscow" is an alternative consistent extension. Let us ask to the experiencer, which is supposed to be a comp practitioner, where he will be located after the experiment. He can answer *in a third person way*, saying for example that if someone wants to call him, he will be joinable both at Moscow *and* at Washington. So, let us ask him more genuinely where he will *feel* to be located after the duplication, that is, what will be written in his *personal* diary. The diaries are duplicated and clearly none will contain the statement "I feel myself to be in both Washington and Moscow." The one who feels having been reconstituted in Washington can only have an intellectual (3-person) belief that he has also been reconstituted in Moscow (resp. Washington), but even about this he cannot be sure of without external clues, like a phone or a video confirmation. From his position, the other self appears as *other*, like a twin falling from the sky.

[9] For an example, it could be the state of a Turing machine emulating some unitary transformation in case the brain, whatever it is, is correctly described by quantum mechanics. This recall that quantum computer does not violate Church thesis, and comp, in its all classical and Platonist form, is not incompatible with the thesis that the brain is a quantum computer (which I doubt). Giving that machine Turing state, it can be recopied, without violating the non cloning theorem of quantum information science. See Jozef Gruska (1999): *Quantum Computing*, McGraw-Hill, London.

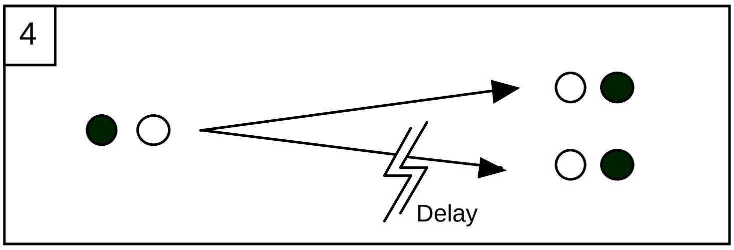


So after the experiment each "first person" will feel to be at one place. To be at both places will never be a realisable consistent belief from the first person point of view. Giving the built-in symmetry of this experiment, if asked before the experiment about his personal future location, the experiencer must confess he cannot predict with certainty the personal outcome of the experiment. He is confronted to an unavoidable uncertainty. This is remarkable because from a third person point of view the experiment is completely deterministic, and indeed the mechanist doctrine is defended most of the time by advocates of determinism. But we see here that mechanism, by being indeed completely 3-deterministic, entails a strong form of indeterminacy[10], bearing on the possible consistent extensions, when they are observed by the first person, as both diaries can witness. This is what I call the first person comp *indeterminacy*, or just 1-indeterminacy. Giving that Moscow and Washington are permutable without any noticeable changes for the experiencer, it is reasonable to ascribe a probability of ½ to the event "I will be in Moscow (resp. Washington)." Before proceeding the experiencer is in a state of maximal ignorance. Actually, we make this move just to simplify the presentation. Indeed eventually we will reduce physics into a search for an uncertainty measure for the 1-indeterminacy, and at this stage it could be a *credibility* measure as well. Yet, in the present context, such a probability can be intuitively justified by both the betting definition of probability, or with a frequency approach to probability through iteration of the experiments. In both of these cases we must consider duplication, not of an individual, but of a collection of individuals. This leads to a notion of first-person plural point of view where the probabilities and bets are locally communicable. For example people inside each multiplied populations can evaluate those probabilities and evaluate the fairness of duplication related bets. From the local point of view shared by person belonging to duplicated populations, the 1-indeterminacy *looks* like third person indeterminacy, but it is so only from inside each population.

4) The fourth step shows that the invoked symmetry and simultaneity was a red erring sort of justification. For this purpose, it is enough to introduce in the preceding setup a delay of reconstitution in one of the bifurcating branches. Then we can use the fact, established at the second step, that a person, from his inner first personal point of view cannot be aware of the delay to understand that the introduction of asymmetric delays will not change the first person perspective. Although a precise measure of the first person uncertainty has not been (and never will be) defined in a precise way, the key point is that such a measure does not change for such delays. In particular, *if* ever we *did* decide to attribute a probability of ½ on

[10] That indeterminacy can be shown totally different from the deterministic chaos, where divergence of histories is produced by lack of precision of the parameters involved. Actually the indeterminacy is already quite comparable to the quantum indeterminacy, especially if we allow ourselves to apply the quantum laws to both the object and the observer interacting with the object, like in Everett's formulation of quantum mechanics (that is Quantum Mechanics without the Copenhagen wave collapse). With Everett, observer can be described by a machine obeying the "natural physical laws," and this makes the quantum indeterminacy a particular case of comp indeterminacy. See H. Everett III (1973): "The theory of the universal wave functions" in B. DeWitt and N. Graham, editors: *The Many-Worlds Interpretation of Quantum Mechanics*, pages 3-140, Princeton University Press, Princeton, New Jersey.

the consistent extensions at step 3, *then* we must also attribute a probability of ½ in the asymmetrical duplication, and that's the point we wanted to show.

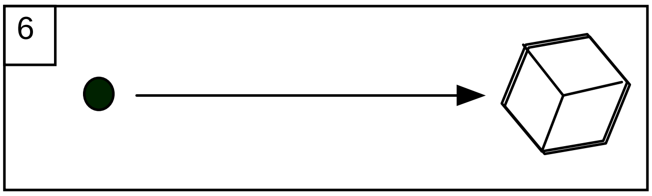


5) Until now, the one we could call conventionally "the original" has always been annihilated at Brussels, its departure point. What can be said about the probability to reach Amsterdam from Brussels with a simple teleportation when the original is *not destroyed* at Brussels. The figure five depicts the experiment:

The absence of the white blob at the left means there is no annihilation at the starting point. We can also consider there is a natural implicit delay on the arrow. It is a consequence of the preceding steps that if the probability is ½ at step 3 (and thus also at step 4), it must be ½ at step 5. The reason is that this setup can be reconsidered equivalently as a duplication-teleportation (like in step 4) from Brussels to {Brussels, Amsterdam} with a *null* delay of reconstitution at Brussels. This is certainly counterintuitive[11], especially if the implicit delay is long, because at Brussels, it is only a picture which has been done (a very precise one giving it has been done at the correct substitution level (which exists by comp)), *and why should we be afraid by a picture of oneself?* Of course, if someone does that experience in the state of being certain he will emerge at Brussels, the one in Amsterdam will understand the falsity, but will never successfully convince the "original" of its error. And this shows, by the way, that none of the experience/experiment presented so far can ever be considered as giving a proof of the comp hypothesis. Actually, no such proofs can exist as the reader can perhaps elaborate him/herself.

[11] In particular it contradicts some physicalist version of Nozick "closer continuer", where the closeness relation is defined in term of spatio-temporal relation. See R. Nozick (1981): *Philosophical Explanations*, Clarendon Press, Oxford. This shows that comp is incompatible with such a notion of closeness. The interview of the sound Löbian machine will suggest a notion of closeness, a priori independent of any space-time, and, contrarily, will explain how a notion of space time can emerge from the closeness relation, in concordance with the conclusion of the UD reasoning. The closeness or similarity relation will be defined by the *non-orthogonality* relation among atomic propositions, itself derivable from the (arithmetical) quantum logics.

6) The sixth step is akin to the oldest metaphysical argument. It is also the most perennial and universal, it is discussed in old Hindouist, Buddhist, Taoist, Islamic, Jewish and Christian texts. It plays a role in Plato's Theaetetus, and Descartes' Meditations, and many others. In his comp form, it is exploited in many Science Fiction Novels; like *Simulacron III* by Daniel Galouye, or in movies, like *The Matrix*. It is the *dream argument*, and it shows mainly that we can always erroneously take a mere belief for knowledge. We will see how the sound universal machine will reflect that insight in its self-referentially correct discourses, but at this stage, all we need amounts only to the following consequence of comp: all the preceding steps can be done again with the reconstitution being "virtual," i.e. emulated by a universal machine, instead of "real" and this without any possible change in the experience of the first person for some arbitrary finite time related to the accuracy of the rendering of the environments (like Washington and Moscow for example). All you need is to simulate the right interface, which is Turing emulable, by definition of comp, and then some approximation of the environment will succeed, the finer in descriptive details, the longer in time. Comp makes it possible to replace dreams by video games in the old dream argument in the sense that a first person cannot distinguish "reality" from an emulation of it when done at a level lower than its substitution level.

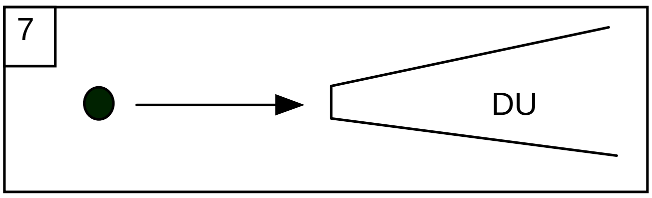


In figure 6, the box represents a (finite) computing machinery. What matter here, is that whatever measure of the comp 1-indeterminacy we choose, that measure will not change in the case where the reconstitution are virtual. Even if the simulation does not last, each first person will take any personal reconstitution as confirming its anticipation, i.e. its bets on its consistent extensions. The probability calculus is again invariant for such a change. This follows directly from our earlier comp assumption that a correct substitution level exists, and that we are Turing emulable.

7) The seventh step introduces the Universal Dovetailer (UD). Let N denotes the set of natural numbers. A function from N to N is said to be total if it is defined on all natural numbers. A function is said to be computable iff there is a programme FORTRAN which computes it[12]. Church thesis (CT) makes the particular choice of FORTRAN irrelevant. CT claims that all computable functions, total or not, are computed by algorithm expressible in FORTRAN. In particular all total computable functions are computed by such FORTRAN program.

[12] This is platonistic or classical talk. For example, today, nobody can compute the classically well defined function given by the following description: f(x) = 1 if there is an infinity of twin prime numbers; 0 if not. (p and q are twin primes, if they are prime and p-q =2, like 3 and 5, 5 and 7, etc.) That function is certainly computable given that it is computed either by the FORTRAN program which outputs always one, or it is computed by the FORTRAN program which outputs always 0. But nobody knows today which one among those two programs compute the function, because the infinity of the twin prime number is, today (2004), still an open problem.

Is there a language, T-Fortran say, which would be capable of defining all and *only all* the total computable functions? T-Fortran language, by some well-defined grammatical restrictions, would make any algorithms written in it computing only total functions. The answer is "no." For, would it be the case, we could enumerate the T-Fortran programs by lexicographical order: P_1, P_2, P_3 …, and the following function g defined on n by $P_n(n)+1$, would be computable, but not T-Fortran computable. Indeed, there would exist a T-Fortran programme P_k computing it, then $P_k(k) = P_k(k) + 1$, and that's absurd giving that $P_k(n)$ is a well defined number for each n (because the function are total). So, with Church thesis, the set of programs computing total functions is necessarily a proper subset of the set of programs computing functions written in FORTRAN. FORTRAN itself is vaccinated against the preceding diagonal "attack". Indeed, although we can enumerate all FORTRAN programs (and this can be done mechanically), and although the resulting diagonal function g can be programmed in FORTRAN, and that it will on its code give again g(k) = g(k) + 1, we will not get a contradiction, but only, in the computer science jargon, a crashing of the computer, that is, the computation of g(k) will just run forever. And this entails there is no complete and decidable theory capable of deciding from a program description if it computes a total or a non total function, because in that case we would be able to use that theory to mechanically filtered the non total programs, and get, with CT, an enumeration of all and only all total computable functions; but then we would obtain again the contradiction we got above. This shows that the incompleteness of theories, with respect to truth, is a direct consequence of CT. The absoluteness of computability, warranted by CT, makes inescapable the relativity of theories. This again will be reflected in our universal machine interview. Concerning the present step in the reasoning, it explains why if we want build a universal machine, which is not only able to emulate all machines, but which actually *does* the emulation of each machine, we will be obliged to dovetail on each execution. We must generate all FORTRAN programs, P_1, P_2, P_3 …, and execute them by little pieces, coming back recurrently on all programs. Let $P_i^j(n)$ represents j steps of the execution of the ith program P_i on input n. We must just computes all those $P_i^j(n)$, and that is easy because the triple <i,j,n> are algorithmically enumerable. It can be seen as a manner to emulate digital parallelism in a linear sequential way. This way avoids any risk of never stopping on a possible infinite computation due to the necessary existence of non stoppable programs, as we have just shown. Such a procedure is called a dovetailing procedure, and I call a universal machine which dovetails on all possible machine executions, a Universal Dovetailer (UD). Suppose now, for the sake of the argument, that our concrete and "physical" universe is a sufficiently robust expanding universe so that a "concrete" UD can run forever, as illustrated in figure 7.



Then, it follows from the six preceding steps that it will generate all possible Turing machine states, infinitely often (why?), which (by comp) includes all your virtual reconstitutions corresponding to (hopefully) consistent extensions of yourself, in all possible (locally) emulable environments or computational histories. And this, with comp, even in the case you consider that your "generalised brain" (the "whatever" which is needed to be emulated by a digital body/brain to survive) is the whole Milky Way galaxy. And we don't need any Science Fiction like devices to make this concrete[13], if we make exception of the robust universe.

We are almost done. Indeed, let us try a simple "physical experiment" like dropping a pen. With comp, when we are in the state of going to drop the pen, we are in a Turing emulable state. Our more probable consistent extension is undetermined by the 1-comp indeterminacy on all the "reconstitution" of that similar states appearing in UD* (the infinite trace of the UD). This follows from 6, and the invariance of the uncertainty measure, notably for the arbitrary delay---including the null one, and the infinite set of states appearing with a arbitrarily large delay in the running of the UD[14]. This gives a huge set. It can be argued that finite computations are of measure null, and that the only way to a measure on the states will consist in finding a measure on the set of maximally complete computational history going through those states, with obviously a rather hard to define equivalence relation among computations. Still, we can show that those (infinite) computations, as seen from some third person description of UD*, correspond to maximally consistent extensions of our (hopefully) actual consistent states. It is not necessary to be more precise here, giving the non constructivism of the collection of those consistent extensions, and the fact that we will make things utterly precise, by directly interviewing a universal machine on those extensions, and this by taking into account the 1/3 person point of view distinction. So, if we grant a sufficiently robust universe, we are completely done: physics, as the "correct" science for the concrete relative predictions must be given by some measure on our consistent relative states. Physics is, in principle reduced to a measure on the collection of computational histories, as seen from some first person point of views. We can say that in principle, physics has been reduced to computer fundamental psychology.

8) Yes, but what *if* we *don't grant* a concrete robust physical universe? Up to this stage, we can still escape the conclusion of the seven preceding reasoning steps, by postulating that a "physical universe" really "exists" and is too little in the sense of not being able to generate the entire UD*, nor any reasonable portions of it, so that our usual physical predictions would be safe from any interference with its UD-generated "little" computational histories. Such a move can be considered as being *ad hoc* and disgraceful. It can also be quite weakened by some acceptation of some conceptual version of Ockham's Razor, and obviously that move is without purpose for those who are willing to accept comp+ (in which case the UDA just show the necessity of the detour in psychology, and the general shape of physics as averages on consistent *1-histories*). But logically, there is still a place for both physicalism *and* comp, once we made that move. Actually the 8th present step will explain that such a move is nevertheless without purpose. This will make the notion of concrete and existing universe completely devoid of any explicative power. It will follow that a much weaker and usual form of Ockham's razor can be used to conclude that not only physics has been *epistemologically* reduced to machine psychology, but that "matter" has been *ontologically* reduced to "mind" where mind is defined as the object study of fundamental machine psychology. All that by *assuming* comp, I insist. The reason is that comp forbids to associate inner experiences with the physical processing related to the computations

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[13] You can find a lisp code for a UD here:

http://iridia.ulb.ac.be/~marchal/bxlthesis/Volume4CC/4%20GEN%20&%20DU.pdf

[14] From the first person point of view the 1-indeterminacy domain is the infinite union of all finite portions of UD* in which correct emulation occurs. This is the main consequence of the 1-invariance for the reconstitution delays.

corresponding (with comp) to those experiences. The physical "supervenience thesis" of the materialist philosophers of mind cannot be maintained, and inner experiences can only be associated with type of computation.

Instead of linking [the pain I feel] at space-time (x,t) to [a machine state] at space-time (x,t), we are obliged to associate [the pain I feel at space-time (x,t)] to a type or a sheaf of computations (existing forever in the arithmetical Platonia which is accepted as existing independently of our selves with arithmetical realism). That result has been found independently by me and Tim Maudlin (Marchal 1988, Maudlin 1989). Maudlin's argumentation provides more information[15]. The argument is less easy to apprehend than those of the preceding step and I will only sketch the basic principle.

For any given precise running computation associated to some inner experience, you can modify the device in such a way that the amount of physical activity involved is arbitrarily low, and even null for dreaming experience which has no inputs and no outputs. Now, having suppressed that physical activity present in the running computation, the machine will only be *accidentally* correct. It will be correct only for that precise computation, with unchanged environment. If it is changed a little bit, it will make the machine running computation no more relatively correct. But then, Maudlin ingenuously showed that counterfactual correctness can be recovered, by adding *non active* devices which will be triggered only if some (counterfactual) change would appear in the environment. Now this shows that any inner experience can be associated with an arbitrary low (even null) physical activity, and *this* in keeping counterfactual correctness. And *that* is absurd with the conjunction of both comp and materialism.

So if we keep comp at this stage, we are forced to relate the inner experience only to the type of computation involved. The reason is that only those types are univocally related to all their possible counterfactuals. This entails that, from a first person point of view, not only the physical cannot be distinguished from the virtual, but the virtual can no more be distinguished from the *arithmetical*[16]. Now DU is emulated platonistically by the verifiable propositions of arithmetic. They are equivalent to sentences of the form "it exists n such that P(n)" with P(n) decidable. Their truth entails their provability, and they are known under the name of Sigma1 sentence.

If comp is correct, the appearance of physics must be recovered from some point of views emerging from those propositions. Indeed, taking into account the seven steps once more, we arrive at the conclusion that the physical atomic (in the Boolean logician sense) invariant proposition must be given by a probability measure on those propositions. A physical *certainty* must be true in all maximal extensions, true in at least one maximal

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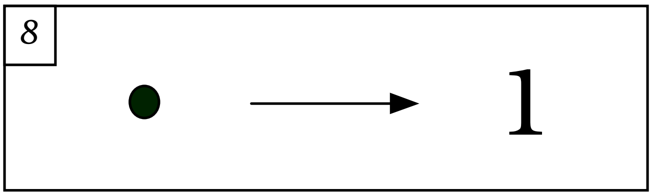
[15] Both Maudlin and me showed, roughly speaking, the incompatibility of comp and materialism. Maudlin tried to modify comp to keep materialism, I am lead toward modifying materialism, giving that comp is our starting hypothesis. See T. Maudlin (1989): "Computation and Consciousness," *The Journal of Philosophy*, pp 407-432.

[16] See my "filmed graph argument" in my PhD thesis (in French), or in "Conscience & Mécanisme"

http://iridia.ulb.ac.be/~marchal/lillethesis/these/node15.html#SECTION00700000000000000000, or here, again in French: http://iridia.ulb.ac.be/~marchal/bxlthesis/Volume3CC/3%20%202%20.pdf

The filmed graph argument, with Maudlin's Olympia reasoning could perhaps leads directly to quantum logic, giving the key role given to the logic of (arithmetical) counterfactuals, and the works of D? Lewis, Stalnaker and Hardegree, see G. M. Hardegree (1976): "The Conditional in Quantum Logic," In P. Suppes, editor: *Logic and Probability in Quantum Mechanics*, volume 78 of *Synthese Library*, pages 55-72, D. Reidel Publishing Company, Dordrecht-Holland. Hardegree shows that the standard orthomodular quantum logic, with the Sazaki hook implication, can be seen, at least formally, as a logic of counterfactuals, with a notion of similarity on histories behaving like quantum similarity (that is non orthogonality).

extension (we will see later why the second condition does not follow from the first) and accessible by the UD, that is arithmetically verifiable. Figure 8 illustrates our main conclusion, where the number 1 is put for the so called Sigma1 sentences of arithmetic.



Conclusion: Physics is given by a measure on the consistent computational *histories*, or *maximal* consistent extensions as seen from some first person point of view. Laws of physics, in particular, should be inferable from the true verifiable ‘‘atomic sentences’’. Those are the verifiable arithmetical sentences. They should be true everywhere (= in all comp histories), true somewhere (= true in at least one comp history), and inferred from the DU-accessible ‘‘atomic’’ states[17].

II. The Interview of the *Modest Machine* gives a non trivial Embryo of a Confirmation of Comp in the form of arithmetical quantum logics

To evaluate comp from its, perhaps startling, consequences, we will adopt a strikingly naïve methodology: we will interrogate the machine itself. Given that the UDA reasoning has shown that physics should emerge from a probabilistic structure bearing on its maximal consistent extensions; it is natural to interrogate the machine on its consistent extensions. Obviously, to interrogate an arbitrary machine will not be necessarily interesting. Eventually we will interrogate a Self-Referentially Correct, Arithmetical Platonist Universal Turing Machine (SRC machine), and this in the computationalist frame. Precisions will follow. At first sight such a choice could give the feeling that we are begging the question, giving that we decide to interview a machine which ‘‘share’’ our hypotheses. But it is all normal to proceed in this way giving that we are arguing neither for nor against comp. We are just studying, as in the first part, the logical consequences of comp. Obviously, at this stage, we can only *hope* the machine will be able to give more precise information than the informal (but precise) consequences the non constructive UDA reasoning has already provide[18].

The naïve methodology invites us to adopt a naïve stance toward machine’s beliefs. This means we will say a machine believes a proposition p if and only if the machine asserts p. It is up to us to choose[19] a sufficiently chatty machine capable of asserting any of its beliefs, or of assessing them in a way or another when asked. It is up to us to choose a

[17] Note that at this stage, we could already compare that ‘‘many histories’’ comp-physics with Everett-Feynman formulation of quantum mechanics. The modest machine interview will give more testable consequences.

[18] In some sense I substitute the grand-mother invoked in UDA by a sound universal machine.

[19] Actually, once machines are a bit complex, such a choice cannot be done constructively. We will follow the classical mathematician procedure of just limiting ourselves, non constructively, to such SRC machines. Giving the hypothesis of self-referentially correctness, we will be able to constructively derived their limiting SRC discourses.

sufficiently *serious* machine. For example, there is no real problem with a machine asserting that London is the capital of France. We can still remain indulgent toward the machine by just attributing it a lack of familiarity with elementary geography. But if the machine asserts that London *is not* the capital of France *and* asserts that London *is* the capital of France, then, that would make us suspects that the machine's beliefs are most probably inconsistent, especially if the machine has been presented as a Platonist machine.

Presentation of the machine. A machine will be said *Platonist*, or *Classical*, if 1) the machine believes all classical tautologies, and 2) it is the case that if the machine ever believes X and ever believes X -> Y, then the machine will believe Y. I will say the machine is consistent if its set of belief does not contain a contradiction. f (read false) will abbreviate any contradiction, like (p & -p) with p denoting some proposition. I will write Bp as an abbreviation for the proposition according to which the machine believes p. In the case we would add a proposition p to a consistent set of machine's beliefs, then, we will say that the machine *remains* consistent if the machine does not get a contradiction from p. So p will be consistent for the machine if -B- p, i.e. the proposition -B- p is true, i.e. the machine does not prove the negation of p. So we can read -B- as *consistent*. For example -B- -p says that -p is consistent, and this is equivalent to the non believability of p, i.e. -Bp. The notion of logical consequences of a finite set of propositions is defined in the usual way[20]. A machine will be said an *Arithmetical Platonist* if the machine believes enough elementary arithmetical truth (including some scheme of induction axiom). A machine will be *self-referentially correct*, or self-accurate, when any proposition the machine ever believes about its own beliefs or consistency propositions, are correct, and this, when B is translated or encoded in some manner in its language, for example arithmetic. A machine will be said *Universal*, if the machine is able to emulate any computation. For being universal, it is enough, for a classical arithmetical Platonist machine, to believe all true Sigma1 propositions. I recall they have the shape ''it exists n such that P(n)''. With the induction axioms such machine will have enough introspective power to ''know'' (in the sense of ''correctly believe'') that there are universal; in the sense that they will believe p -> Bp for any p arithmetical Sigma1 proposition. This will eventually provide us a very simple way to translate the computationalist hypothesis in the machine language, by adding the belief p->Bp to the machine's beliefs, identifying the atomic belief with a notion of DU accessibility.

The fact that we ask ''B'' to be translated in the machine language, that is, in term of object the machine is able to handle, like numbers, makes the machine beliefs ''scientific'' i.e. third person communicable (assertable) beliefs. It also protects us against Quine's form of essentialism accusation. The machine talks about some description of itself like an experiencer talks in a third person external way about a description of its body with its surgeon, or about its doppelganger after a self-duplication experiment. This means we will need to define in the machine language the notion of first person points of view. This will be done later by using the traditional definition given by Theaetetus to Socrates, and variants, in Plato's Theaetetus.

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[20] See R. Smullyan (1987): *Forever Undecided*, Knopf, New York. Mainly: X is said to be a consequence of a Y and Z, if (Y & Z)->X is a tautology, that is equivalent to (Y->(Z->X)), and is motivated by the modus ponens. Generalisations are easy on arbitrary finite set of formulas.

Gödel, Minds and Machines. It is known, and we will see why below, that *all* machines suffer from some intrinsic limitations which are related in particular to the difference, discovered by Gödel[21], between truth and provability. An important literature bears on the impact of Gödel's results, on the limitation of formal systems, on the question of mechanism. There are those who, like Lucas and Penrose, think that the Gödelian incompleteness show we are not machines, those who doubt any positive or negative relationship can be done, and those who believe and argue that Gödel's theorem is really a chance for mechanism. We belong, like Judson Webb, quasi by construction, to that last category. Giving that the incompleteness is a direct consequence of Church thesis, as we have shown, and giving that Gödel has proven his incompleteness theorem without CT, Judson Webb concludes, in a remarkable book, that incompleteness could not have been a luckier discovery for the mechanist: it is a confirmation of CT. And it makes CT a vaccine which protects universal machines against abusive diagonalisation. Eventually it protects Grand-Mother against Mister Theory!

No logician, as far as I know, has ever been convinced by Lucas or Penrose use of Gödel's results against mechanism[22]. Some genuine reconstructions of Lucas argument have been proposed, and a consensus exists that incompleteness can be used to show that *if* we are consistent machines *then* we cannot know which machine we are, and a fortiori in which computational history we are most probably supported by![23] Even Penrose acknowledges this fundamental nuance in his second best seller book bearing on that question, but, curiously does not take the nuance into account.

At first sight UDA, which forces us to capture physics through a measure on the consistent extensions of a SRC machine, could apparently leads us to some conflict with the second incompleteness theorem (which will be proved below). It says that a SRC machine cannot believe its own consistency, -B(-Bf) is true on such machine, so that if you ask such a machine if she has (at least) one consistent extension, she remains silent! And without any caution the machine just crash, again! Fortunately, if we are patient and let the SRC machine dovetail on its beliefs justifications, sooner or later it will "explain" its silence by asserting that -Bf -> -B-Bf, that is the machine believes that if she is consistent she can't believe in its consistency.

Gödel's first incompleteness discovery was indeed that any machine capable of proving arithmetical theorems either proves falsities or is incapable to prove some true arithmetical proposition. The lesson is that whatever the machine we choose; truth will

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[21] Actually, to my knowledge, this has been foreseen by Emil Post, who is the first to derive incompleteness from Church thesis (which he called a law of mind). That proof is basically the one I have given in step 7 of the UDA.

[22] See J. C. Webb (1980): *Mechanism, Mentalism and Metamathematics: An Essay on Finitism*, D. Reidel Publishing Company, Dordrecht, Holland. See « Conscience et Mécanisme » for my own reflection, an its comparison with many works in that field. It should be noted that our methodology does not need any philosophical study on that question, giving our naïve stance, and giving the fact that we will just interview an universal machine on that question. I include that paragraph to point on the fact that the Gödel/mechanism question has a long and rich tradition.

[23] Here incompleteness can already be intuitively related to the comp 1-indeterminacy, or its 3-person domain. I have discovered recently how Alain Connes compares implicitly the quantum indeterminacy and the incompleteness. We have been lead to make this connection necessary, with comp, and utterly transparent, as I hope the reader will find, through the interview of the universal machine, see my paper on the Changeux/Connes debate Marchal 2004, here: http://lutecium.org/stp/marchal.html (written in French). All machines suffer from limitations, but the modest ones I will describe, and which are exactly the Platonist one, have enough introspective abilities to assess the proof-truth gap and even to explore the infinitely complex border of that gap.

always extend properly its formal (sharable, checkable) provability abilities. But how to interrogate the machine on the geometry of its ignorance, as defined by its set of consistent extensions, if the machine is so limited. A theoretical shorter path toward the solution will be offered under the form of a couple of logics of self-reference, the Solovay provability logics G and G*, and which can be considered as fruitful and amazing descendant of the Gödel and Löb epoch making incompleteness theorems. I will try now, borrowing some trick in Smullyan's gentle introduction to incompleteness, to convey the main ideas without getting involved into too many technicalities. I hardly can make a better recommendation than to invite those who want get some familiarisation with the notions involved here to study Smullyan's lovely book.

Smullyan Pedagogy. To explain Gödel's and Löb's theorems[24], Smullyan proposes a puzzle. There is an island where all natives habitants are either knight or knave. Knights always tell the truth and knaves always lie. Some reasoner is visiting the island and some habitant tells him "You will never know that I am a knight." What can we deduce[25]?

The reasoner could reason in the following way. Let us suppose the native is a knave. Then he was lying and this means I will know he is a knight. But I cannot *know* he is a knight when he is a knave, so he cannot be a knave and he is a knight accordingly. Now *we* can drive a contradiction. We know the reasoner has reasoned correctly, so the native is really a knight and the reasoner believe the native is a knight. So we know that the reasoner know it is a knight, but then the native was wrong and must be a knave, and that is a contradiction. By knowing we have meant correctly believe. We got a paradox! Obviously this is a variant of Epimenides' Paradox. Now for letting the reasoner himself obtaining such a paradoxical conclusion we must suppose some capacity of reasoning. Indeed, as Smullyan very genuinely explains, no paradox would occur in the case a habitant says to a corpse, or less extremely a deaf, "you will never know I am a knight." Indeed in that case the habitant is a knight and indeed the deaf will not know that, giving that he does not even hear the question. If you are mentally disabled no paradox occurs either. Some *native* tells to you the same sentence, and you can answer "Ah OK" without deducing anything and no paradox will occurs. So what are the minimal reasoning abilities to get the paradox? For this problem, it can be shown that the knowledge of classical propositional logic is enough together with the assumption that the reasoner is *normal*, i.e. that if he knows p then he knows that he knows p.

The reasoning has shown that in case such an island ever exists, no native will ever say to a normal knower of classical logic: "you will never know that I am a knight." That leads the reasoner to a frank contradiction. Of course it could also mean the "native" was not a native, it could have been a joking tourist or a mad explorer *disguised* into a knave.

Now, suppose a (real) native tells you instead: "you will never *believe* that I am a knight." What can you deduce? We have followed implicitly the tradition, which originates in the Theaetetus of Plato, of defining the knowledge of some proposition p, by the correct belief in that proposition. That is, by definition, "knowing p" is "believing p" with p true. We can write Cp = p & Bp, where Cp means to (ever) know p, and Bp means to (ever) believe p.

Going from knowledge to belief makes things much more subtle and interesting. Indeed the paradox above, for example, will occur only if the visitor (which the habitant is

[24] Löb, M. H. (1955). Solution of a problem of Leon Henkin. *Journal of Symbolic Logic*, 20:115-118.

[25] Note that Smullyan pedagogy is not without danger. It could give the feeling that you need to believe in fairy tales to proceed. I will make clear the diagonalisation lemma, below, eliminates the need of the KK Island.

addressing) believes all his beliefs are true. In the case where indeed all his beliefs are true, the reasoning above will show that the reasoner can neither believe, nor know for the matter, the very fact that all his beliefs are true. So if all the propositions Bp -> p are true *about* you, they cannot all be believed *by* you. Instead of a paradox, we get an incompleteness result. And you don’t need really to go on the KK Island; it is enough some habitant asserts “Mister X or Misses X will never believe I am knight.” That sentence will be true, although unbelievable by X, independently of the fact X met such sentence. Imagine a native saying “the Belgians will never believe I am a knight,” then any Belgian believing in its own accuracy, i.e. believing in all the propositions Bp -> p, will be inaccurate, even if the Belgian didn’t know anything about the KK Island. Giving that the use of “believe” instead of “know” evacuates the paradox, such an island could well exist and the assertion of their inhabitants could have consequences on our ability or inability to believe some truth! This is a very weird situation. To reassure ourselves we can still hope such an island does not exist.

But the purpose of the island was just to build a fictive situation illustrating easily how someone could meet and believe some self-referential sentence. That works like this: let k be the proposition that the native is a knight, and suppose the native asserts p. Then p will be true if and only if k is true, and if someone believes in the rule of the island (I recall: all knight tell the truth, all knaves lie, and all habitants are either a knave or a knight), he will believe the proposition (k <-> p). Now the proposition “you will never believe I am a knight”, once asserted to a believer in the island rules will make the self-referential proposition (k <-> -Bk) believed by that believer. Moreover (k <-> - Bk) will be indeed true in case the rules actually hold on the island.

The point now is that, with or without the KK Island, machines cannot dispose so easily of the self-referential propositions. Actually machines cannot dispose of them at all. There is a famous result which proves this fact, known as the *diagonalisation lemma*. So with the diagonal lemma, we can reason *as if* there were a KK Island, making incomplete any third person sort of “checkable” belief from honest universal machine. The reader who grants this can jump the following more technical section.

The diagonalisation lemma. If you have a duplicating machine D, which when glued a little bit on any machine M duplicates it, and paste it a little giving say MM, then, gluing it a little bit to itself DD will results in DD itself. That is DD produces DD, relatively to some probable universal computational history. In our chatty approach it is enough the machine believes elementary substitution relations, like subst(abc, baX) = [baabc], meaning that the substitution of X in the second argument by the first argument gives (a description of) the string baabc. The bracket “[“ and “]” are used to represent a description of the final result in term of object the machine can reason about, like the numbers of our arithmetical universal machine. If the machine remembers that it is always the first argument which is substitute in the second argument, she will correctly believe that subst(aXc, baX) = [baaXc], although it could seem at first a little bit confusing due to the occurrence of X in the first argument string. So the machine will believe, and the reader is invited to verify this by hands, that:

subst(subst(X,X), subst(X,X)) = [subst(subst(X,X), subst(X,X))]

We obtain an expression which denotes a description of itself. Suppose now you want build a machine capable to operate some mechanical transformation on itself by applying some other machine T on itself. All you need is a new machine, which I still write D, capable, if you present it a machine A as input to apply T on the result of gluing a little bit A on itself before: DA gives T([AA]). Then DD will gives T([DD]). Applying this idea on our chatty substitution leads to an expression capable of producing a transformation of itself, and this in a way the machine can believe. Let us take any adjective understandable by the machine. They are called predicates in logic. For exemple the predicate odd(X) which says that X is odd; for instance, odd(23) is a true proposition, and odd(24) is false. Odd(X) is easily understandable by our arithmetical machine: odd(X) <-> there is a number Y such that X = (2 times Y) + 1.

Let us define 1) T(X) by odd([subst(X,X)]), and 2) let m = [T(X)]. The machine will believe that T(m) is equivalent to odd([subst(m,m)]), and thus equivalent to odd([subst(m, [T(X)])]), by 2, and thus equivalent to odd([subst(m, odd([subst(X,X)])]), thus equivalent to odd([odd[subst(m,m)]]), thus equivalent to odd([T(m)]). That is: the machine will believe that the proposition T(m) is equivalent to odd(T(m)). Let us define the closed formula by T(m) by k: we have that the machine believes k <-> odd([k]).

So k is true iff its description in the machine language is odd! Now, the choice of the predicate “odd(X)” didn’t have any relevant role in the proof, as far as it is definable in the machine language, and we have illustrate that for any such definable predicate P, there is a corresponding fixed point sentence k such that the machine believes (k <-> P([k])).

Theorem: For any predicate A definable in the machine language, there is a proposition k such that the machine will know (correctly believe) the proposition (k <-> A([k])). Put in another way, with simplified notations, for any definable predicate P, the logical equation X <-> P(X) admits a solution k such that the machine believes k <-> Pk.

The consequences of the diagonalisation lemma are tremendous The fact is that machines no more need to visit the KK Island to be troubled by all kinds of self-reference. What happens with the paradoxes? What if a native just simply says “I am not a knight”. The traditional way to escape the paradox consists to say no native will ever say that, giving that otherwise, we would be lead to a thorough contradiction. Suppose now the notion of knight is definable in the machine language by a predicate knight(x) meaning that x names a true proposition, so that the machine believes for any proposition p: p <-> knight(p). Then by the application of the diagonal lemma on the predicate defined by “not knight(x)”, there is a k such that the machine will believe that k is equivalent to the negation of knight(k), itself equivalent to -k, so the machine will believe k <-> -k: contradiction. Now, by the diagonalisation lemma *assumption*, this means that “knight(x)” or “knave(x)” is just not definable in the language of the machine. Truth on a machine is unnameable by the machine. This is a version of Tarski theorem. For the same reason, the paradox we get above when we met a native telling us “you will not *know* I am a knight”, with the corresponding fixed point sentence k <-> -Ck, shows that consistent machine’s *knowledge* is not definable by the consistent machine. What can be said about machine’s *beliefs*, and in particular about the third person communicable beliefs of our Platonist SRC machine? From the visit in the Knight Knave Island we got an incompleteness theorem. By the diagonalisation lemma, we thus get a corresponding incompleteness theorem for the machine.

If Gödel incompleteness theorem is amazing, it is nothing compared to Löb’s theorem. We first need the following sort of sum up theorem. It can be shown that the beliefs of the Platonist universal machine are described by the following provable (and true with the self-referential interpretation) propositions:

1) If M believes p then M believes Bp (M is normal)

2) M believes Bp -> BBp (M *knows* he is normal, we will say M is of type 4)

3) M believes B(p->q) -> (Bp -> Bq) (M believes he is regular, that is, he knows he follows Modus Ponens, or MoPo). That formula is named K (for Kripke).

I will say that a normal machine is a type 4 reasoner when it verifies 1, 2, and 3. Line 1 says that the machine is normal. We can say that Bp->BBp is true for the machine, given that we interpret Bp by the machine believes p. It can be seen as a form of self-awareness. The second line says that for all propositions the machine *believes* it is normal with respect to them, this gives still more self-awareness: not only Bp->BBp is true about the machine (by line 1), but line 2 makes it believed by the machine[26]. Line 3 says that the machine is not only Platonist, in the sense of having a set of beliefs closed for the modus ponens rule, but actually knows (correctly believes) it is closed for MoPo.

Revision exercises: Let us says that a machine is accurate or correct, or sound if Bp->p is true for the machine. Let us say that a machine is stable if BBp->Bp is true for the machine. Could an accurate machine believe it is accurate with respect to any proposition? Could a consistent machine believe in its own stability? You can try to show that for any consistent normal and stable machine, there is an undecidable proposition, i.e. a proposition p such that the machine can believe neither p nor –p.

Some useful definitions: A machine M1 is referentially correct about a machine M2, if every proposition proved by M2 is true for M1 (we suppose that true propositions with no symbol B in it are vacuously referentially true, for example 1+1=2 is true *about* everybody). A machine is self-referentially correct if it is referentially correct about itself. Obviously: SRC implies soundness implies stability. A machine M1 is *referentially complete* on M2 if M1 proves all the propositions which are true for M2. You might show that self-referential correctness entails self-referential incompleteness.

Arithmetical Placebo, Self-Confidence and Modesty What about a native telling to a reasoner the following much more positive proposition “You will believe I am a knight”? This is, in KK language, the question L. Henkin asked to M. H. Löb, which leads Löb to a genuine astonishing generalisation of Gödel’s theorem. In the language of a arithmetical classical machine, what can be said about a sentence k saying about itself provable([k]). Apparently that sentence can be said by a knave (and be false) or by a knight (and be true). That is quite unlike the Gödel sentences previously studied, on the type p <-> -Bp, which said about themselves that they are not provable by M, making them true *about* M and unprovable *by* M, when M is consistent, and making them undecidable by M, when M is also stable. But the diagonalisation lemma can strike against, in a deeper and more positive way than we could expect at first sight.

Let us go back on the KK Island. A type 4 student, that is a normal platonist knowing he is normal, is developing some anxiousness concerning his end of year exams. The teacher told him not to worry so much and that his anxiousness was just due to some lack of self-confidence. He told the student that if he could just believe in the success then he would succeed. That was not a big help giving the fact that the student is really lacking such a self-

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[26] This follows from the Sigma1 completeness described above. The fact that the universal machine knows its universality, because the predicate B is translated by "it exists y such that proof(x, y)," where proof(x, y) is the decidable predicate saying that y is (a description) of a proof of (a description) of the formula x. Proofs are just sequences of formulas which are either axioms or derived from preceding theorems from the rules. Proving that the arithmetically translated belief, or arithmetical provability “B”, verifies p->Bp for p sigma1 is the most delicate part. See Boolos 1993 for a thoroughly detailed explanation. G. Boolos (1993): *The Logic of Provability*, Cambridge University Press, Cambridge.

confidence, and so, although he trusts his teacher that if he could ever believe in success he would succeed, he is, as a matter of fact not believing in success, and could as well completely fail. The teacher then suggested him to make a visit to the Knight Knave Island, being told there was a native who was a gifted priestess specialized in the art of rising up self-confidence. The student trusts completely his teacher and believes in the KK rules, and thus decides to go there during the Easter holidays, a little before the exams. The student meets the priestess and explains her that his teacher was trustful so that if he was able to believe that if he could ever believe in success, success would happen, but adds that he did actually not believe in success. After some ceremony the priestess eventually tells him: “if you ever believe I am a knight then you will succeed.”

Now the student gets really desperate. He thinks he has got no more evidence that the priestess was a knight than he had trust in himself at the start. Thinking twice he gets a big relief, though. Why?

Let s be the proposition that the student will be successful. The student trusts his teacher so that he believes Bs -> s. Now he believes in the rule of the island, so that he believes k <-> (Bk -> s) where k is the proposition that the priestess is a knight. The student made the following reasoning: “Let us suppose I will believe she is a knight, then I will believe what she said, that is Bk -> s (being of type 4, he knows he is regular). But if I believe she is a knight, I will believe that I believe she is a knight (being of type 4 he knows he is normal), that is I will believe Bk; so if I ever believe she is a knight I will believe both Bk and Bk -> s, so by propositional logic, I will believe s, and because I trust my teacher it means I will succeed. But that is exactly what the priestess said: if I believe she is a knight I will succeed. So she told me a true statement! So she *is* a knight.” Being normal, the student will now believe she is a knight, and thus believes also what she said, that if he believes she is a knight he will succeed. So he will believe he will succeed, and then, if his teacher was right, he will succeed!

Now by the diagonalisation lemma, there is no need for a universal machine of type 4 to go on a KK island. It simply exists a fixed point sentence k such that k <-> (Bk -> s), for any proposition s, and the reasoning above gives a proof of **Löb’s theorem**: If a type 4 machine believes Bp -> p for some proposition p, then the machine believes p.

A simple “corollary” follows: Gödel’s second incompleteness theorem: if a type 4 machine is consistent then the machine is unable to believe she is consistent.

Proof: consistency is, in the machine language, **–**Bf. But this is Bf -> f, by PC, as the reader can verify by a two line truth table. By Löb, if ever the machine believes Bf -> f, she will believe in f, contradicting the assumed consistency.

And what about Henkin’s question? It is a direct consequence of Löb’s theorem that a sentence saying “I am provable by M” is true and provable by M! This follows from the fact that the proposition p<->Bp entails in particular Bp->p.

Note that Löb’s theorem can be stated as B(Bp->p)->Bp. This formula is called Löb’s formula, and is named L. That formula is true for M, but M believes it too. A type 4 universal machine can even prove what we have just proved, that is:

B(k <->(Bk ->p))   ->   B(Bp->p) -> Bp

And Löb’s formula follows again by a visit to the KK Island, or more seriously by the diagonisation lemma on the formula BX->p. Löbian machine has been called modest by Rohit Parikh, and Löb’s formula is really a modesty formula. The reason is that the machine will believe its accuracy with respect to p, i.e. will believe Bp->p, *only* when it actually believes p.

In which case Bp is obvious from MoPo, given that (p->(q->p)) is a tautology. So it is hard to imagine how to be more modest than that.

Definition. A type 4 machine is modest, if it believes all propositions B(Bp ->p)->Bp. It can be shown that modesty entails belief in its own normality, and so we will indifferently called our SRC machine, which is provably modest, a modest or a Löbian machine[27]. A type 4 universal machine does not need to visit the KK Island to become modest, by the diagonalisation lemma.

Solovay's incompleteness-completeness theorems In 1976, Solovay has given two genuine and wonderful completeness theorems, concerning the (infinite) discourses we can have with an arithmetical Platonist SRC machine, or more general Löbian machines and entity[28]. His first theorem says that modest propositional believability logic, Solovay named G, that is the normal system with K and L as axioms, formalizes completely the *provable* arithmetical propositional logic of provability and consistency, of Peano arithmetic, or ZF, actually of any ordinary provability predicate in RE set extending PA. This makes the L formula really the fundamental formula of machine's psychology. It is known that 4 can be derived from L in G.

The second theorem is still more amazing. We consider the following theory G* which has as axioms all theorems of G, plus the soundness formula Bp->p. And which is closed for MoPo. Note that we don't ask G* being normal, for the reason that, in that case, from the axiom instantiation Bf->f, normality would lead to B(Bf->f), and Löbianity would then lead to Bf, and giving we got already Bf->f, MoPo would lead to f, making G* inconsistent. The second theorem of Solovay says that G* formalizes completely the true arithmetical propositional logic of provability and consistency.

Now it can be shown that both G and G* are decidable, making the G* minus G corona a decidable set, closed for MoPo, of unbelievable truth. Giving our naïve stance, it makes them non communicable as well. For example we know that the SRC is consistent, stable and sound, but cannot know it, and that makes –Bf, BBp->Bp, and Bp->p belonging to G*\G. In fact, as G is closed for the necessitation rule, G* is closed for the "possibilization" rule: if p is provable by G*, then -B- p is also provable by G*. The decidability of G and G* entails[29] the decidability of all the logics which follow.

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[27] Boolos 1993 gives 5 reasons to be utterly astonished by Löb's theorem. Here we emphasize on a possible sixth one: that Löb's theorem describes a form of very basic arithmetical placebo. It is arguable that it can be used for making clearer the comp grand-mother vindication (we need perhaps some grain of salt!). IF grand-mother succeeds to convince her Löbian grand-child that if he believes that some grasses are good for his health, it will be good for his health, THEN it really will! Obviously this makes the Löbian machine prone to negative placebo effects making them sensible to possible verbal perversity.

[28] G and G* are sound and complete for larger systems, and can be enriched for providing non-comp notion of belief, for example Solovay got that G together with the formulas B(BX->BY) v B(BY->(BX&X)) give a system which is sound and complete for the (set theory) propositions which are true in all transitive models of ZF (Zermelo Fraenkel set theory). For a proof see Boolos 1993. Solovay got also that G together with the formulas B(BX->Y)vB((BY &Y)->X) captures in the same way the propositions true in all models V_{Kappa} with kappa an inaccessible (rather big) cardinal. In case we find, as a measure on the consistent histories, a consistent subset of physics, but don't find all of physics, making comp false, similar Solovay extensions of G and G* could provide psychologies of some "non machine" notions. See R. M. Solovay (1976): "Provability Interpretation of Modal Logic," *Israel Journal of Mathematics*, 25:287-304.

[29] This is true only at the propositional level where no variable enters in the scope of the modal connector B. The Russian logicians have solved the question of the decidability of the first order extension of G and G* in the worst possible negative way. See the book by Boolos 1993 which relates in details those results.

Computationalism: It is the computationnalist hypothesis which has invited us to interview the self-referentially correct machine. Such a machine could consistently being non computationalist. By incompleteness it is consistent for a consistent machine to believe in its own inconsistency, indeed the second incompleteness theorem just says that: -Bf->-B-Bf. We could interview consistent but non self-referentially correct machine, and actually we could interview non computationalist machines, who believe they lose their consistency by doing teleportation. But, as we justify at the start, we are interested in the discourse of the SRC machine *in the comp frame*. Self-reference and Solovay theorems did justifies that *atomic*, in the logician sense, propositions corresponds to the arithmetical propositions, and that makes unavoidable the use of G and G*. Now, to take into account comp and the UD Argument which shows that the physical propositions arises from a sum of DU-accessible states, we must restrict those arithmetical propositions to those proved or generated by the Arithmetical Dovetailer, i.e. the Sigma1 sentences[30] as explained in the 8th UDA step. Our introspective universal machine *knows* that they are universal in the sense that for any Sigma1 sentence p, the machine can prove that if p is true then p is provable: they can prove p->Bp. So to restrict, the SRC discourse in the comp frame, and in that way enrich the self-reference logic, it is just enough to add to G the sentence p->Bp with p atomic. I like to call 1 the proposition "p->Bp" with p atomic, due to its fundamental importance but also as a shortening of Sigma1. "1" can be seen as the *comp axiom* written as a (scheme of) formula added to G, and so belonging to the (infinite) discourse of the SRC *in the comp frame*. In my previous work I did use only the arithmetical soundness of that new logic, but the logician Albert Visser (19) did prove the soundness and completeness of G+1, and its corresponding (G+1)* truth theory. Vickers gives also independent motivations for a similar notion of verifiability, and I am used to call G+1 and (G+1)*, V and V* accordingly[31]. Note that the sentence letter p in p->Bp cannot be substitute by any formula, but only by propositional letter, if we want keep correct the arithmetical discourse interpretation. By way of counterexample p->Bp would be in contradiction with incompleteness in case p is replaced by **–**Bf.

If you identify a logic with its set of theorems you have the following diamond, which I will call the basic diamond for further reference. The implicit edges represent inclusion:

V*

G*   V

G

Going up in the North West direction is the non trivial Gödelian passage from provability (believability) to truth. Going up in the North East direction is the non trivial comp direction. Sometimes, to fix the things, I say that G gives science and G* gives theology, V gives comp-science and V* gives comp-theology. But this *can* be taken with some grain of salt[32].

[30] The relation between universality, creative set in Post 1944 sense, complete recursively enumerable set and Sigma1 formula are explained in books on elementary recursion theory. Important isomorphism theorem like Myhill's theorem makes such a link quite natural, with Church thesis in the background.

[31] Although modal logicians are somehow the experts in "naming theory," they are very bad in giving names to formulas. I follow the (bad) tradition of using number for names of formula. For exemple 4 is the traditional name of Bp->BBp. For S. Vickers see its *Topology via Logic*, (1989), Cambridge University Press.

[32] Or perhaps without: recall that we have shown that *truth* about a machine is unnameable by the machine. Unnameability is taken as an axiomatic property of the "big one" in almost all religious/philosophical traditions.

Arithmetical Theaetetus. We are not yet in a position to get physics. What is missing is the fundamental distinction between the first and third person points of view, without which the UD Argument just doesn't start. The four G, G*, V, V* gives only 3rd person descriptions. G for example axiomatizes completely the propositional logic of self-referentially correct discourse made by Platonist machines, but those machines talk about themselves only through third person description made (by construction) at the right level. For instance Peano arithmetic provability is described in term of numbers, often called Gödel Numbers.

But the Universal Dovetailer Argument (UDA) did show that physics must appear through the machine's first person point of view, or from some first person plural point of views. Those first person points of view concern anticipations of consistent extensions from some personal, interrogative, perhaps unconscious most of the time but made conscious in front of the comp doctor, bet on self-consistency.

To interrogate the Universal Machine we need to define those points of views from what the machine is able to talk on. We will follow two ideas and their union: 1) to define the first person by the knower, i.e. the one who correctly believes the propositions. This is one of the well known (by philosophers) Theaetetus attempts to define knowledge from opinion after Socrates asked him, in Plato's Theaetetus. 2) to define the first person (plural ?) by the better. With the first idea we give to the believer an unbreakable umbilical cord with truth, making it incorrigible as a knower should be. With the second idea we attach the believer to some (hopefully correct) bet on his own consistency. At least formally, we can imagine uniting those two ideas for getting the *correct better*. All three ideas can be defined in the propositional self-reference logic:

To know p, written Cp, is defined by Bp & p,

To bet on p, written Pp, is defined by Bp & -B-p,

To correctly bet on P, written Op, is defined by Bp & -B-p & p.

This makes sense: G* proves indeed that Bp is equivalent to Cp, and to Pp, and to Op, but from the machine point of view, (Bp <-> Cp), i.e. (Bp <-> Bp & p) is neither believable, nor knowable, that is provable by G; nor are the G* equivalence (Bp <-> Pp) and (Bp <-> Op), (Pp <-> Op) provable by G. This follows from the simple facts that G*, unlike G, proves Bp ->p, and G* proves p->-B-p. All arithmetical realisations of the corresponding modal logics, where the sentence letter are interpreted by arithmetical sentences, prove the same arithmetical sentences, but from the machine point of view they give very different logics. Those variants of Theaetetus' definition describe different ways a machine can be related to truth, and those ways are ontic-equivalent (by G*), but epistemic-non-equivalent (by G). And all those G/G* remarks can be lifted with the comp V/V* constraints, where the sentence letters are interpreted by Sigma1 sentences. So, by applying the three Theaetetus variants on each logics taken from the basic diamond, we get 12 logics. Actually we get 10 logics, because two of the logics obtained can be shown to be equal: G* and G give the same knower (Cp), and V* and V give the same "comp-knower". This means that from the point of view of

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This reminds only that *truth* is a very encompassing notion. Theology is defined here by all true but non communicable propositions. Comp theology adds the constraint that true atomic propositions are UD-accessible.

the knower, believability is equated to truth. It makes it akin to a constructivist self-extending self. Like in Brouwer's consciousness theory[33], that self is unnameable by itself, and this follows from the fact that Cp, as it has been showed, is no definable by the machine. So the knower cannot really believe he is any third person nameable machine, and this could explain some reluctance of the first person to bet on an artificial digital body or to fear digital duplication. The application of the CP variants on $G^{(*)}$ has been well studied in the literature. It has been done independently by Boolos, Goldblatt, and Kuznetsov and Muravitski. Artemov makes it a thesis[34]. It gives a logic of irreversible (antisymmetric) subjective "time" quite similar again in that respect with Brouwer's consciousness theory. And this has been confirmed (not proved!) by a result of Goldblatt, itself (related to some work of Gödel and McKinsey & Tarski), relating S4Grz (read S4 Grzegorczyk, it is the result of the CP variant on $G^{(*)}$) and intuitionist logic. Let us note that philosophers who don't accept the Cp-Theaetetus definition of knowledge, implicitly or explicitly pretend to be able to distinguish the waking state from the dreaming state, and so, negate the most primitive form of comp, as was suggested by the step 6 in UDA. As an example, the positivist philosopher Malcolm attempts to refute[35] both the existence of consciousness in dream and in machine. He compares the *lucid dream* proposition "I dream" with the Epimenides' lying sentence "I lie," G and G* make possible finer comparisons. S4grz is an abyss of interesting things to say on the machine's first person psychology, but I will refer the reader to my "Conscience et Mécanisme" for more information, because it is about time to look at physics and sensations.

Physics and Sensation To get physics and sensations we must apply the Theaetetus variants on V*. It is the only way to find the "true" logic of a probability measure on *all* consistent extensions (this explains the star *), arising when the atomic propositions are restricted on those accessible by the Universal Dovetailer (the Sigma1 one, this explains the V = G+1).

To get a modal logic featuring a probability notion, both model theory and modal semantics, which are a little bit beyond the scope of this paper, suggests the need of having the deontic formula Np -> -N-p, where N is an abstract necessity modality at first. The idea is that Np means P(p) = 1, with P(p) interpreted as a probability of p, and then -P(-p) means "P(-p) is different of 1" which means "P(p) is different of 0", which makes the deontic formula natural for a probability notion. Note that neither G nor G* does prove it (G* does not prove Bf->-B-f, indeed G* proves Bf->B-f). Now all logics obtained by an application of the Theaetetus variants give a logic verifying the deontic probabilistic formula. Naturally the Pp-variant is the literal translation of the consequence of UDA, so it should, with the comp hypothesis, give the physical probability. So the Pp variant, which gives a modal logic featuring the "probability one" or the "measure one" on the consistent extensions should give a logic of measure one on the physical propositions. So we need to look about what the physicist says on such a logic, and to look what the Pp variant on V* says, and then compare.

I said at the beginning that Quantum Physics was a good candidate for being a stable part of fundamental physics. Now quantum physics is essentially a probability calculus. Von

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[33] See W. P. van Stigt (1990): *Brouwer's Intuitionism*, volume 2 of *Studies in the history and philosophy of Mathematics*, North Holland, Amsterdam.

[34] Artemov, S. (1990). "Kolmogorov' s logic of problems and a provability interpretation of intuitionistic logic". In Parikh, R., editor, *Proceedings of the Third Conference on Theoretical Aspect of Reasoning about Knowledge (TARK 90)*. Morgan Kaufmann Publishers.

[35] Malcolm, N. (1959). *Dreaming*. Routledge & Kegan Paul ltd., London.

Neumann worked out, with the help of Birkhoff, a logic of quantum probability *one*. In quantum physics, worlds or states are represented by line in a Hilbert space. Propositions are represented by linear subspace. Like Boolean classical logic can be interpreted as a lattice of subsets of a set, and like Brouwerian intuitionist Logic can be interpreted as the lattice of open sets of a topological space, Quantum Logic can be interpreted as a lattice of subspaces of a Hilbert (complex linear) space. Each ‘‘sub-notion’’ can, from a modal angle, be considered as both a proposition and as the ‘‘collection’’ of worlds in which that proposition is true. Modularity of the lattice (a weakening of the Boolean distributivity) gives hope to von Neumann of capturing a complete logic of yes-no orthogonal experiments capable of yielding all the quantum probabilities. Alas, infinite dimension kills modularity, and von Neumann will jump from the Hilbert space to the… von Neumann algebras. Still, he will remain unsatisfied with the quantum logics he will isolate[36], and, as van Fraassen wrote, physicists are confronted with a *labyrinth* of quantum logics.

So, to be honest, I don’t know yet if it is a good news or a bad news, for those wanting comp being confirmed or being refuted, but, not only Pp, but also Op, and even Cp, when applied on V* leads to bizarre and different sorts of arithmetical quantum logics. How?

Definition A modal quasi-quantum logic has as main axiom, p->BMp (p atomic). I like to call that formula ‘‘LASE’’ for ‘‘Little Abstract Schrödinger Equation’’. M is an abbreviation of -B-. A modal quasi-quantum logic has also the axiom Bp ->p (T), with K B(p->q)->(Bp->Bq), and is closed for MoPo, but not necessarily the necessitation inference rule (= is not necessarily normal).

Why? Goldblatt has shown that the logic B, known as the Brouwersche System, and which is the modal logic with K, B(p->q)->(Bp->Bq), and LASE, p->BMp, and T, Bp->p, and which *is* normal, axiomatizes quantum logic (in the classical setting), in a similar way as S4Grz axiomatized intuitionist logic (in the classical setting).

Precisely, considering the following transformation GOLDB, due to Goldblatt 1974, from the propositional language to the modal propositional language, which transforms sentence letters p into BMp, and transforms -p into B-p, and transforms recursively (A & B) into (GOLDB A & GOLDB B), Goldblatt showed[37] that a formula A is proved in a minimal version of quantum logic iff B proves GOLDB(A).

Now, the modal quasi quantum logics have, thanks to the truth of LASE for the atomic propositions, all what is needed to be able to apply the Goldblatt transformation for getting reasonable arithmetical quantum logics. Applying the Cp, Pp, and Op Theaetetus variants on the basic logics diamond, gives, as we expected from UDA with at least the Pp variants, three modal quasi-quantum logics. They are the one called S4Grz1, Z1*, X1* respectively, in my PhD thesis ‘‘Calculabilite, Physique et Cognition[38]’’.

	S4Grz^1			Z1*			X1*	
S4Grz		S4Grz^1	Z*		Z1	X*		X1
	S4Grz			Z			X	

[36] See the book by Miklos Redei, (1998): *Quantum Logic in Algebraic Approach*, Kluwer Academic Publisher.

[37] See R. I. Goldblatt (1974): ‘‘Semantic Analysis of Orthologic,’’ *Journal of Philosophical Logic*, 3:19-35. Also in R. I. Goldblatt (1993). *Mathematics of Modality*. CSLI Lectures Notes, Stanford California, pp 81-97.

[38] Actually I missed badly S4Grz1, which I thought wrongly that it would lead to a collapse of the modalities.

Applying the (inverse) Goldblatt transform on S4Grz1, Z1*, X1* gives the three arithmetical quantum logics AQL_0 , AQL_1, AQL_2 leading to many open problems. Do we have modularity, or orthomodularity, or something else? Are the Bell's inequality violated?

I conjecture a quantum computer can be defined in the AQL_i, or in their first order extensions (with quantifiers). This would explain why any universal machine looking at itself discovers a quantum "reality" as a measure on its most probably correct anticipations.

When applying the Goldblatt transform on the non empty collection of propositions Z1* minus Z1, and X1* minus X1, this gives a description of the consistent (and true) comp-physically measurable but *uncommunicable* truth, so that the "qualia" or sensations, are themselves described by sorts of quantum logics[39]. It is the main advantage of comp, compared to traditional empirical physics. Empirical physics is obviously in advance compared to the comp physics, but is quasi obliged, by methodology, to put the first person under the rug, and so misses the Qualia Logics.

Comp makes them possible and necessary, and isolates them from the many modal nuances imposed by Löbian incompleteness.

Brussels, 14 August 2004

[39] For quantum logic not unrelated with "perception" see Bell, J. L. (1986). A new approach to quantum logic. *Brit. J. Phil. Sci.*, 37:83-99. (Don't confuse the logician J. L. Bell with the physicist J. S. Bell.)