# Advanced ACO Algorithms

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# The Ant System Algorithm

**For** #iterations

   **For each** ant

      Choose a random starting city

      **While** tour is not complete

         Select next city using random proportional transition rule

      **End While**

   **End for**

   Update pheromone information

**End for**

# Successors and Extensions of the AS

- MAX-MIN Ant System (MMAS)

    - Only iteration best or best-so-far ants update pheromone

    - Pheromone trails have explicit upper and lower limits

    - Pheromone trails initialized to upper limit

    - Pheromone trails are re-initialized when stagnated

- Ant Colony System (ACS)

    - Pheromone is also updated while building solution

    - Only iteration best or best-so-far ants update pheromone

# MAX-MIN Ant System (MMAS)

☐ Only iteration best or best-so-far ants update pheromone

$$\tau_{ij} \leftarrow (1 - \rho) \cdot \tau_{ij} \quad \forall(i,j)$$

$$\tau_{ij} \leftarrow \tau_{ij} + \Delta\tau_{ij}^{best} \quad \forall(i,j)$$

$$\Delta\tau_{ij}^{best} = \begin{cases} \dfrac{1}{L'} & \text{if } arc(i,j) \in T^{best} \\ 0 & \text{otherwise} \end{cases}$$

# MAX-MIN Ant System (MMAS)

☐ Pheromone trail values are subject to bounds

$$\tau_{min} \leq \tau_{ij} \leq \tau_{max}$$

$$\tau_{max} = \frac{1}{\rho \cdot L^{opt}} \qquad\qquad \hat{\tau}_{max} = \frac{1}{\rho \cdot L^{bs}}$$

$$\tau_{min} = \frac{\hat{\tau}_{max}}{a} \qquad\qquad \tau_0 = \infty$$

# MAX-MIN Ant System (MMAS)

☐ Pheromone trails are reinitialized:

   ☐ When the algorithm converges;

   ☐ When no improved solutions has been generated for a certain number of consecutive iterations.

# Ant Colony System (ACS)

☐ Three main ideas:

  ☐ Different state transition rule;

  ☐ Different global pheromone update rule;

  ☐ New local pheromone update rule

# ACS - State Transition Rule

□ A.k.a. pseudo-random proportional rule:

  □ with probability $q_0$, exploitation ;

  □ with probability $1 - q_0$, biased exploration

$$
j = \begin{cases}
\arg\max_{j \in \mathcal{N}_i^k} \left( \tau_{ij} \cdot \eta_{ij}^{\beta} \right) & \text{if } q \leq q_0 \quad \text{(Exploitation)} \\
J & \text{otherwise} \quad \text{(Exploration)}
\end{cases}
$$

# ACS - Global Update Rule

☐ Pheromone modified only on edges of the best tour so far

$$\tau_{ij} \leftarrow (1 - \rho) \cdot \tau_{ij} \quad \forall (i, j) \in T^{bs}$$

$$\tau_{ij} \leftarrow \tau_{ij} + \rho \cdot \Delta\tau_{ij}^{bs} \quad \forall (i, j) \in T^{bs}$$

$$\text{where} \quad \Delta\tau_{ij}^{bs} = \frac{1}{L^{bs}}$$

# ACS - Local Update Rule

☐ While building a solution each ants updates pheromone on visited edges:

$$\tau_{ij} \leftarrow (1 - \rho) \cdot \tau_{ij} + \rho \cdot \tau_0$$

☐ This update rule introduces diversification

# ACS - Pseudocode

```
Loop
    Randomly select starting cities
    For step=1 to #cities
        For k=1 to #ants
            Apply the state transition rule

            Apply the online trail updating rule
        End-for
    End-for
    Apply the offline trail updating rule
Until End_condition
```