Local Search under Uncertainty

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Outline

- Introduction
- State-of-the-Art
- Proposed Approach
- Preliminary Results
- Conclusion and Future Work

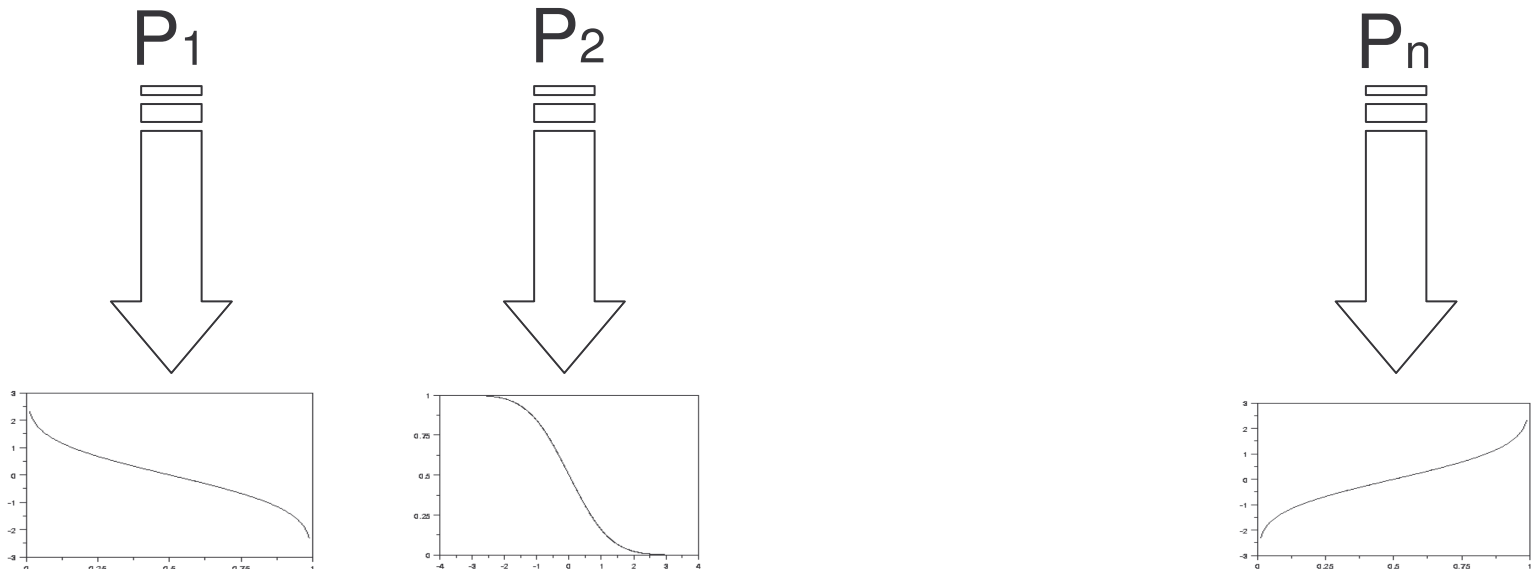
Goal and Motivation

- Local search based on empirical estimation
  - Tackling CO under uncertainty using sampling techniques

CO under Uncertainty

- The objective function is affected by uncertainty
- Cost of the solution is random variable
- Customary to optimize w.r.t expectation
- The goal is to minimize the expected cost
- Requires computationally expensive procedures

Example of a Portfolio Investment

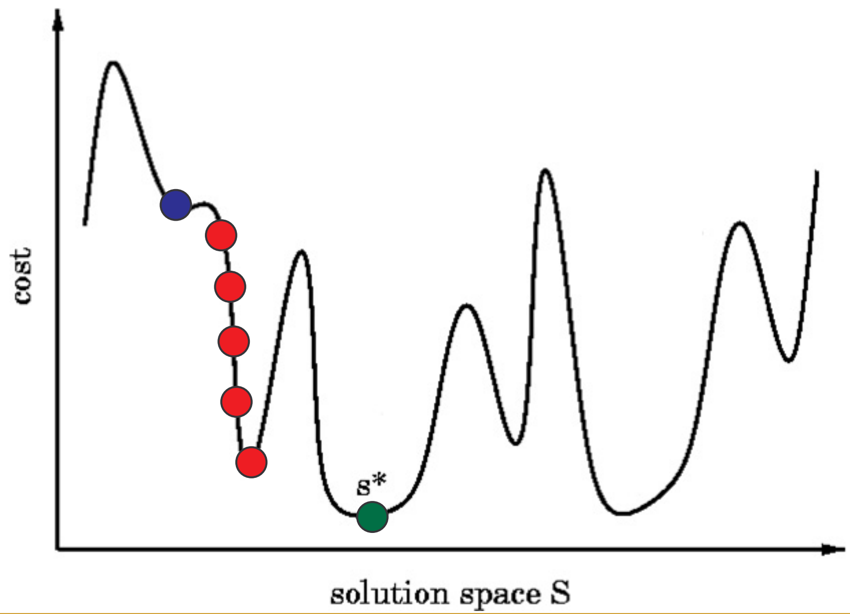


Solution techniques

- Analytical approximation – Probabilistic model
- Empirical estimation – Sampling or Simulation
- Advantages of estimation
  - Generality – Expected cost by averaging a number of realizations
  - Profitable approximation is complex and problem specific

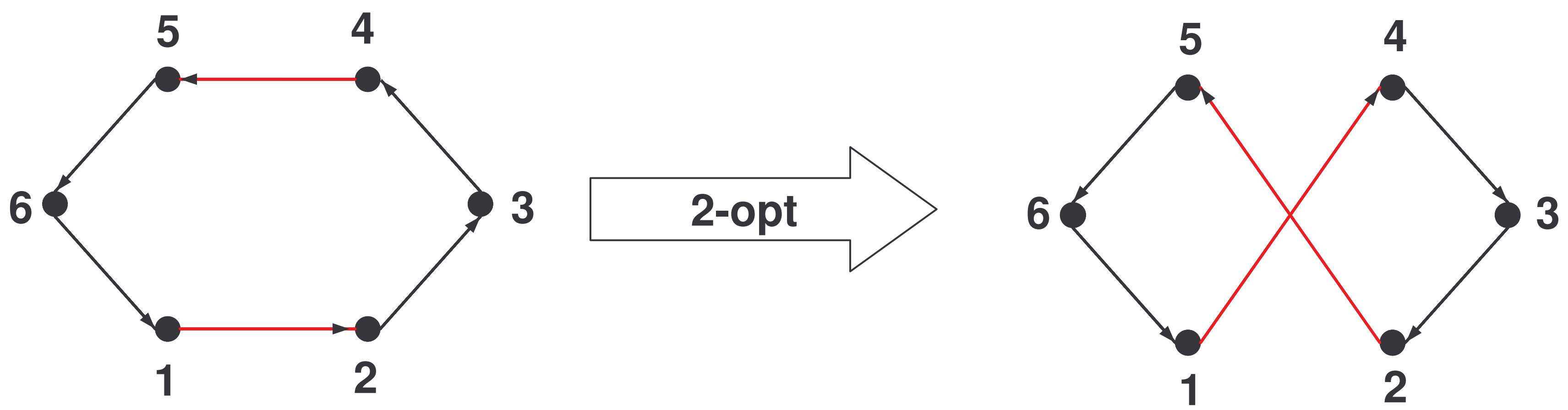
Local Search for TSP

- Initialization: Complete tour
- In each step, move from current solution to neighboring solution



K-exchange neighborhood

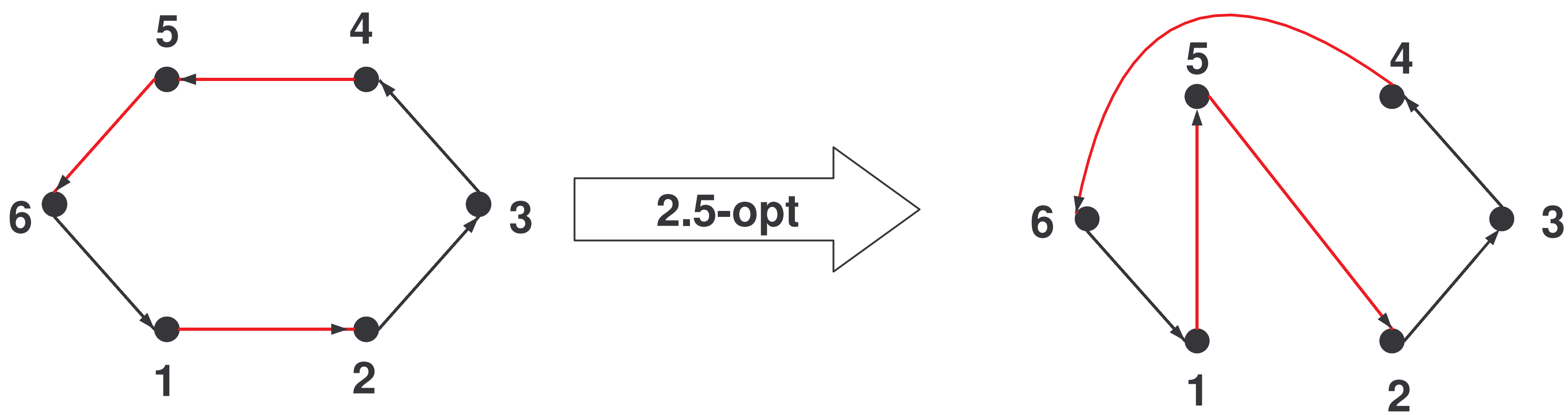
- Solution differ by atmost *k* components
- 2-opt, 2.5-opt, 3-opt



Delta=(D[1][2]+D[4][5])-(D[1][4]+D[2][5])

K-exchange neighborhood

- Solution differ by atmost *k* components
- 2-opt, 2.5-opt, 3-opt

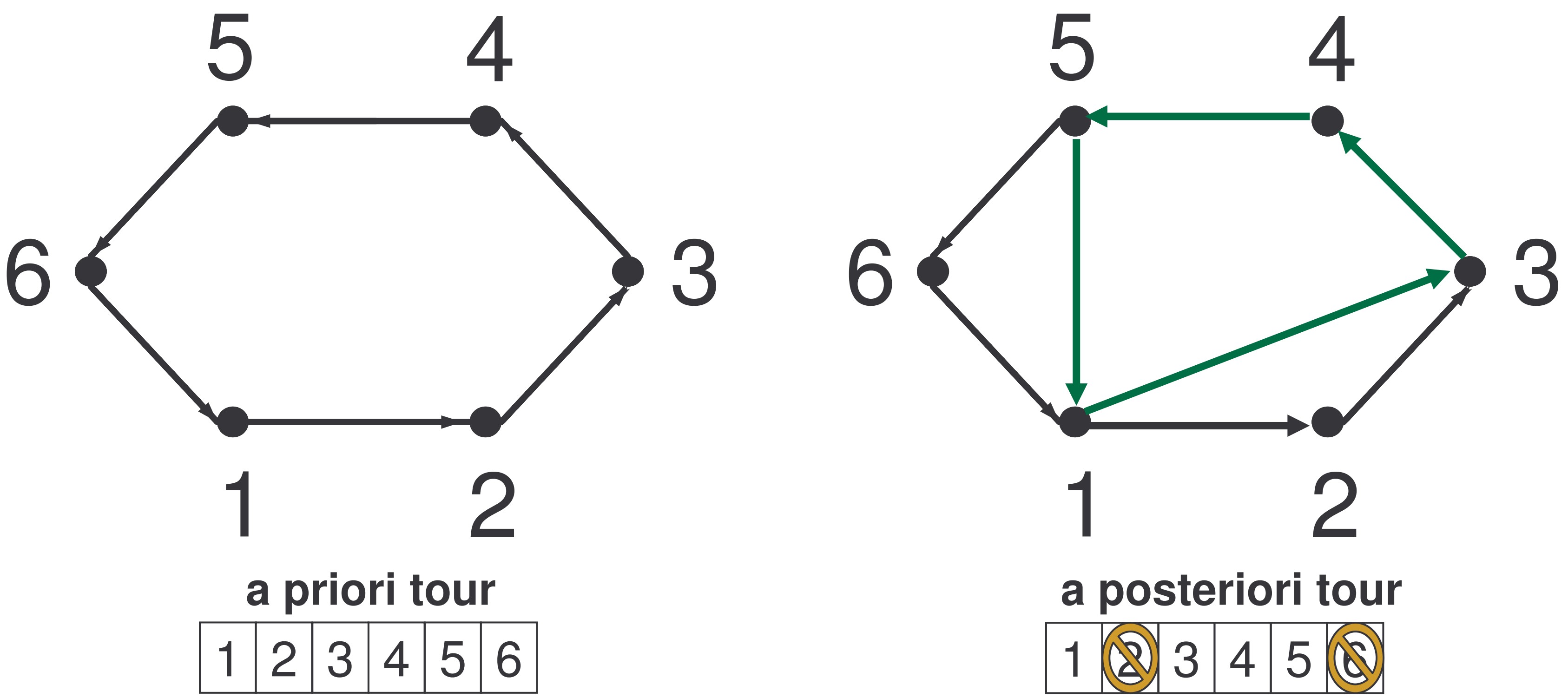


Delta=(D[1][2]+D[4][5]+D[5][6])-(D[1][5]+D[5][2]+D[4][6])

Probabilistic TSP

- PTSP as a stochastic CO problem
- Each city has a probability of requiring a visit
- Homogeneous PTSP
- Solution by *apriori* optimization

a priori Optimization



Goal: To find an apriori tour of minimal expected a posteriori tour length

Empirical estimation of the costs

- Formal definition of a stochastic CO problem
  - *x* : solution
  - S : set of feasible solutions
  - E : Mathematical expectation
  - ω : random influence
  - f (*x*,ω) : random cost function on *x* and ω
  - *F(x) :* expected cost of *x*

$$\text{Minimize } F(x) = E[f(x,\omega)], \text{ subject to } x \in S$$

  - Estimation by Sampling

$$\text{Minimize } \widehat{F_M}(x) = \frac{1}{M} \Sigma_{i=0}^{M} f(x,\omega_i)$$

$\widehat{F_M}(x)$ is an *unbiased estimator* of $F(x)$

2-opt Delta by Approximation

- 1-opt move for each city – swapping of two consecutive cities

$$\Delta E_{i,i+1} = p^3[q^{-1}A_{i,2} - (B_{i,1} - B_{i,n-1}) - (A_{i+1,1} - A_{i+1,n-1}) + q^{-1}B_{i+1,2}].$$

- Make a 2-opt move
  - Compute delta

$$\begin{aligned}\Delta E_{i,i+k} = \Delta E_{i+1,i+k-1} + p^2[&(q^{-k} - 1)A_{i,k+1} + (q^k - 1)(B_{i,1} - B_{i,n-k})\\ &+ (q^k - 1)(A_{i+k,1} - A_{i+k,n-k}) + (q^{-k} - 1)B_{i+k,k+1}\\ &+ (1 - q^{n-k})(A_{i,1} - A_{i,k}) + (1 - q^{k-n})B_{i,n-k+1}\\ &+ (1 - q^{k-n})A_{i+k,n-k+1} + (1 - q^{n-k})(B_{i+k,1} - B_{i+k,k})].\end{aligned}$$

  - If improvement found, make 1-opt move

2.5-opt Delta by Approximation

- 1-opt move for each city – swapping of two consecutive cities

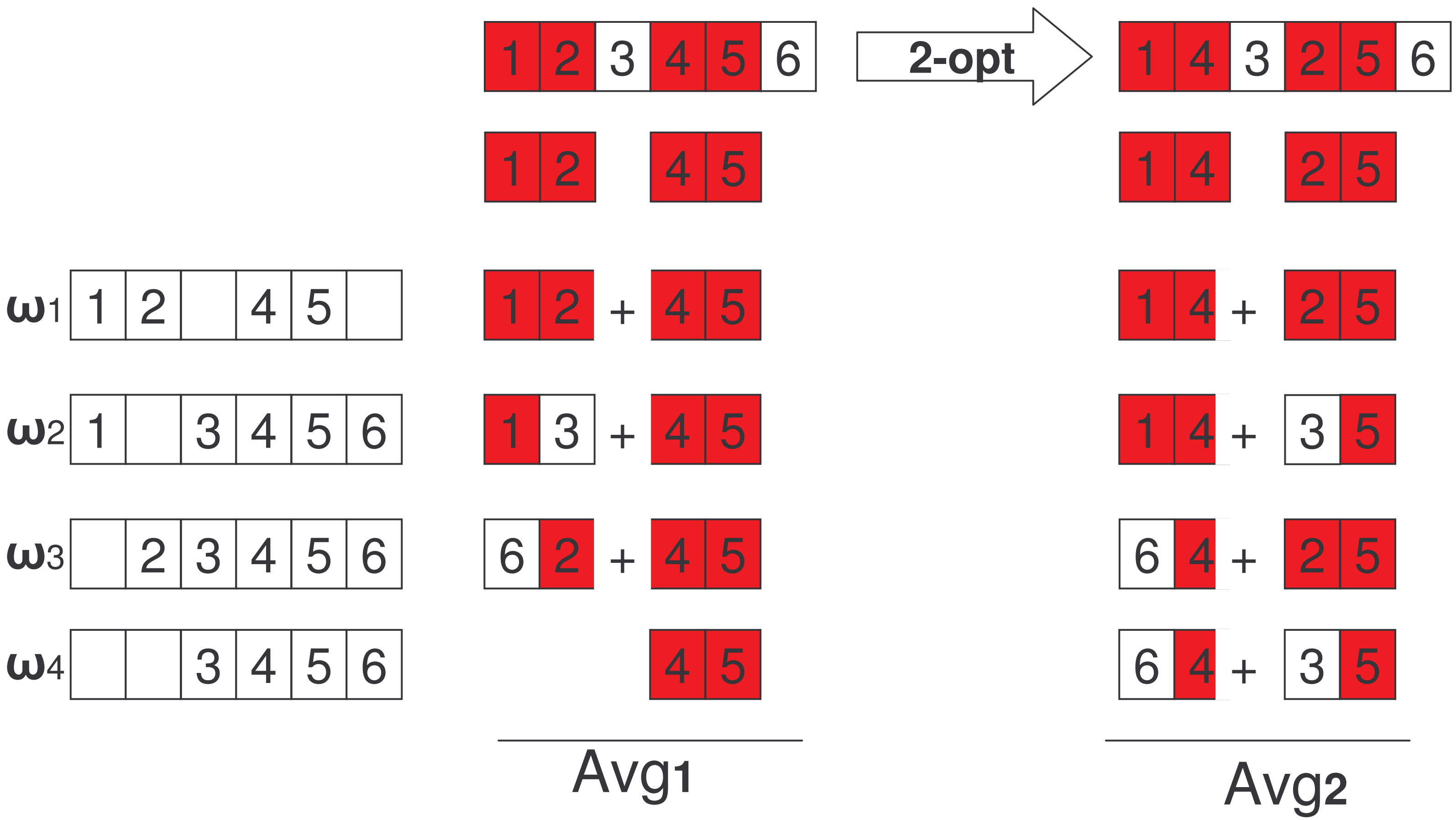
$$\Delta E_{i,i+1} = p^3[q^{-1}A_{i,2} - (B_{i,1} - B_{i,n-1}) - (A_{i+1,1} - A_{i+1,n-1}) + q^{-1}B_{i+1,2}].$$

- Make a 2.5-opt move
  - Compute delta

$$\begin{aligned}\Delta' E_{i,i+k} = \Delta' E_{i,i+k-1} + p^2[&(q^{n-k} - q^{-k})(qA_{i,k} - A_{i,k+1})\\ &+(q^{k-n} - q^{k-1})(qB_{i,n-k} - B_{i,n-k+1})\\ &+(1 - q^{-1})(q^k B_{i,1} - B_{i+k,k+1})\\ &+(q^{-1} - 1)(q^{n-k} A_{i,1} - A_{i+k,n-k+1})\\ &+(q - 1)(A_{i+k,1} - A_{i+k,n-k})\\ &+(1 - q)(B_{i+k,1} - B_{i+k,k})].\end{aligned}$$

  - If improvement found, make 1-opt move

Delta Estimation by Sampling



Delta = Avg1-Avg2

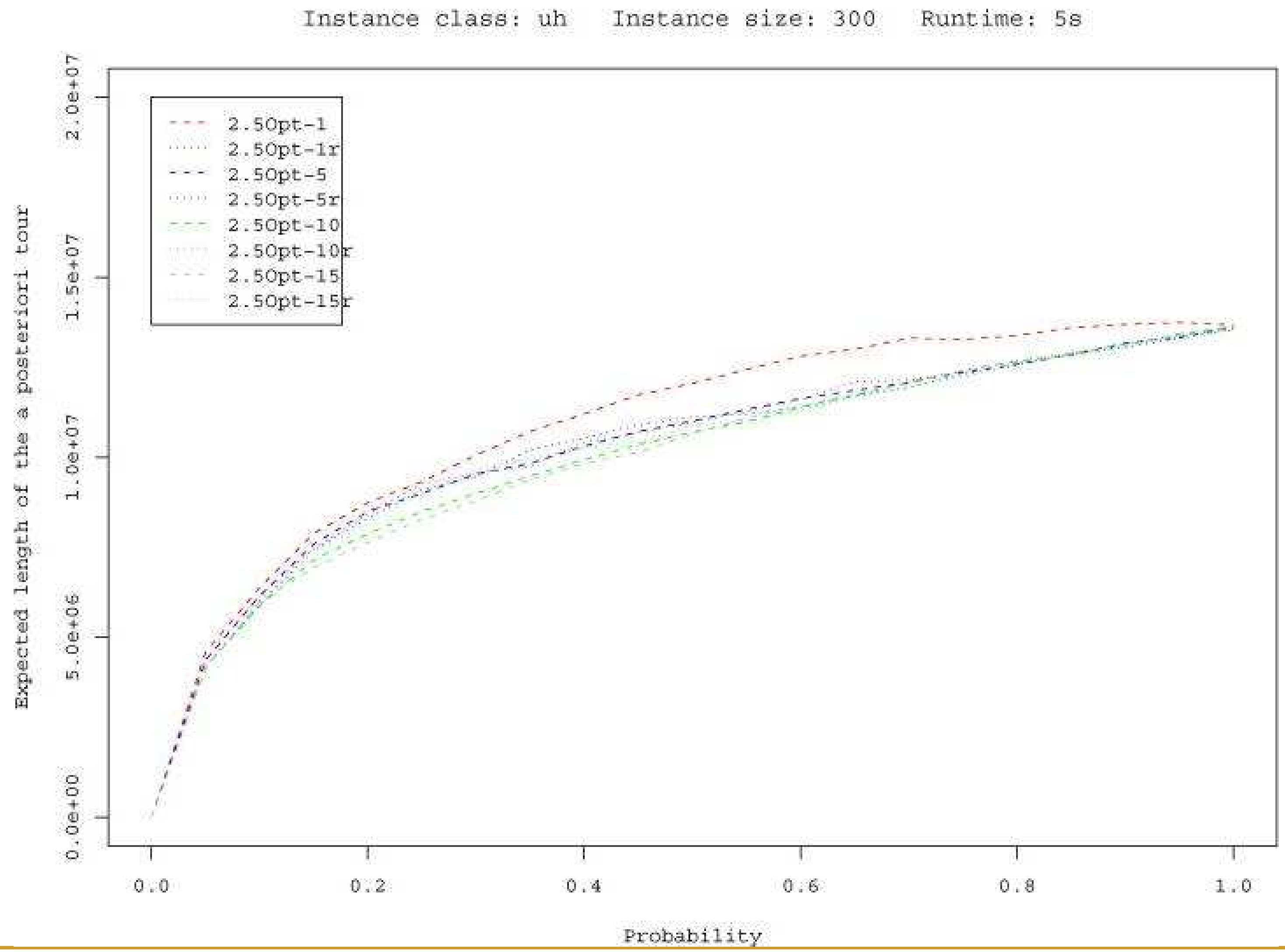
Delta Estimation by Sampling

- 2-opt and 2.5-opt local search
- First improvement algorithms
- Nearest neighbor exploration and don‘t look bits as a speed up
- Computational complexity
  - Sampling: O(n) – worst case, O(1) – best case, times no. of samples
  - Approximation: $O(n^2)$

Experimental setup

- Homogenous PTSP from DIMACS TSP generator
- Each TSP x {0.05,0.10,0.15,.......0.95,1.0 } ⇨ 20 PTSP instances
- Initialization by nearest neighbor heuristic
- Number of nearest neighbor = 20
- Solution quality evaluation with 300 freshly generated realizations

Results on the Number of Realizations



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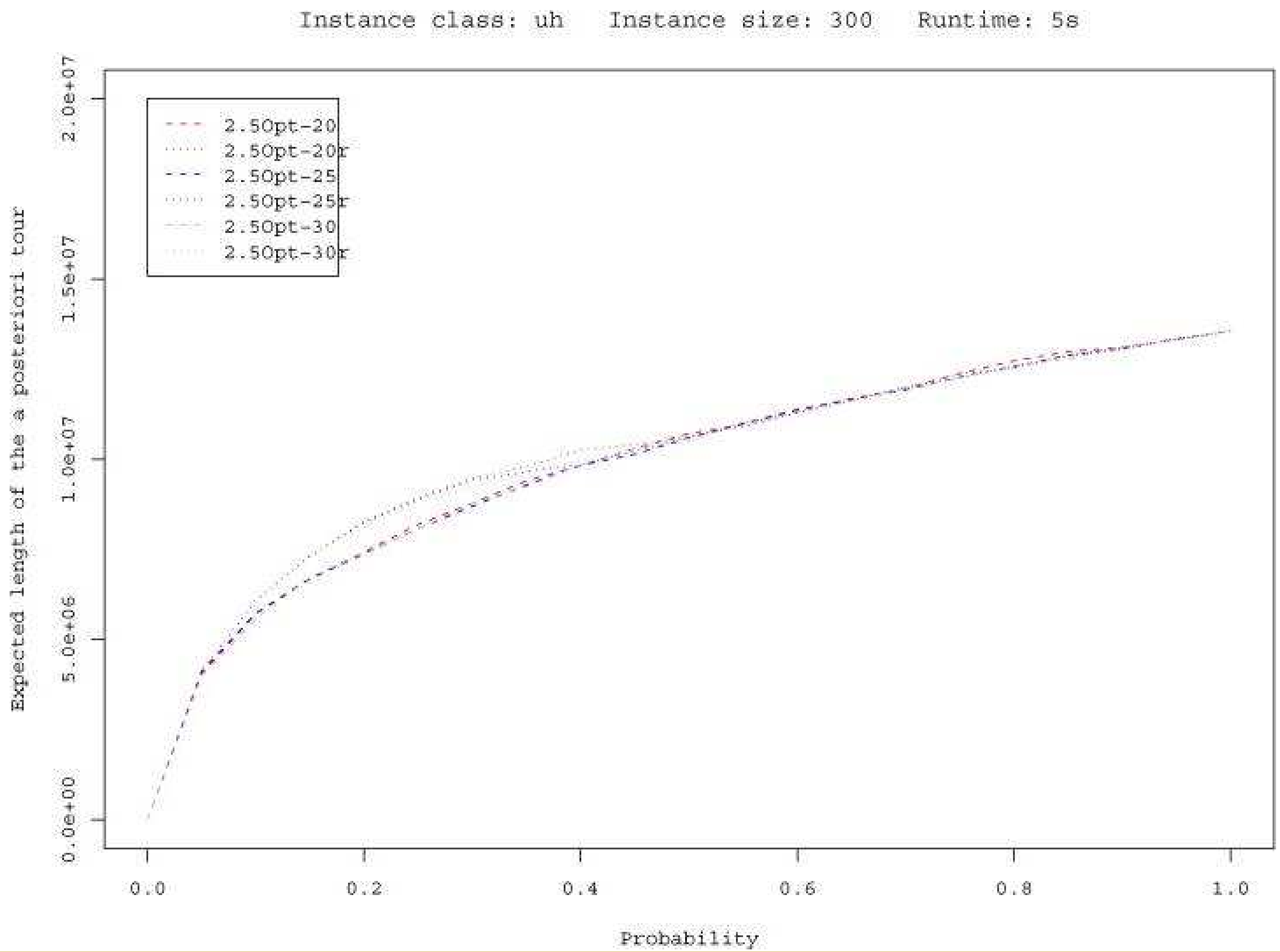
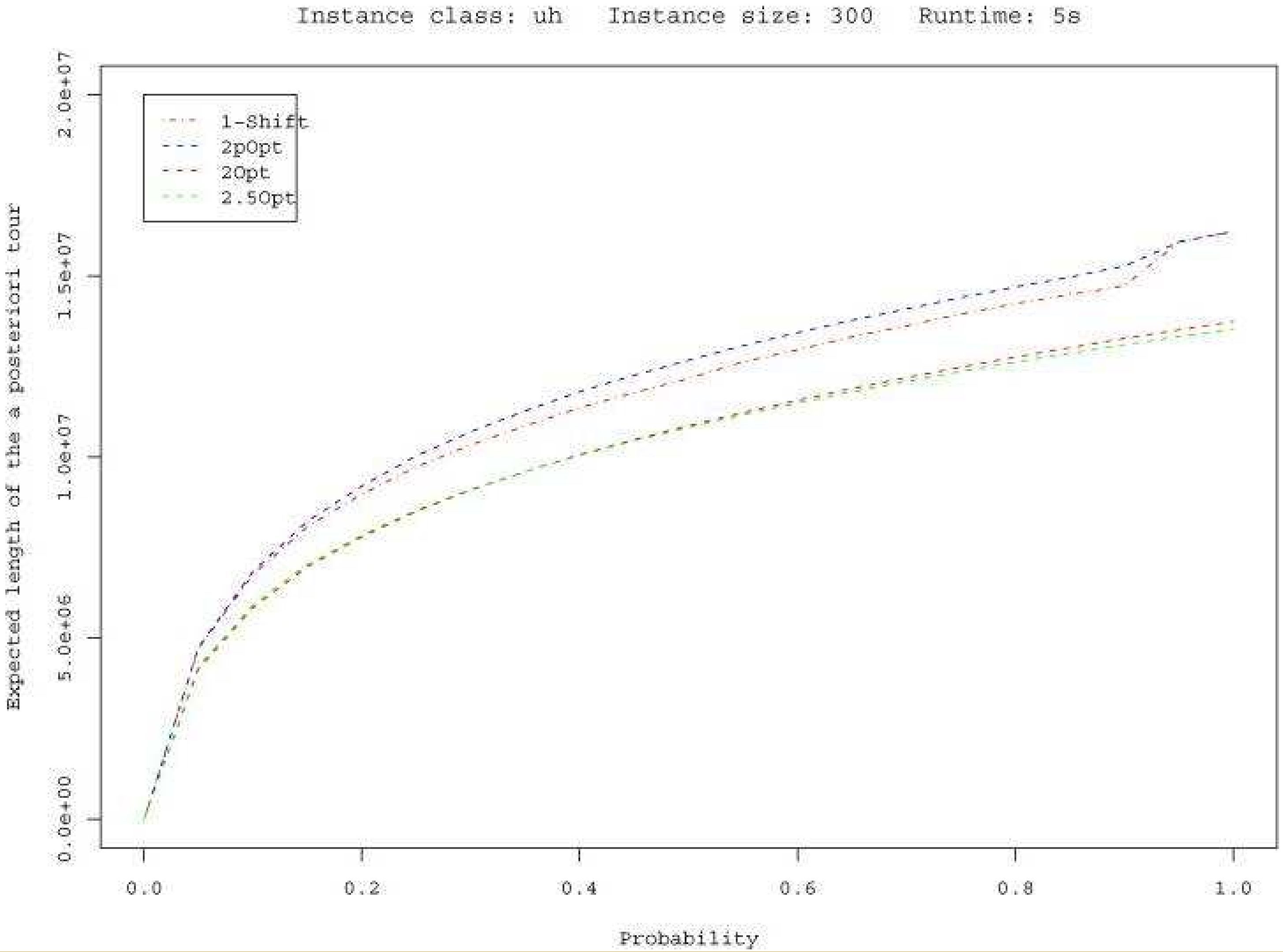


Results on the Delta Evaluation Strategy



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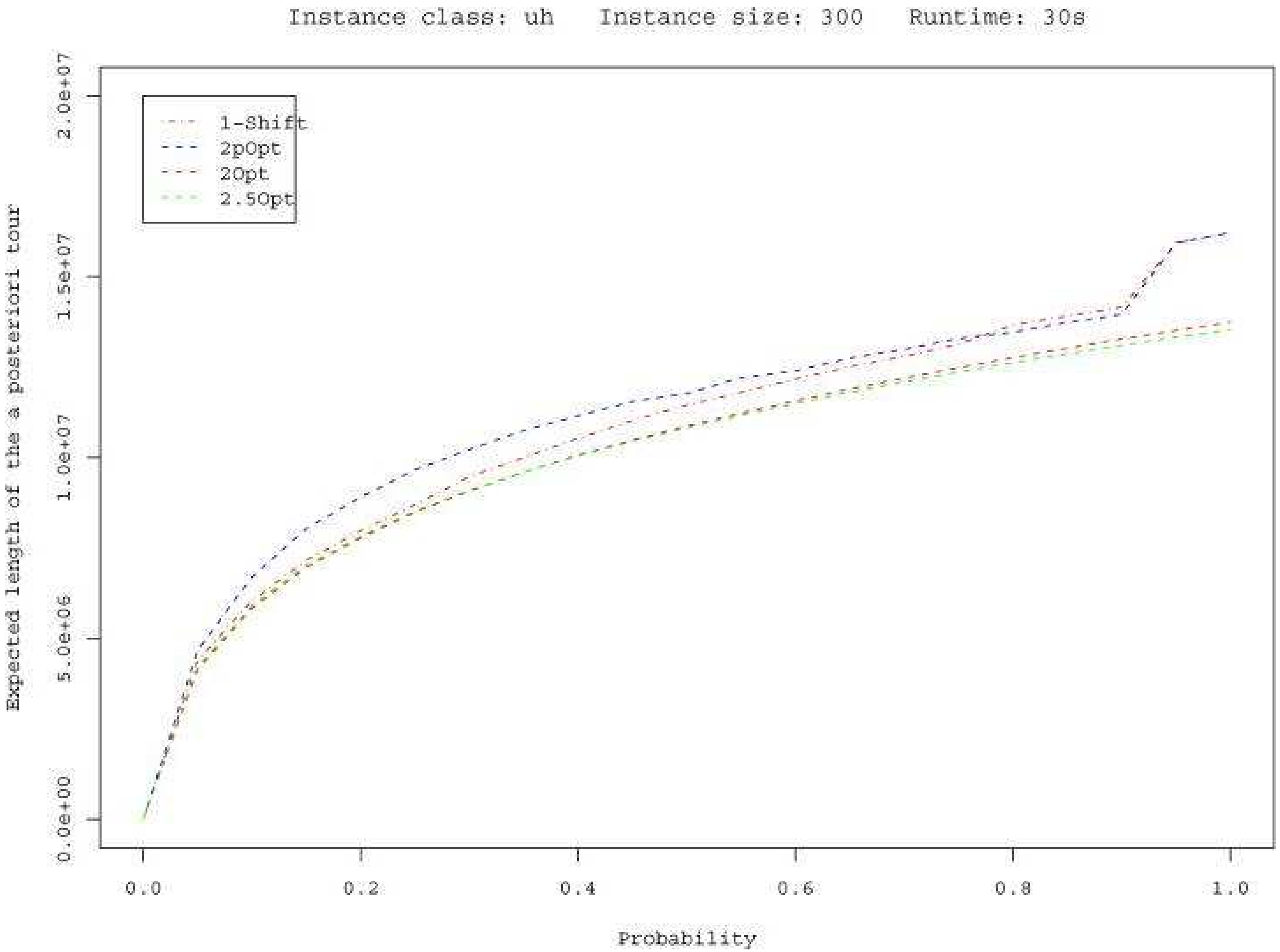

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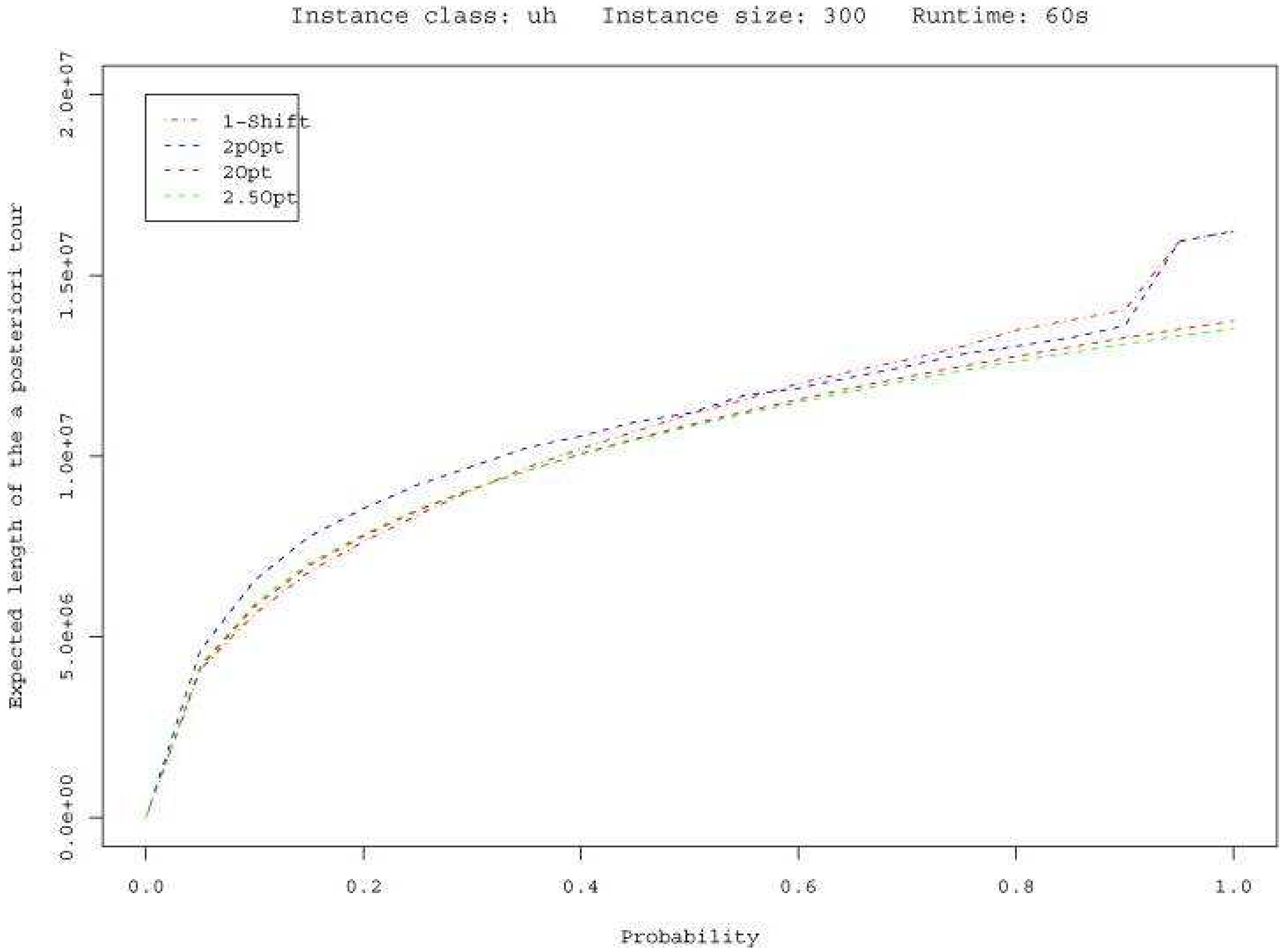

Conclusion

- Local search based on sampling technique
- Generality of the empirical estimation
- Heterogenous PTSP without any modification
- Empirical estimation performs better than analytical approximation

Work in Progress

- Tuning each algorithm and to carry out final experiments
- First Vs. Best improvement
- Nearest neighbor and don‘t look bit speed up for approximation
- Exhaustive Vs. Nearest neighbor and don‘t look bit
- Nearest neighbor sampling Vs. Nearest neighbor approximation

Future Work

- Implementation of approximation local search for heterogenous PTSP
- Experiments on Heterogenous PTSP
- Coupling empirical local search with ACO/F-Race
- Unified ACO framework (pACS, SACO, ACO/F-Race) with local search (approximation and empirical) for PTSP
- Application to realistic problems such as vehicle routing