

```

In[1]:= (* ----- Majority Rule Group Size G odd ----- *)
(* Parameters setting and initial conditions *)
G = 3;
NS = G - 1;
ρA = 1;
α = (ρA * g) ^ -1;
β = (ρB * g) ^ -1;

In[6]:= (* pXY:probability that an agent X switch opinion
towards Y as a result of the Majority Rule group size G=NS+1 *)

pA :=  $\frac{d_A}{(d_A + d_B)}$ ;
pB :=  $\frac{d_B}{(d_A + d_B)}$ ;

(* pAA := Sum[NS!/(i!*(NS-i)!)*pA^i*pB^(NS-i),{i,Floor[NS/2],NS}];*)
pBA :=  $\sum_{i=0}^{\text{Floor}[\frac{NS}{2}]-1} \frac{NS!}{i! * (NS - i)!} * p_A^{(NS-i)} * p_B^i$ ;
pAB :=  $\sum_{i=0}^{\text{Floor}[\frac{NS}{2}]-1} \frac{NS!}{i! * (NS - i)!} * p_A^i * p_B^{(NS-i)}$ ;

pAA := 1 - pAB;
pBB := 1 - pBA;

In[12]:= (* Definition of the Steady State System *)
steadySystem = {
σ * eA - α * dA == 0,
σ * eB - β * dB == 0,
pAA * α * dA + pBA * β * dB - σ * eA == 0,
pAB * α * dA + pBB * β * dB - σ * eB == 0};
conditions = {
dA + dB + eA + eB - 1 == 0,
dA ≥ 0,
dB ≥ 0,
eA ≥ 0,
eB ≥ 0,
σ > 0,
g > 0,
1 > ρB > 0};
variables = {dA, dB, eA, eB};

In[15]:= (* Compute steady states *)
rule = ConditionalExpression[a_, b_] → a;
equilibria =
Simplify[Solve[Rationalize[Join[steadySystem, conditions]], variables] /. rule];
Print["There are ", Length[equilibria], " steady states"];
Print[equilibria];

```

There are 3 steady states

$$\left\{ \left\{ d_A \rightarrow \frac{g \sigma}{1 + g \sigma}, d_B \rightarrow 0, e_A \rightarrow \frac{1}{1 + g \sigma}, e_B \rightarrow 0 \right\}, \right. \\ \left. \left\{ d_A \rightarrow 0, d_B \rightarrow \frac{g \sigma \rho_B}{1 + g \sigma \rho_B}, e_A \rightarrow 0, e_B \rightarrow \frac{1}{1 + g \sigma \rho_B} \right\}, \left\{ d_A \rightarrow \frac{g \sigma \rho_B^2}{1 + g \sigma \rho_B + (1 + g \sigma) \rho_B^2}, \right. \right. \\ \left. \left. d_B \rightarrow \frac{g \sigma \rho_B}{1 + g \sigma \rho_B + (1 + g \sigma) \rho_B^2}, e_A \rightarrow \frac{\rho_B^2}{1 + g \sigma \rho_B + (1 + g \sigma) \rho_B^2}, e_B \rightarrow \frac{1}{1 + g \sigma \rho_B + (1 + g \sigma) \rho_B^2} \right\} \right\}$$

```
In[19]:= (* Compute eigenvalues *)
steadySystemLHS = steadySystem /. Equal[x_, y_] -> x;
jac = Outer[D, steadySystemLHS, variables];
eigenValues = Map[Eigenvalues[jac /. #] &, equilibria];

In[24]:= (* Stability analysis *)
For[eq = 1, eq < Length[eigenValues] + 1, eq++,
  ev = eigenValues[[eq]];
  Print["Equilibrium[" , eq, "]:", equilibria[[eq]];
  Print["Eigenvalues:", ev];
  For[evIt = 2, evIt < Length[ev] + 1, evIt++,
    eval = Rationalize[ev[[evIt]]];
    conditions = {FullSimplify[eval] < 0, σ > 0, g > 0, 1 > ρ_B > 0};
    variables = {σ, g, ρ_B};
    red = Reduce[conditions, variables];
    Print["Condition satisfied(ev[" , evIt, "]<0): ", red];];
  Print["-----"];
];
```

$$\text{Equilibrium}[1]: \left\{ d_A \rightarrow \frac{g \sigma}{1 + g \sigma}, d_B \rightarrow 0, e_A \rightarrow \frac{1}{1 + g \sigma}, e_B \rightarrow 0 \right\}$$

$$\text{Eigenvalues}: \left\{ -\frac{1}{g \rho_B}, 0, -\sigma, \frac{-\rho_B - g \sigma \rho_B}{g \rho_B} \right\}$$

Condition satisfied(ev[2]<0): False

Condition satisfied(ev[3]<0):  $\sigma > 0 \ \&\& \ g > 0 \ \&\& \ 0 < \rho_B < 1$

Condition satisfied(ev[4]<0):  $\sigma > 0 \ \&\& \ g > 0 \ \&\& \ 0 < \rho_B < 1$

-----

$$\text{Equilibrium}[2]: \left\{ d_A \rightarrow 0, d_B \rightarrow \frac{g \sigma \rho_B}{1 + g \sigma \rho_B}, e_A \rightarrow 0, e_B \rightarrow \frac{1}{1 + g \sigma \rho_B} \right\}$$

$$\text{Eigenvalues}: \left\{ 0, -\frac{1}{g}, -\sigma, \frac{-1 - g \sigma \rho_B}{g \rho_B} \right\}$$

Condition satisfied(ev[2]<0):  $\sigma > 0 \ \&\& \ g > 0 \ \&\& \ 0 < \rho_B < 1$

Condition satisfied(ev[3]<0):  $\sigma > 0 \ \&\& \ g > 0 \ \&\& \ 0 < \rho_B < 1$

Condition satisfied(ev[4]<0):  $\sigma > 0 \ \&\& \ g > 0 \ \&\& \ 0 < \rho_B < 1$

-----









$$\left( \frac{g^3 \sigma^3 \rho_B^9}{\left( (1 + g \sigma \rho_B + \rho_B^2 + g \sigma \rho_B^2)^2 \left( \frac{g \sigma \rho_B}{1 + g \sigma \rho_B + \rho_B^2 + g \sigma \rho_B^2} + \frac{g \sigma \rho_B^2}{1 + g \sigma \rho_B + \rho_B^2 + g \sigma \rho_B^2} \right)^2 \right)} \right) \&, 3 \}$$

Condition satisfied(ev[2]<0):  $\sigma > 0 \ \&\& \ g > 0 \ \&\& \ 0 < \rho_B < 1$

Condition satisfied(ev[3]<0):  $\sigma > 0 \ \&\& \ g > 0 \ \&\& \ 0 < \rho_B < 1$

Condition satisfied(ev[4]<0): False

-----