Ant Colony Optimization for Mixed-Variable Optimization Problems: Supplementary Pages

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I. The best solutions found by $\mathrm{ACO}_{\mathrm{MV}}$ in the engineering optimization problems

A. *Welded beam design problem case A*

Please see Table I.

B. *Pressure vessel design problem case A, B, C*

Please see Table II.

C. *Pressure vessel design problem case D*

Please see Table III.

D. *Coil spring design problem*

Please see Table IV.

E. *Thermal insulation systems design problem*

Please see Table V.

F. *Welded beam design problem case B*

Please see Table VI.

II. CPU Timing experiment of $\mathrm{ACO}_{\mathrm{MV}}$ in the engineering optimization problems

We run $\mathrm{ACO}_{\mathrm{MV}}$ on each engineering optimization problem until 1000 function evaluations have passed. These experiments were conducted with Pentium(R) Dual-Core CPU E6300(2.80 GHz) on Linux (kernel 2.6.32). Over five independent runs, we compute the average CPU time per function evaluation on each engineering optimization problem. Please see Table VII.

III. The comparative studies about two $\text{ACO}_{\mathbf{MV}}$ variants with Equation (9) and (10) that directly use the fine-tuned parameter values of $\text{ACO}_{\mathbf{MV}}$ with Equation (6)

The comparative studies about two $\text{ACO}_{\mathbf{MV}}$ variants with Equation (9) and (10) that directly use the fine-tuned parameter values of $\text{ACO}_{\mathbf{MV}}$ with Equation (6). See Fig. 1 and 2.

IV. The comparative studies about five different scales of q values in Equation (6)

The comparative studies about five different scales of q values in Equation (6). See Fig. 3

TABLE I

THE BEST SOLUTIONS FOR WELDED BEAM DESIGN PROBLEM CASE A

Methods	$x_1(h)$	$x_2(l)$	$x_3(t)$	$x_4(b)$	$f(x)$
GA1 [1]	0.208800	3.420500	8.997500	0.210000	1.748309
GA2 [2]	0.205986	3.471328	9.020224	0.206480	1.728226
EP [3]	0.205700	3.470500	9.036600	0.205700	1.724852
$(\mu+\lambda)$ES [4]	0.205730	3.470489	9.036624	0.205729	1.724852
CPSO [5]	0.202369	3.544214	9.048210	0.205723	1.728024
HPSO [6]	0.205730	3.470489	9.033624	0.205730	1.724852
CLPSO [7]	0.205730	3.470489	9.033624	0.205730	1.724852
DELC [8]	0.205730	3.470489	9.033624	0.205730	1.724852
ABC [9]	0.205730	3.470489	9.033624	0.205730	1.724852
ACO$_{\mathbf{MV}}$	0.205729	3.470489	9.033624	0.205730	1.724852

TABLE II

THE BEST SOLUTIONS FOR PRESSURE VESSEL DESIGN PROBLEM CASE A, B AND C

Methods	$x_1(T_s)$	$x_2(T_h)$	$x_3(R)$	$x_4(L)$	$f(x)$
In case A: DE [10]	1.100	0.600	56.9948	51.0013	7019.031
In case A: ACO$_{\mathbf{MV}}$	1.100	0.600	56.9948	51.0013	7019.031
In case B: DE [10]	1.125	0.625	58.2902	43.6927	7197.729
In case B: ACO$_{\mathbf{MV}}$	1.125	0.625	58.2902	43.6927	7197.729
In case C: DE [10]	1.000	0.625	51.8135	84.5785	7006.358
In case C: ACO$_{\mathbf{MV}}$	1.000	0.625	51.8135	84.5785	7006.358

TABLE III

THE BEST SOLUTIONS FOR PRESSURE VESSELDESIGN PROBLEM CASE D

Methods	$x_1(T_s)$	$x_2(T_h)$	$x_3(R)$	$x_4(L)$	$f(x)$
GA1 [1]	0.8125	0.4375	40.3239	200.0000	6288.7445
GA2 [2]	0.8125	0.4375	42.0974	176.6540	6059.9463
$(\mu+\lambda)$ES [4]	0.8125	0.4375	42.0984	176.6366	6059.7143
CPSO [5]	0.8125	0.4375	42.0913	176.7465	6061.0777
HPSO [6]	0.8125	0.4375	42.0984	176.6366	6059.7143
RSPSO [11]	0.8125	0.4375	42.0984	176.6366	6059.7143
CLPSO [7]	0.8125	0.4375	42.0984	176.6366	6059.7143
DELC [8]	0.8125	0.4375	42.0984	176.6366	6059.7143
ABC [9]	0.8125	0.4375	42.0984	176.6366	6059.7143
ACO$_{\mathbf{MV}}$	0.8125	0.4375	42.0984	176.6366	6059.7143

TABLE IV

THE BEST SOLUTIONS FOR THE COIL SPRING DESIGN PROBLEM

Algs	NLIDP [12]	GA [13]	GA [14]	DE [10]	HSIA [15]	DE [16]	$ACO_{\mathbf{MV}}$
N	10	9	9	9	9	9	9
D [inch]	1.180701	1.2287	1.227411	1.223041	1.223	1.223044	1.223041
d [inch]	0.283	0.283	0.283	0.283	0.283	0.283	0.283
SRf_{Best}	2.7995	2.6709	2.6681	2.65856	2.659	2.658565	2.65856

TABLE V

THE BEST SOLUTIONS FOR THE THERMAL INSULATION SYSTEMS

Solution information	MVP [17]	FMGPS [18]	ACO_{MV}
Continuous variable			
$x_i(cm)$			
1	0.3125	4.5313	4.9506
2	5.4688	6.7188	7.9729
3	3.9062	4.8437	12.8448
4	6.5625	4.2188	17.07978
5	5.7812	7.3438	9.4420
6	5.1562	9.8438	10.1077
7	13.2812	24.948	0.02811
8	21.4062	12.135	7.3080
9	8.5938	7.5	11.9592
10	9.2188	6.4063	12.1872
11	20.3125	11.5105	6.1197
$T_i(K)$			
1	4.2188	6.125	6.1003
2	7.3438	10.55	11.0841
3	10	14.35	21.2509
4	15	17.994	38.2608
5	20	24.969	51.8508
6	25	36.006	70.1000
7	40	71.094	71.0001
8	71.0938	116.88	99.4475
9	101.25	156.88	153.1701
10	146.25	198.44	236.8358
11	300	300	300
Categorical variable			
I_i			
1	N	N	N
2	N	N	N
3	N	N	N
4	N	N	N
5	N	N	T
6	N	N	E
7	N	T	T
8	E	E	E
9	E	E	E
10	E	T	T
11	T	T	T
Power($\frac{PL}{A}(\frac{W}{cm})$)	25.294	25.58	24.299

TABLE VI

THE BEST SOLUTIONS FOR WELDED BEAMDESIGN DESIGN PROBLEM CASE B

Methods	$x_1(h)$	$x_2(l)$	$x_3(t)$	$x_4(b)$	$x_5(M)$	$x_6(Joint)$	$f(x)$
GeneAS [19]	0.1875	1.6849	8.2500	0.2500	Steel	4-sided	1.9422
RSPSO [11]	0.1875	1.6842	8.2500	0.2500	Steel	4-sided	1.9421
PSOA[20]	0.2500	2.2219	8.2500	0.2500	Steel	2-sided	1.7631
CLPSO [7]	0.2500	1.1412	8.2500	0.2500	Steel	4-sided	1.5809
ACO_{MV}	0.2250	1.2724	8.2500	0.2250	Steel	4-sided	1.5029

TABLE VII

SECONDS PER FUNCTION EVALUATION ON THE ENGINEERING OPTIMIZATION PROBLEMS

Welded beam design problem case A	4.94E−03
Pressure vessel design problem case A	4.69E−03
Pressure vessel design problem case B	4.96E−03
Pressure vessel design problem case C	4.81E−03
Pressure vessel design problem case D	5.00E−03
Coil spring design problem	1.32E−03
Thermal insulation systems design problem	3.73E−02
Welded beam design problem case B	5.74E−03

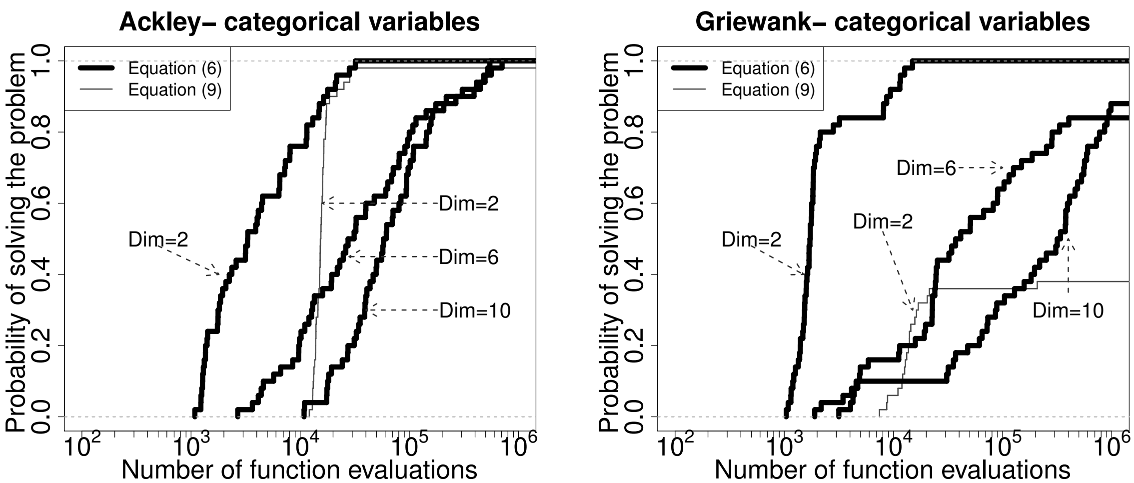

Fig. 1. The RLDs obtained by the two $\mathrm{ACO}_{\mathbf{MV}}$ variants with Equation (6) and (9) in 50 independent runs. The solution quality threshold is 1.00E−10. Dim indicates the dimensionality of the benchmark problem. Half of the dimensions are categorical variables and the other half are continuous variables.

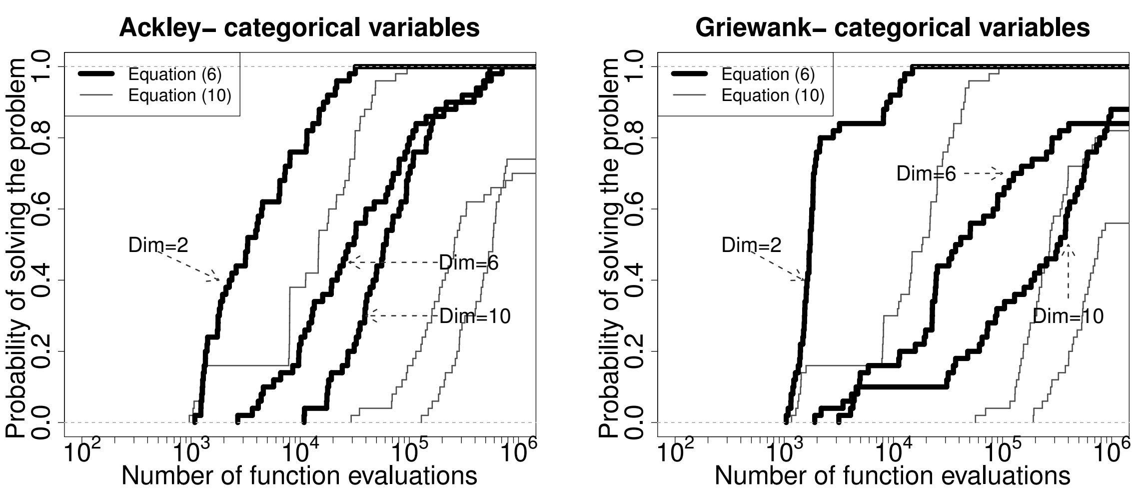

Fig. 2. The RLDs obtained by the two $\mathrm{ACO}_{\mathbf{MV}}$ variants with Equation (6) and (10) in 50 independent runs. The solution quality threshold is 1.00E−10. Dim indicates the dimensionality of the benchmark problem. Half of the dimensions are categorical variables and the other half are continuous variables. The RLDs obtained by $\mathrm{ACO}_{\mathbf{MV}}$ with Equation (10) in dimensions two, six and ten are sequentially shown as the increasing number of function evaluations to solve the problem at the first time.

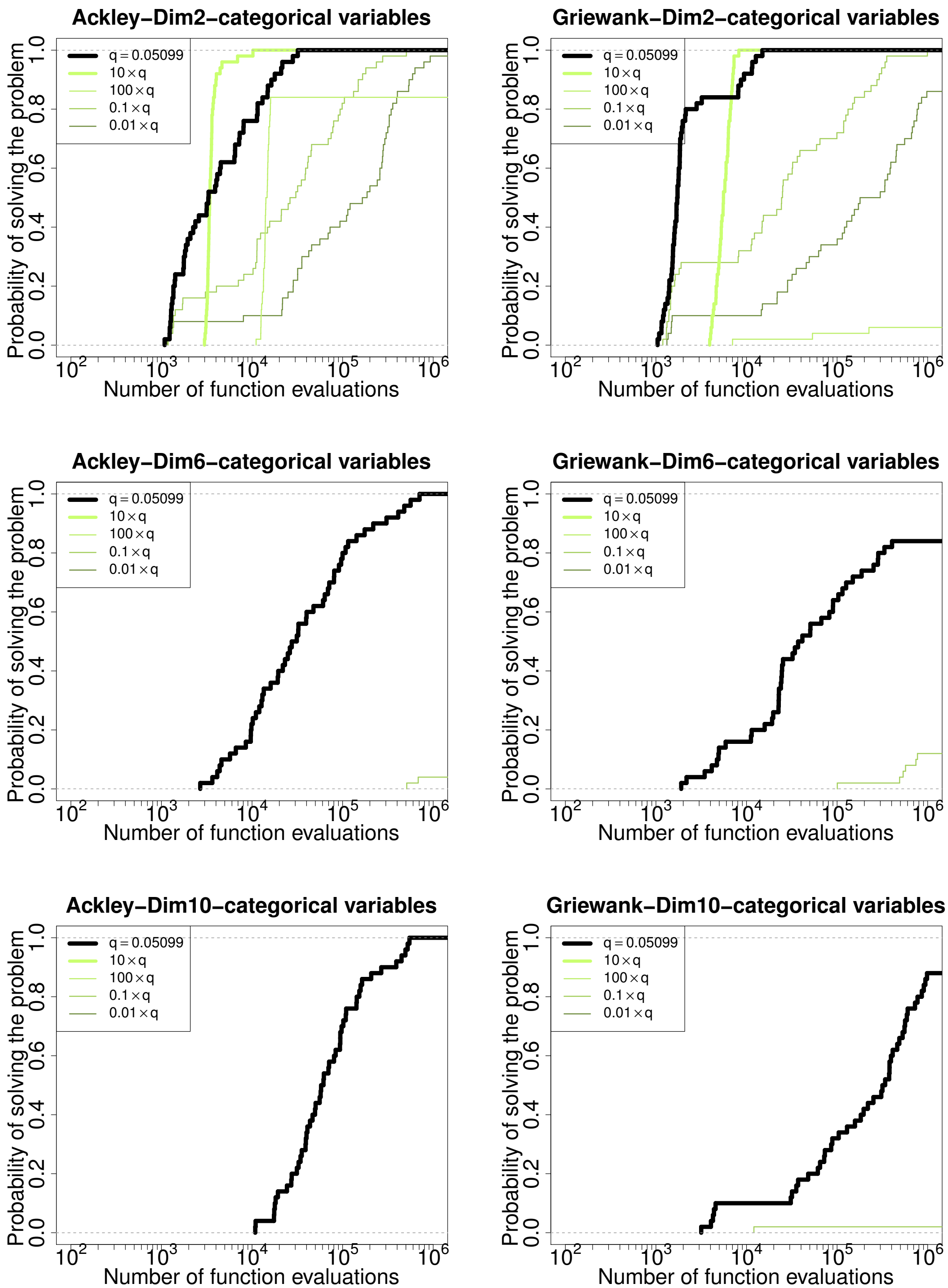

Fig. 3. The RLDs obtained by $\mathrm{ACO_{MV}}$ with five different scales of the q values in 50 independent runs. $q = 0.05099$ is the tuned value used for $\mathrm{ACO_{MV}}$ in the paper. The solution quality threshold is 1.00E−10. Dim indicates the dimensionality of the benchmark problem. Half of the dimensions are categorical variables and the other half are continuous variables.

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