Complete Numerical Results for the Paper Adaptive Sample Size and Importance Sampling in Estimation-based Local Search for the Probabilistic Traveling Salesman Problem

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1 Experiments on *estimation-based* algorithms

Table 1: Experimental results for `2.5-opt-EEas`, `2.5-opt-EEais`, `2.5-opt-EEs-10`, `2.5-opt-EEs-100`, and `2.5-opt-EEs-1000` on clustered instances of size 1000. Each algorithm is allowed to run until it reaches a local optimum. The table gives, for each probability level, the mean and the standard deviation (s.d.) of the final solution cost and the computation time in seconds over 50 instances (Part I).

	Algorithm	Solution Cost		Computation Time	
		mean	s.d.	mean	s.d.
$p = 0.050$	`2.5-opt-EEais`	4020433	437996	11.100	1.794
	`2.5-opt-EEas`	4137855	430945	2.970	0.497
	`2.5-opt-EEs-1000`	4014200	455410	41.104	6.821
	`2.5-opt-EEs-100`	4168788	434760	4.309	0.713
	`2.5-opt-EEs-10`	4713400	491452	0.482	0.035
$p = 0.075$	`2.5-opt-EEais`	4576520	444295	6.215	0.668
	`2.5-opt-EEas`	4701385	440533	2.526	0.310
	`2.5-opt-EEs-1000`	4587142	465301	22.288	2.827
	`2.5-opt-EEs-100`	4713313	439795	3.353	0.396
	`2.5-opt-EEs-10`	5408878	526909	0.541	0.040
$p = 0.100$	`2.5-opt-EEais`	5103869	508867	4.119	0.451
	`2.5-opt-EEas`	5179648	486450	2.230	0.249
	`2.5-opt-EEs-1000`	5108555	503921	14.096	1.972
	`2.5-opt-EEs-100`	5183844	470288	2.629	0.306
	`2.5-opt-EEs-10`	5922935	509627	0.591	0.053
$p = 0.125$	`2.5-opt-EEais`	5530407	500157	3.074	0.341
	`2.5-opt-EEas`	5638332	529218	1.956	0.215
	`2.5-opt-EEs-1000`	5581993	494767	10.111	1.123
	`2.5-opt-EEs-100`	5579436	479260	2.163	0.228
	`2.5-opt-EEs-10`	6414131	534911	0.627	0.051
$p = 0.150$	`2.5-opt-EEais`	5959120	496566	2.495	0.301
	`2.5-opt-EEas`	6050183	505229	1.702	0.161
	`2.5-opt-EEs-1000`	5966002	479174	8.104	1.172
	`2.5-opt-EEs-100`	6007125	500754	1.827	0.187
	`2.5-opt-EEs-10`	6808184	555047	0.648	0.045
$p = 0.175$	`2.5-opt-EEais`	6344650	507027	2.088	0.210
	`2.5-opt-EEas`	6412816	527670	1.545	0.160
	`2.5-opt-EEs-1000`	6356009	504451	6.575	0.834
	`2.5-opt-EEs-100`	6364219	516539	1.575	0.146
	`2.5-opt-EEs-10`	7121641	607449	0.667	0.049
$p = 0.200$	`2.5-opt-EEais`	6701562	545366	1.776	0.147
	`2.5-opt-EEas`	6734587	558760	1.407	0.120
	`2.5-opt-EEs-1000`	6720197	543464	5.596	0.574
	`2.5-opt-EEs-100`	6758117	563812	1.349	0.114
	`2.5-opt-EEs-10`	7416077	612763	0.661	0.053

Table 2: Experimental results for `2.5-opt-EEas`, `2.5-opt-EEais`, `2.5-opt-EEs-10`, `2.5-opt-EEs-100`, and `2.5-opt-EEs-1000` on clustered instances of size 1000. Each algorithm is allowed to run until it reaches a local optimum. The table gives, for each probability level, the mean and the standard deviation (s.d.) of the final solution cost and the computation time in seconds over 50 instances (Part II).

	Algorithm	Solution Cost		Computation Time	
		mean	s.d.	mean	s.d.
$p = 0.300$	`2.5-opt-EEais`	7894796	576228	1.176	0.096
	`2.5-opt-EEas`	7905909	604695	1.015	0.070
	`2.5-opt-EEs-1000`	7921041	559782	3.576	0.342
	`2.5-opt-EEs-100`	7904281	606021	0.918	0.070
	`2.5-opt-EEs-10`	8250174	596289	0.624	0.046
$p = 0.400$	`2.5-opt-EEais`	8801025	619952	0.901	0.075
	`2.5-opt-EEas`	8830392	623723	0.820	0.075
	`2.5-opt-EEs-1000`	8841115	629460	2.733	0.312
	`2.5-opt-EEs-100`	8840586	636550	0.744	0.060
	`2.5-opt-EEs-10`	9004327	620836	0.553	0.041
$p = 0.500$	`2.5-opt-EEais`	9554051	683756	0.762	0.056
	`2.5-opt-EEas`	9557156	666743	0.693	0.050
	`2.5-opt-EEs-1000`	9611587	694012	2.202	0.202
	`2.5-opt-EEs-100`	9603635	665133	0.647	0.044
	`2.5-opt-EEs-10`	9689039	650254	0.498	0.029

Table 3: The p-values of the pairwise comparisons of `2.5-opt-EEas`, `2.5-opt-EEais`, `2.5-opt-EEs-10`, `2.5-opt-EEs-100`, and `2.5-opt-EEs--1000` on clustered instances of size 1000 for probability levels less than 0.5. Values in bold mean that the algorithm in the row performs significantly better than the algorithm in the column, while values in italic mean that the algorithm in the column performs significantly better than the algorithm in the row.

		p-values				
		2.5-opt-EEais	2.5-opt-EEas	2.5-opt-EEs-1000	2.5-opt-EEs-100	2.5-opt-EEs-10
$p = 0.050$	2.5-opt-EEais	-	**0.000**	0.678	**0.000**	**0.000**
	2.5-opt-EEas	*0.000*	-	*0.000*	0.425	**0.000**
	2.5-opt-EEs-1000	0.678	**0.000**	-	**0.000**	**0.000**
	2.5-opt-EEs-100	*0.000*	0.425	*0.000*	-	**0.000**
	2.5-opt-EEs-10	*0.000*	*0.000*	*0.000*	*0.000*	-
		2.5-opt-EEais	2.5-opt-EEas	2.5-opt-EEs-1000	2.5-opt-EEs-100	2.5-opt-EEs-10
$p = 0.075$	2.5-opt-EEais	-	**0.000**	0.951	**0.000**	**0.000**
	2.5-opt-EEas	*0.000*	-	*0.000*	0.951	**0.000**
	2.5-opt-EEs-1000	0.951	**0.000**	-	**0.000**	**0.000**
	2.5-opt-EEs-100	*0.000*	0.951	*0.000*	-	**0.000**
	2.5-opt-EEs-10	*0.000*	*0.000*	*0.000*	*0.000*	-
		2.5-opt-EEais	2.5-opt-EEas	2.5-opt-EEs-1000	2.5-opt-EEs-100	2.5-opt-EEs-10
$p = 0.100$	2.5-opt-EEais	-	**0.001**	1.000	**0.005**	**0.000**
	2.5-opt-EEas	*0.001*	-	*0.005*	1.000	**0.000**
	2.5-opt-EEs-1000	1.000	**0.005**	-	**0.009**	**0.000**
	2.5-opt-EEs-100	*0.005*	1.000	*0.009*	-	**0.000**
	2.5-opt-EEs-10	*0.000*	*0.000*	*0.000*	*0.000*	-
		2.5-opt-EEais	2.5-opt-EEas	2.5-opt-EEs-1000	2.5-opt-EEs-100	2.5-opt-EEs-10
$p = 0.125$	2.5-opt-EEais	-	**0.000**	0.101	0.101	**0.000**
	2.5-opt-EEas	*0.000*	-	0.101	0.101	**0.000**
	2.5-opt-EEs-1000	0.101	0.101	-	0.922	**0.000**
	2.5-opt-EEs-100	0.101	0.101	0.922	-	**0.000**
	2.5-opt-EEs-10	*0.000*	*0.000*	*0.000*	*0.000*	-
		2.5-opt-EEais	2.5-opt-EEas	2.5-opt-EEs-1000	2.5-opt-EEs-100	2.5-opt-EEs-10
$p = 0.150$	2.5-opt-EEais	-	**0.001**	0.707	**0.034**	**0.000**
	2.5-opt-EEas	*0.001*	-	*0.000*	0.072	**0.000**
	2.5-opt-EEs-1000	0.707	**0.000**	-	**0.023**	**0.000**
	2.5-opt-EEs-100	*0.034*	0.072	*0.023*	-	**0.000**
	2.5-opt-EEs-10	*0.000*	*0.000*	*0.000*	*0.000*	-
		2.5-opt-EEais	2.5-opt-EEas	2.5-opt-EEs-1000	2.5-opt-EEs-100	2.5-opt-EEs-10
$p = 0.175$	2.5-opt-EEais	-	**0.002**	1.000	1.000	**0.000**
	2.5-opt-EEas	*0.002*	-	*0.029*	0.209	**0.000**
	2.5-opt-EEs-1000	1.000	**0.029**	-	1.000	**0.000**
	2.5-opt-EEs-100	1.000	0.209	1.000	-	**0.000**
	2.5-opt-EEs-10	*0.000*	*0.000*	*0.000*	*0.000*	-
		2.5-opt-EEais	2.5-opt-EEas	2.5-opt-EEs-1000	2.5-opt-EEs-100	2.5-opt-EEs-10
$p = 0.200$	2.5-opt-EEais	-	0.630	0.639	0.106	**0.000**
	2.5-opt-EEas	0.630	-	0.639	0.639	**0.000**
	2.5-opt-EEs-1000	0.639	0.639	-	0.630	**0.000**
	2.5-opt-EEs-100	0.106	0.639	0.630	-	**0.000**
	2.5-opt-EEs-10	*0.000*	*0.000*	*0.000*	*0.000*	-
		2.5-opt-EEais	2.5-opt-EEas	2.5-opt-EEs-1000	2.5-opt-EEs-100	2.5-opt-EEs-10
$p = 0.300$	2.5-opt-EEais	-	1.000	1.000	1.000	**0.000**
	2.5-opt-EEas	1.000	-	1.000	1.000	**0.000**
	2.5-opt-EEs-1000	1.000	1.000	-	1.000	**0.000**
	2.5-opt-EEs-100	1.000	1.000	1.000	-	**0.000**
	2.5-opt-EEs-10	*0.000*	*0.000*	*0.000*	*0.000*	-
		2.5-opt-EEais	2.5-opt-EEas	2.5-opt-EEs-1000	2.5-opt-EEs-100	2.5-opt-EEs-10
$p = 0.400$	2.5-opt-EEais	-	1.000	0.514	1.000	**0.000**
	2.5-opt-EEas	1.000	-	1.000	1.000	**0.000**
	2.5-opt-EEs-1000	0.514	1.000	-	1.000	**0.000**
	2.5-opt-EEs-100	1.000	1.000	1.000	-	**0.000**
	2.5-opt-EEs-10	*0.000*	*0.000*	*0.000*	*0.000*	-
		2.5-opt-EEais	2.5-opt-EEas	2.5-opt-EEs-1000	2.5-opt-EEs-100	2.5-opt-EEs-10
$p = 0.500$	2.5-opt-EEais	-	1.000	0.328	0.724	**0.000**
	2.5-opt-EEas	1.000	-	0.153	0.724	**0.000**
	2.5-opt-EEs-1000	0.328	0.153	-	1.000	0.078
	2.5-opt-EEs-100	0.724	0.724	1.000	-	0.078
	2.5-opt-EEs-10	*0.000*	*0.000*	0.078	0.078	-

Table 4: Experimental results for `2.5-opt-EEas`, `2.5-opt-EEais`, `2.5-opt-EEs-10`, `2.5-opt-EEs-100`, and `2.5-opt-EEs-1000` on clustered instances of size 1000. Each algorithm is allowed to run until it reaches a local optimum. The table gives, for each probability level, the mean and the standard deviation (s.d.) of the final solution cost and the computation time in seconds over 50 instances.

	Algorithm	Solution Cost		Computation Time	
		mean	s.d.	mean	s.d.
$p = 0.050 - 16$	2.5-opt-EEais	3949356	409824	13.494	3.379
	2.5-opt-EEas	4119370	461810	1.473	0.207
	2.5-opt-EEs-1000	3984725	441295	38.069	7.455
	2.5-opt-EEs-100	4082128	435499	2.806	0.405
	2.5-opt-EEs-10	4449540	447232	0.418	0.028
$p = 0.050 - 50$	2.5-opt-EEais	3976942	474990	9.440	2.127
	2.5-opt-EEas	4109346	465125	0.759	0.117
	2.5-opt-EEs-1000	3973894	466992	25.631	4.789
	2.5-opt-EEs-100	4039728	470275	1.442	0.239
	2.5-opt-EEs-10	4244445	492875	0.347	0.020
$p = 0.050 - 83$	2.5-opt-EEais	3876139	483153	6.251	2.457
	2.5-opt-EEas	4005424	550905	0.511	0.054
	2.5-opt-EEs-1000	3914341	522037	19.476	3.409
	2.5-opt-EEs-100	3990014	519289	1.033	0.163
	2.5-opt-EEs-10	3996970	531437	0.305	0.011
$p = 0.075 - 16$	2.5-opt-EEais	4522979	470694	6.826	1.229
	2.5-opt-EEas	4646785	437910	1.583	0.178
	2.5-opt-EEs-1000	4537593	470270	24.838	4.220
	2.5-opt-EEs-100	4655615	457299	2.536	0.378
	2.5-opt-EEs-10	5082578	485366	0.473	0.033
$p = 0.075 - 50$	2.5-opt-EEais	4455673	487464	6.199	1.239
	2.5-opt-EEas	4582790	489497	0.797	0.071
	2.5-opt-EEs-1000	4463345	467608	19.308	3.423
	2.5-opt-EEs-100	4521628	476766	1.415	0.210
	2.5-opt-EEs-10	4723969	477824	0.377	0.023
$p = 0.075 - 83$	2.5-opt-EEais	4365460	523685	4.590	1.191
	2.5-opt-EEas	4472406	479261	0.511	0.051
	2.5-opt-EEs-1000	4430461	541723	15.046	2.561
	2.5-opt-EEs-100	4487755	532021	0.942	0.111
	2.5-opt-EEs-10	4520934	492132	0.317	0.013
$p = 0.100 - 16$	2.5-opt-EEais	5020798	490919	4.551	0.597
	2.5-opt-EEas	5144669	476145	1.545	0.201
	2.5-opt-EEs-1000	5040520	514632	17.288	2.159
	2.5-opt-EEs-100	5124233	524701	2.190	0.255
	2.5-opt-EEs-10	5639296	515016	0.506	0.034
$p = 0.100 - 50$	2.5-opt-EEais	4862631	495358	5.108	1.068
	2.5-opt-EEas	5051261	474155	0.795	0.081
	2.5-opt-EEs-1000	4895334	508215	15.428	2.407
	2.5-opt-EEs-100	4963919	524821	1.354	0.158
	2.5-opt-EEs-10	5222524	535871	0.395	0.023
$p = 0.100 - 83$	2.5-opt-EEais	4810587	528808	3.562	0.718
	2.5-opt-EEas	4985247	555381	0.494	0.033
	2.5-opt-EEs-1000	4846572	539555	12.524	2.043
	2.5-opt-EEs-100	4932663	564888	0.886	0.097
	2.5-opt-EEs-10	4987284	584701	0.325	0.012
$p = 0.150 - 16$	2.5-opt-EEais	5887167	466783	2.628	0.311
	2.5-opt-EEas	5991627	484034	1.404	0.178
	2.5-opt-EEs-1000	5872681	496464	9.898	1.385
	2.5-opt-EEs-100	5982149	505366	1.748	0.174
	2.5-opt-EEs-10	6466842	519981	0.554	0.045
$p = 0.150 - 50$	2.5-opt-EEais	5706583	537448	2.922	0.396
	2.5-opt-EEas	5857982	563963	0.835	0.094
	2.5-opt-EEs-1000	5722413	585130	11.167	1.276
	2.5-opt-EEs-100	5802041	595699	1.273	0.148
	2.5-opt-EEs-10	6061067	612648	0.431	0.028
$p = 0.150 - 83$	2.5-opt-EEais	5567535	583671	2.656	0.405
	2.5-opt-EEas	5688096	584020	0.528	0.043
	2.5-opt-EEs-1000	5599325	571210	9.728	1.663
	2.5-opt-EEs-100	5693890	626506	0.834	0.094
	2.5-opt-EEs-10	5756341	591266	0.349	0.017
$p = 0.200 - 16$	2.5-opt-EEais	6589342	537399	1.910	0.203
	2.5-opt-EEas	6641879	547318	1.283	0.118
	2.5-opt-EEs-1000	6601665	537940	6.674	0.890
	2.5-opt-EEs-100	6620425	553190	1.447	0.114
	2.5-opt-EEs-10	7100032	569439	0.579	0.049
$p = 0.200 - 50$	2.5-opt-EEais	6346298	502390	2.105	0.323
	2.5-opt-EEas	6478208	516958	0.835	0.078
	2.5-opt-EEs-1000	6337073	497589	8.004	1.340
	2.5-opt-EEs-100	6414726	509602	1.171	0.124
	2.5-opt-EEs-10	6690508	520244	0.458	0.037
$p = 0.200 - 83$	2.5-opt-EEais	6244761	605180	1.991	0.284
	2.5-opt-EEas	6348075	607690	0.546	0.039
	2.5-opt-EEs-1000	6230550	602679	7.217	1.128
	2.5-opt-EEs-100	6306303	604002	0.811	0.090
	2.5-opt-EEs-10	6375420	613433	0.367	0.020

Table 5: The p-values of the pairwise comparisons of `2.5-opt-EEas`, `2.5-opt-EEais`, `2.5-opt-EEs-10`, `2.5-opt-EEs-100`, and `2.5-opt-EEs--1000` on clustered instances of size 1000 for probability levels less than 0.5. Values in bold mean that the algorithm in the row performs significantly better than the algorithm in the column, while values in italic mean that the algorithm in the column performs significantly better than the algorithm in the row (Part I).

		p-values				
		2.5-opt-EEais	2.5-opt-EEas	2.5-opt-EEs-1000	2.5-opt-EEs-100	2.5-opt-EEs-10
$p = 0.050 - 16$	2.5-opt-EEais	-	**0.000**	**0.039**	**0.000**	**0.000**
	2.5-opt-EEas	*0.000*	-	*0.000*	*0.025*	**0.000**
	2.5-opt-EEs-1000	*0.039*	**0.000**	-	**0.000**	**0.000**
	2.5-opt-EEs-100	*0.000*	**0.025**	*0.000*	-	**0.000**
	2.5-opt-EEs-10	*0.000*	*0.000*	*0.000*	*0.000*	-
		2.5-opt-EEais	2.5-opt-EEas	2.5-opt-EEs-1000	2.5-opt-EEs-100	2.5-opt-EEs-10
$p = 0.050 - 50$	2.5-opt-EEais	-	**0.000**	0.844	**0.007**	**0.000**
	2.5-opt-EEas	*0.000*	-	*0.000*	*0.001*	**0.000**
	2.5-opt-EEs-1000	0.844	**0.000**	-	**0.007**	**0.000**
	2.5-opt-EEs-100	*0.007*	**0.001**	*0.007*	-	**0.000**
	2.5-opt-EEs-10	*0.000*	*0.000*	*0.000*	*0.000*	-
		2.5-opt-EEais	2.5-opt-EEas	2.5-opt-EEs-1000	2.5-opt-EEs-100	2.5-opt-EEs-10
$p = 0.050 - 83$	2.5-opt-EEais	-	**0.001**	0.206	**0.000**	**0.000**
	2.5-opt-EEas	*0.001*	-	*0.017*	1.000	1.000
	2.5-opt-EEs-1000	0.206	**0.017**	-	**0.004**	**0.003**
	2.5-opt-EEs-100	*0.000*	1.000	*0.004*	-	1.000
	2.5-opt-EEs-10	*0.000*	1.000	*0.003*	1.000	-
		2.5-opt-EEais	2.5-opt-EEas	2.5-opt-EEs-1000	2.5-opt-EEs-100	2.5-opt-EEs-10
$p = 0.075 - 16$	2.5-opt-EEais	-	**0.000**	0.674	**0.000**	**0.000**
	2.5-opt-EEas	*0.000*	-	*0.000*	0.674	**0.000**
	2.5-opt-EEs-1000	0.674	**0.000**	-	**0.000**	**0.000**
	2.5-opt-EEs-100	*0.000*	0.674	*0.000*	-	**0.000**
	2.5-opt-EEs-10	*0.000*	*0.000*	*0.000*	*0.000*	-
		2.5-opt-EEais	2.5-opt-EEas	2.5-opt-EEs-1000	2.5-opt-EEs-100	2.5-opt-EEs-10
$p = 0.075 - 50$	2.5-opt-EEais	-	**0.000**	0.722	**0.006**	**0.000**
	2.5-opt-EEas	*0.000*	-	*0.000*	*0.002*	**0.000**
	2.5-opt-EEs-1000	0.722	**0.000**	-	**0.011**	**0.000**
	2.5-opt-EEs-100	*0.006*	**0.002**	*0.011*	-	**0.000**
	2.5-opt-EEs-10	*0.000*	*0.000*	*0.000*	*0.000*	-
		2.5-opt-EEais	2.5-opt-EEas	2.5-opt-EEs-1000	2.5-opt-EEs-100	2.5-opt-EEs-10
$p = 0.075 - 83$	2.5-opt-EEais	-	**0.004**	0.154	**0.002**	**0.000**
	2.5-opt-EEas	*0.004*	-	0.578	0.578	0.162
	2.5-opt-EEs-1000	0.154	0.578	-	0.215	**0.020**
	2.5-opt-EEs-100	*0.002*	0.578	0.215	-	0.578
	2.5-opt-EEs-10	*0.000*	0.162	*0.020*	0.578	-
		2.5-opt-EEais	2.5-opt-EEas	2.5-opt-EEs-1000	2.5-opt-EEs-100	2.5-opt-EEs-10
$p = 0.100 - 16$	2.5-opt-EEais	-	**0.000**	0.719	**0.000**	**0.000**
	2.5-opt-EEas	*0.000*	-	*0.001*	0.719	**0.000**
	2.5-opt-EEs-1000	0.719	**0.001**	-	**0.000**	**0.000**
	2.5-opt-EEs-100	*0.000*	0.719	*0.000*	-	**0.000**
	2.5-opt-EEs-10	*0.000*	*0.000*	*0.000*	*0.000*	-
		2.5-opt-EEais	2.5-opt-EEas	2.5-opt-EEs-1000	2.5-opt-EEs-100	2.5-opt-EEs-10
$p = 0.100 - 50$	2.5-opt-EEais	-	**0.000**	0.100	**0.000**	**0.000**
	2.5-opt-EEas	*0.000*	-	*0.000*	*0.001*	**0.000**
	2.5-opt-EEs-1000	0.100	**0.000**	-	**0.003**	**0.000**
	2.5-opt-EEs-100	*0.000*	**0.001**	*0.003*	-	**0.000**
	2.5-opt-EEs-10	*0.000*	*0.000*	*0.000*	*0.000*	-
		2.5-opt-EEais	2.5-opt-EEas	2.5-opt-EEs-1000	2.5-opt-EEs-100	2.5-opt-EEs-10
$p = 0.100 - 83$	2.5-opt-EEais	-	**0.000**	0.182	**0.000**	**0.000**
	2.5-opt-EEas	*0.000*	-	*0.001*	0.218	0.946
	2.5-opt-EEs-1000	0.182	**0.001**	-	**0.011**	**0.000**
	2.5-opt-EEs-100	*0.000*	0.218	*0.011*	-	**0.034**
	2.5-opt-EEs-10	*0.000*	0.946	*0.000*	*0.034*	-

Table 6: The p-values of the pairwise comparisons of `2.5-opt-EEas`, `2.5-opt-EEais`, `2.5-opt-EEs-10`, `2.5-opt-EEs-100`, and `2.5-opt-EEs--1000` on clustered instances of size 1000 for probability levels less than 0.5. Values in bold mean that the algorithm in the row performs significantly better than the algorithm in the column, while values in italic mean that the algorithm in the column performs significantly better than the algorithm in the row (Part II).

		p-values				
		2.5-opt-EEais	2.5-opt-EEas	2.5-opt-EEs-1000	2.5-opt-EEs-100	2.5-opt-EEs-10
$p = 0.150 - 16$	2.5-opt-EEais	-	**0.000**	1.000	**0.000**	**0.000**
	2.5-opt-EEas	*0.000*	-	*0.000*	1.000	**0.000**
	2.5-opt-EEs-1000	1.000	**0.000**	-	**0.001**	**0.000**
	2.5-opt-EEs-100	*0.000*	1.000	*0.001*	-	**0.000**
	2.5-opt-EEs-10	*0.000*	*0.000*	*0.000*	*0.000*	-
		2.5-opt-EEais	2.5-opt-EEas	2.5-opt-EEs-1000	2.5-opt-EEs-100	2.5-opt-EEs-10
$p = 0.150 - 50$	2.5-opt-EEais	-	**0.000**	0.502	**0.014**	**0.000**
	2.5-opt-EEas	*0.000*	-	*0.000*	0.065	**0.000**
	2.5-opt-EEs-1000	0.502	**0.000**	-	**0.014**	**0.000**
	2.5-opt-EEs-100	*0.014*	0.065	*0.014*	-	**0.000**
	2.5-opt-EEs-10	*0.000*	*0.000*	*0.000*	*0.000*	-
		2.5-opt-EEais	2.5-opt-EEas	2.5-opt-EEs-1000	2.5-opt-EEs-100	2.5-opt-EEs-10
$p = 0.150 - 83$	2.5-opt-EEais	-	**0.000**	0.437	**0.000**	**0.000**
	2.5-opt-EEas	*0.000*	-	*0.014*	0.873	0.090
	2.5-opt-EEs-1000	0.437	**0.014**	-	**0.014**	**0.000**
	2.5-opt-EEs-100	*0.000*	0.873	*0.014*	-	0.165
	2.5-opt-EEs-10	*0.000*	0.090	*0.000*	0.165	-
		2.5-opt-EEais	2.5-opt-EEas	2.5-opt-EEs-1000	2.5-opt-EEs-100	2.5-opt-EEs-10
$p = 0.200 - 16$	2.5-opt-EEais	-	0.236	1.000	0.686	**0.000**
	2.5-opt-EEas	0.236	-	0.686	1.000	**0.000**
	2.5-opt-EEs-1000	1.000	0.686	-	1.000	**0.000**
	2.5-opt-EEs-100	0.686	1.000	1.000	-	**0.000**
	2.5-opt-EEs-10	*0.000*	*0.000*	*0.000*	*0.000*	-
		2.5-opt-EEais	2.5-opt-EEas	2.5-opt-EEs-1000	2.5-opt-EEs-100	2.5-opt-EEs-10
$p = 0.200 - 50$	2.5-opt-EEais	-	**0.000**	0.647	**0.013**	**0.000**
	2.5-opt-EEas	*0.000*	-	*0.000*	*0.025*	**0.000**
	2.5-opt-EEs-1000	0.647	**0.000**	-	**0.008**	**0.000**
	2.5-opt-EEs-100	*0.013*	**0.025**	*0.008*	-	**0.000**
	2.5-opt-EEs-10	*0.000*	*0.000*	*0.000*	*0.000*	-
		2.5-opt-EEais	2.5-opt-EEas	2.5-opt-EEs-1000	2.5-opt-EEs-100	2.5-opt-EEs-10
$p = 0.200 - 83$	2.5-opt-EEais	-	**0.010**	0.825	0.217	**0.001**
	2.5-opt-EEas	*0.010*	-	*0.010*	0.654	0.825
	2.5-opt-EEs-1000	0.825	**0.010**	-	0.178	**0.000**
	2.5-opt-EEs-100	0.217	0.654	0.178	-	0.178
	2.5-opt-EEs-10	*0.001*	0.825	*0.000*	0.178	-

2 Experiments with the analytical computation algorithm

Table 7: Experimental results for `2.5-opt-EEais` and `2.5-opt-ACs`, on clustered homogenous instances of size 1000. Each algorithm is allowed to run until it reaches a local optimum. The table gives, for each probability level, the mean and the standard deviation (s.d.) of the final solution cost and the computation time in seconds over 50 instances.

	Algorithm	Solution Cost		Computation Time	
		mean	s.d.	mean	s.d.
$p = 0.050$	`2.5-opt-EEais`	4039978	411034	11.308	1.787
	`2.5-opt-ACs`	4018046	385868	781.901	124.675
$p = 0.075$	`2.5-opt-EEais`	4630425	443827	6.339	0.765
	`2.5-opt-ACs`	4619711	425864	575.683	59.632
$p = 0.100$	`2.5-opt-EEais`	5153129	446026	4.106	0.540
	`2.5-opt-ACs`	5138511	433649	455.202	64.556
$p = 0.125$	`2.5-opt-EEais`	5579797	450779	3.080	0.300
	`2.5-opt-ACs`	5598429	438755	372.142	48.367
$p = 0.150$	`2.5-opt-EEais`	6003726	437115	2.524	0.294
	`2.5-opt-ACs`	6023435	465723	307.357	42.294
$p = 0.175$	`2.5-opt-EEais`	6398470	501005	2.079	0.182
	`2.5-opt-ACs`	6401986	471953	259.875	38.194
$p = 0.200$	`2.5-opt-EEais`	6783718	512967	1.789	0.163
	`2.5-opt-ACs`	6755178	477283	225.713	28.359
$p = 0.300$	`2.5-opt-EEais`	7986030	537662	1.197	0.095
	`2.5-opt-ACs`	7953217	509601	154.554	22.035
$p = 0.400$	`2.5-opt-EEais`	8899923	587034	0.918	0.051
	`2.5-opt-ACs`	8936331	566315	113.896	18.515
$p = 0.500$	`2.5-opt-EEais`	9704330	597566	0.771	0.039
	`2.5-opt-ACs`	9731076	641231	93.510	11.571

Table 8: Experimental results for `2.5-opt-EEais` and `2.5-opt-ACs`, on clustered heterogeneous instances of size 1000. `2.5-opt-ACs` use a library for arbitrary precision arithmetics. Each algorithm is allowed to run until it reaches a local optimum. The table gives, for each probability level, the mean and the standard deviation (s.d.) of the final solution cost and the computation time in seconds over 50 instances.

	Algorithm	Solution Cost		Computation Time	
		mean	s.d.	mean	s.d.
p =0.050-16	`2.5-opt-EEais`	3975485	377343	36.355	6.604
	`2.5-opt-ACs`	3956801	370360	22608.919	2988.255
p =0.050-50	`2.5-opt-EEais`	3942676	348364	25.172	5.980
	`2.5-opt-ACs`	3910902	347699	26440.638	3987.424
p =0.050-83	`2.5-opt-EEais`	3894077	383960	18.593	4.242
	`2.5-opt-ACs`	3878939	368562	27494.180	3989.230
p =0.075-16	`2.5-opt-EEais`	4509766	423895	24.269	3.840
	`2.5-opt-ACs`	4465208	381561	17389.714	2313.659
p =0.075-50	`2.5-opt-EEais`	4413041	350060	19.076	3.276
	`2.5-opt-ACs`	4404631	376652	21091.996	2661.850
p =0.075-83	`2.5-opt-EEais`	4402750	434125	14.899	3.042
	`2.5-opt-ACs`	4368684	435844	23467.139	2892.983
p =0.100-16	`2.5-opt-EEais`	4986256	384359	17.316	2.713
	`2.5-opt-ACs`	4988090	368263	13239.152	1561.241
p =0.100-50	`2.5-opt-EEais`	4869802	374161	15.482	2.567
	`2.5-opt-ACs`	4879515	375595	17261.794	2048.723
p =0.100-83	`2.5-opt-EEais`	4784504	420709	12.466	2.014
	`2.5-opt-ACs`	4767701	438382	19752.553	3357.073
p =0.150-16	`2.5-opt-EEais`	5851321	409612	9.635	1.005
	`2.5-opt-ACs`	5861570	477616	8831.015	796.226
p =0.150-50	`2.5-opt-EEais`	5682382	392742	10.923	1.623
	`2.5-opt-ACs`	5697001	412345	11946.901	1206.764
p =0.150-83	`2.5-opt-EEais`	5550580	469042	9.402	1.240
	`2.5-opt-ACs`	5524463	440260	14743.172	2136.242
p =0.200-16	`2.5-opt-EEais`	6584162	471175	6.572	0.862
	`2.5-opt-ACs`	6618176	481065	6414.619	685.362
p =0.200-50	`2.5-opt-EEais`	6348657	397729	7.819	1.090
	`2.5-opt-ACs`	6416145	410262	8635.364	1086.996
p =0.200-83	`2.5-opt-EEais`	6170392	405031	7.270	1.053
	`2.5-opt-ACs`	6175680	411698	10995.424	1670.946

Table 9: Comparison of the average cost obtained by `2.5-opt-EEais` and by `2.5-opt-ACs`, on clustered instances of size 1000. For each level of probability, the table reports the observed relative difference and a 95% confidence interval (CI) obtained through the t-test on the relative difference. Concerning the relative difference, if the value is positive, `2.5-opt-EEais` obtained an average cost that is larger than the one obtained by the other algorithm considered; if it is negative, `2.5-opt-EEais` reached solutions of lower average cost. In both cases, a value is typeset in boldface if it is significantly different from zero according to the t-test, at a confidence of 95%.

homogeneous PTSP		
	`2.5-opt-EEais` vs. `2.5-opt-ACs`	
p	Difference	95% CI
0.050	$+0.546\%$	$[-0.157, +1.248]\%$
0.075	$+0.232\%$	$[-0.675, +1.139]\%$
0.100	$+0.284\%$	$[-0.645, +1.214]\%$
0.125	-0.333%	$[-1.122, +0.456]\%$
0.150	-0.327%	$[-1.132, +0.478]\%$
0.175	-0.055%	$[-0.813, +0.703]\%$
0.200	$+0.422\%$	$[-0.386, +1.231]\%$
0.300	0.413%	$[-0.261, 1.086]\%$
0.400	-0.407%	$[-1.052, 0.237]\%$
0.500	-0.275%	$[-0.870, +0.320]\%$

heterogenous PTSP		
	`2.5-opt-EEais` vs. `2.5-opt-ACs`	
p	Difference	95% CI
0.050-16	$+0.472\%$	$[-0.243, +1.187]\%$
0.050-50	$+\mathbf{0.812}\%$	$[+0.147, +1.478]\%$
0.050-83	$+0.390\%$	$[-0.648, +1.428]\%$
0.075-16	$+0.998\%$	$[-0.081, +2.077]\%$
0.075-50	$+0.191\%$	$[-0.372, +0.754]\%$
0.075-83	$+0.780\%$	$[-0.128, +1.688]\%$
0.100-16	-0.037%	$[-0.868, +0.795]\%$
0.100-50	-0.199%	$[-1.091, +0.693]\%$
0.100-83	$+0.352\%$	$[-0.494, +1.199]\%$
0.150-16	-0.175%	$[-1.327, +0.977]\%$
0.150-50	-0.257%	$[-1.093, +0.580]\%$
0.150-83	$+0.473\%$	$[-0.848, +1.794]\%$
0.200-16	-0.514%	$[-1.394, +0.366]\%$
0.200-50	$-\mathbf{1.052}\%$	$[-1.781, -0.323]\%$
0.200-83	-0.086%	$[-0.800, +0.629]\%$

3 Experiments with iterated local search

Table 10: Experimental results for `ILS-2.5-opt-EEais` and `ILS-2.5-opt-ACs` on clustered instances of size 1000. The table gives mean and standard deviation (s.d.) of final solution cost and computation time in seconds. The results are given for 10 instances at each probability level.

	Algorithm	Solution Cost		Computation Time	
		mean	s.d.	mean	s.d.
$p = 0.050$	`ILS-2.5-opt-EEais`	3807580	461280	88.770	8.823
	`ILS-2.5-opt-ACs`	3822518	485976	8877.042	882.311
$p = 0.075$	`ILS-2.5-opt-EEais`	4317220	481569	47.074	3.472
	`ILS-2.5-opt-ACs`	4370470	505924	4707.405	347.211
$p = 0.100$	`ILS-2.5-opt-EEais`	4768619	504847	32.824	2.340
	`ILS-2.5-opt-ACs`	4835552	538804	3282.372	233.997
$p = 0.125$	`ILS-2.5-opt-EEais`	5195562	531549	25.425	1.882
	`ILS-2.5-opt-ACs`	5248933	542716	2542.515	188.213
$p = 0.150$	`ILS-2.5-opt-EEais`	5567761	516760	20.493	1.799
	`ILS-2.5-opt-ACs`	5609656	516846	2049.330	179.894
$p = 0.175$	`ILS-2.5-opt-EEais`	5912404	544093	17.430	1.582
	`ILS-2.5-opt-ACs`	6194239	803708	1743.016	158.233
$p = 0.200$	`ILS-2.5-opt-EEais`	6240723	580072	15.660	1.572
	`ILS-2.5-opt-ACs`	6842928	751783	1566.023	157.196

Table 11: Experimental results for `ILS-2.5-opt-EEais` and `ILS-2.5-opt-ACs` on clustered instances of size 1000. The table gives mean and standard deviation (s.d.) of final solution cost and computation time in seconds. The results are given for 10 instances at each probability level. Note that `ILS-2.5-opt-ACs` uses a library for arbitrary precision arithmetics.

	Algorithm	Solution Cost		Computation Time	
		mean	s.d.	mean	s.d.
p =0.050-16	`ILS-2.5-opt-EEais`	3840549	339955	125.152	14.834
	`ILS-2.5-opt-ACs`	4216734	357264	12515.188	1483.376
p =0.050-50	`ILS-2.5-opt-EEais`	3847541	290172	93.981	12.941
	`ILS-2.5-opt-ACs`	4539817	399929	9398.082	1294.118
p =0.050-83	`ILS-2.5-opt-EEais`	3806451	372408	51.436	14.696
	`ILS-2.5-opt-ACs`	4911136	802057	5143.589	1469.553
p =0.075-16	`ILS-2.5-opt-EEais`	4339270	379237	60.196	7.066
	`ILS-2.5-opt-ACs`	5264820	740131	6019.556	706.595
p =0.075-50	`ILS-2.5-opt-EEais`	4300136	311940	57.126	13.869
	`ILS-2.5-opt-ACs`	5230935	532444	5712.643	1386.902
p =0.075-83	`ILS-2.5-opt-EEais`	4196582	396801	38.524	8.695
	`ILS-2.5-opt-ACs`	5395641	1010625	3852.365	869.511
p =0.100-16	`ILS-2.5-opt-EEais`	4763102	326458	35.926	2.956
	`ILS-2.5-opt-ACs`	6096620	785467	3592.555	295.560
p =0.100-50	`ILS-2.5-opt-EEais`	4604943	291749	40.127	8.148
	`ILS-2.5-opt-ACs`	5943761	591347	4096.402	628.338
p =0.100-83	`ILS-2.5-opt-EEais`	4497071	328981	30.314	6.553
	`ILS-2.5-opt-ACs`	6060358	803229	3031.430	655.297
p =0.150-16	`ILS-2.5-opt-EEais`	5550129	378229	22.329	1.667
	`ILS-2.5-opt-ACs`	7083197	877009	2232.942	166.654
p =0.150-50	`ILS-2.5-opt-EEais`	5366079	326513	25.512	2.655
	`ILS-2.5-opt-ACs`	6992901	668879	2551.223	265.522
p =0.150-83	`ILS-2.5-opt-EEais`	5263152	393994	21.045	2.393
	`ILS-2.5-opt-ACs`	7320704	751214	2104.531	239.278
p =0.200-16	`ILS-2.5-opt-EEais`	6237114	386162	16.514	1.872
	`ILS-2.5-opt-ACs`	7840650	806949	1651.370	187.190
p =0.200-50	`ILS-2.5-opt-EEais`	5939087	309994	18.586	2.080
	`ILS-2.5-opt-ACs`	7849354	683990	1858.618	207.963
p =0.200-83	`ILS-2.5-opt-EEais`	5794719	372376	16.870	1.313
	`ILS-2.5-opt-ACs`	7822102	652433	1687.035	131.296

Table 12: Comparison of the average cost obtained by `ILS-2.5-opt-EEais` and by `ILS-2.5-opt-ACs`, on clustered instances of size 1000. For each level of probability, the table reports the observed relative difference and a 95% confidence interval (CI) obtained through the t-test on the relative difference. Concerning the relative difference, if the value is positive, `2.5-opt-EEs-100` obtained an average cost that is larger than the one obtained by the other algorithm considered; if it is negative, `2.5-opt-EEs-100` reached solutions of lower average cost. In both cases, a value is typeset in boldface if it is significantly different from zero according to the t-test, at a confidence of 95%.

homogeneous PTSP		
	`ILS-2.5-opt-EEais` vs. `ILS-2.5-opt-ACs`	
p	Difference	95% CI
0.050	−0.391%	[−01.301, +00.520]%
0.075	**−1.218%**	[−02.079, −00.357]%
0.100	**−1.384%**	[−02.602, −00.166]%
0.125	**−1.017%**	[−01.763, −00.271]%
0.150	−0.747%	[−01.512, +00.018]%
0.175	−4.550%	[−11.891, +02.791]%
0.200	−8.800%	[−17.984, +00.383]%

heterogenous PTSP		
	`ILS-2.5-opt-EEais` vs. `ILS-2.5-opt-ACs`	
p	Difference	95% CI
0.050-16	**−08.921%**	[−12.549, −05.293]%
0.050-50	**−15.249%**	[−19.178, −11.320]%
0.050-83	**−22.493%**	[−31.942, −13.045]%
0.075-16	**−17.580%**	[−24.485, −10.675]%
0.075-50	**−17.794%**	[−23.158, −12.430]%
0.075-83	**−22.223%**	[−31.615, −12.830]%
0.100-16	**−21.873%**	[−28.571, −15.175]%
0.100-50	**−22.525%**	[−27.941, −17.108]%
0.100-83	**−25.795%**	[−33.350, −18.240]%
0.150-16	**−21.644%**	[−27.865, −15.422]%
0.150-50	**−23.264%**	[−27.749, −18.779]%
0.150-83	**−28.106%**	[−33.490, −22.722]%
0.200-16	**−20.452%**	[−25.451, −15.452]%
0.200-50	**−24.337%**	[−28.864, −19.809]%
0.200-83	**−25.919%**	[−30.913, −20.924]%