Implementation exercises for the course Heuristic Optimization

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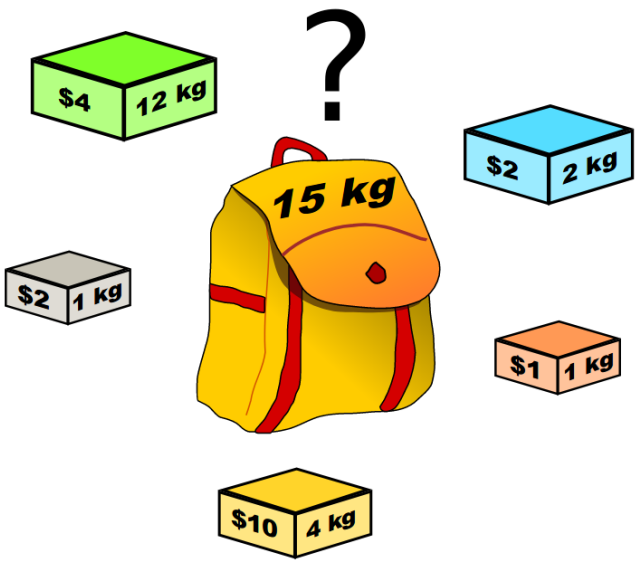
[1] Slides based on 2017 exercises by Federico Pagnozzi.

Exercise 1.1: Heuristics for the MKP

Implement constructive and perturbative local search algorithms for the MKP

1. Multidimensional 0-1 Knapsack Problem (MKP)
2. Constructive Heuristics
3. First-improvement and Best-improvement
4. Variable Neighbourhood Descent
5. Statistical Analysis

The 0-1 Knapsack Problem



Models many real-life allocation problems:

- a subset of item has to be chosen from a larger set
- each item has a *value*
- but items have also a *cost*.
- Resources are scarce: select the subset of items that give the highest cumulative value, while respecting the budget.

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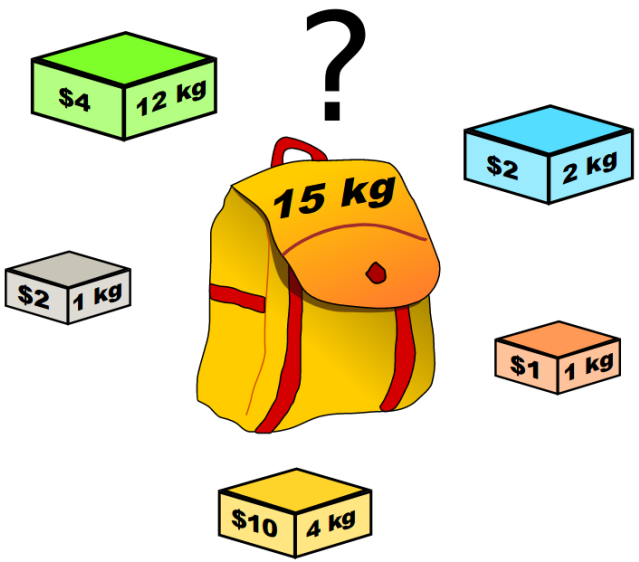

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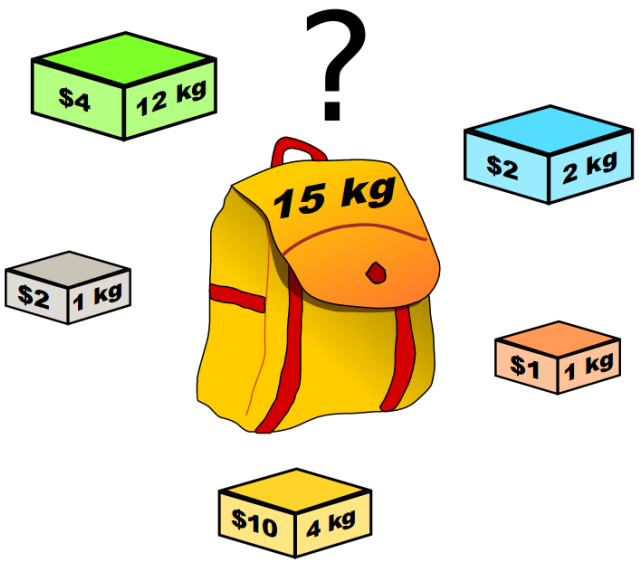

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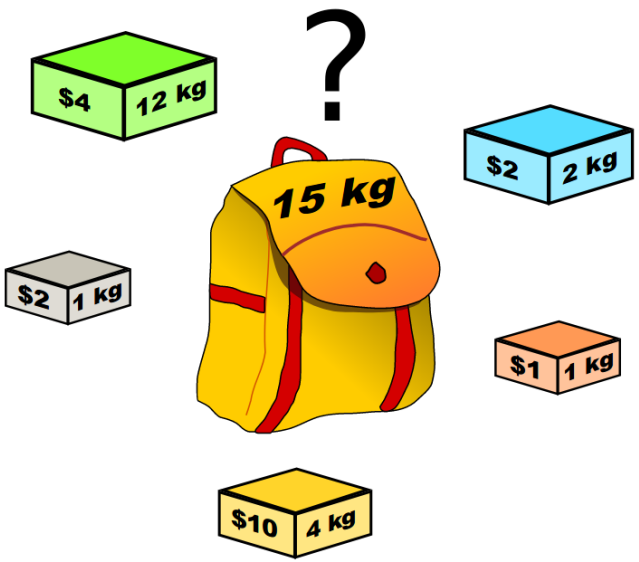

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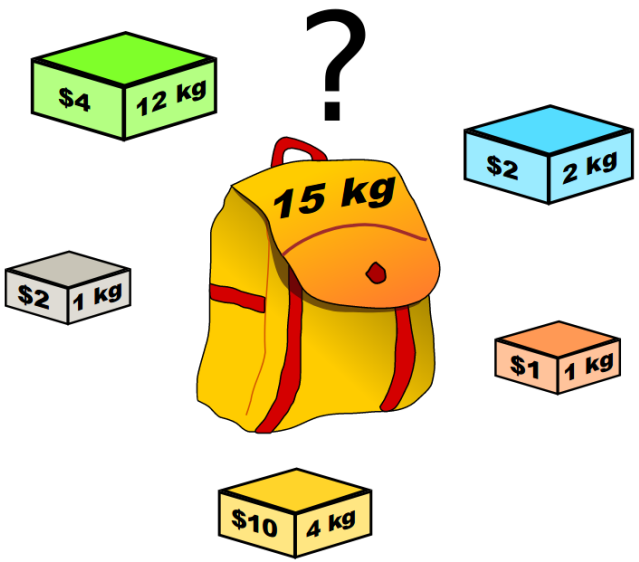

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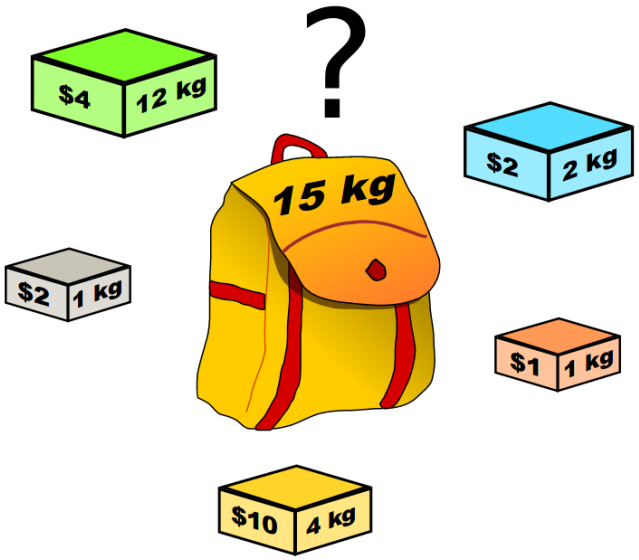

Multidimensional: several resources involved

- each item is associated to a value and a set of resources needed: cost, weight, volume, . . .
- for every resource, the knapsack has a maximum capacity
- Goal: select the subset of items that give the highest profit, while respecting all the constraints.

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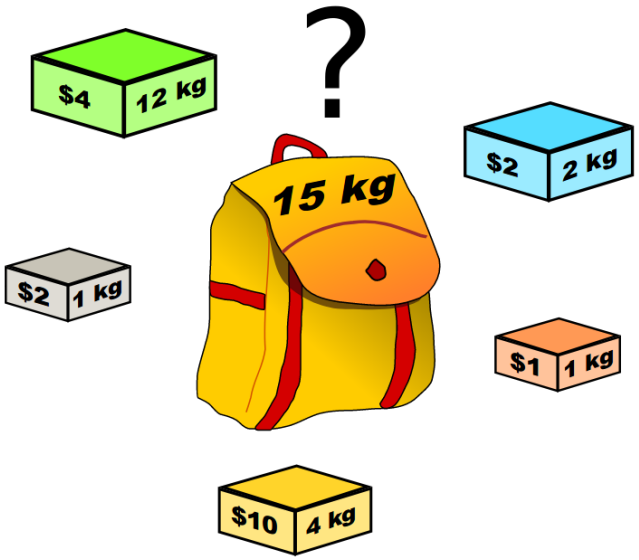

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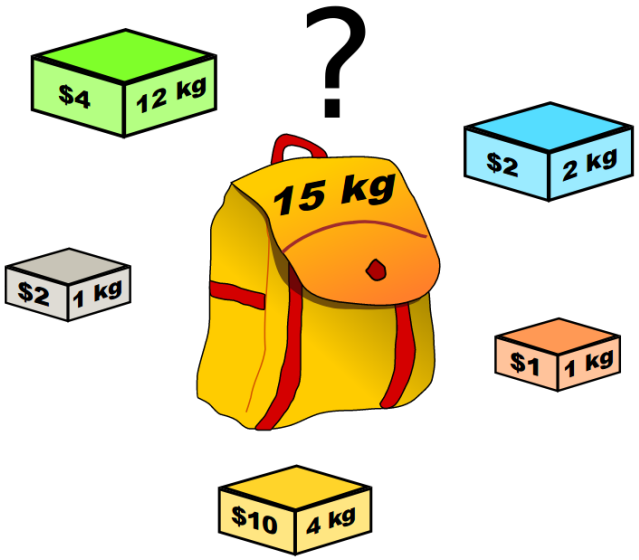

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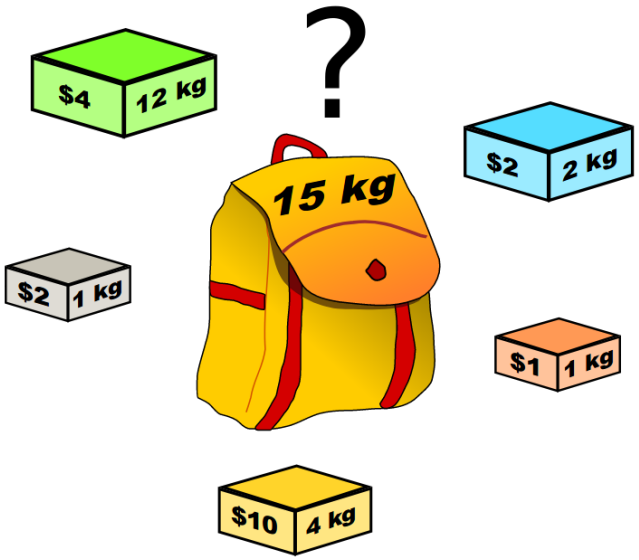

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The Multidimensional 0-1 Knapsack Problem

More precisely:

- we have *n items* and *m resources*
- we have a knapsack that can contain some (but not all) our items
  - the knapsack has a maximum *capacity* for every resource
- each item has associated *profit* or *value* p, and consumes a certain amount w_i for every resource i
- profits and weights are non-negative (so-called knapsack constraints)
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The Multidimensional 0-1 Knapsack Problem

Given

A set of n items and m resources, where each item j has a profit $p_j \geq 0$ and consumes an amount $w_{ij} \geq 0$ of each resource i, and each resource i has a maximum capacity L_i that cannot be exceeded.

Objective

Find the subset of items that maximizes

$\sum_{j=1}^{n} p_j \cdot x_j$

subject to

$\sum_{j=1}^{n} w_{ij} \cdot x_j \leq L_i \; \forall i$

$x_j \in \{0, 1\}$.

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Example

6 items, 10 constraints

Object profits: 100 600 1200 2400 500 2000

Constraints:

$$
\begin{array}{lcr}
8x_0 + 12x_1 + 13x_2 + 64x_3 + 22x_4 + 41x_5 & \leq & 80 \\
8x_0 + 12x_1 + 13x_2 + 75x_3 + 22x_4 + 41x_5 & \leq & 96 \\
3x_0 + 6x_1 + 4x_2 + 18x_3 + 6x_4 + 4x_5 & \leq & 20 \\
5x_0 + 10x_1 + 8x_2 + 32x_3 + 6x_4 + 12x_5 & \leq & 36 \\
5x_0 + 13x_1 + 8x_2 + 42x_3 + 6x_4 + 20x_5 & \leq & 44 \\
5x_0 + 13x_1 + 8x_2 + 48x_3 + 6x_4 + 20x_5 & \leq & 48 \\
0x_0 + 0x_1 + 0x_2 + 0x_3 + 8x_4 + 0x_5 & \leq & 10 \\
3x_0 + 0x_1 + 4x_2 + 0x_3 + 8x_4 + 0x_5 & \leq & 18 \\
3x_0 + 2x_1 + 4x_2 + 0x_3 + 8x_4 + 4x_5 & \leq & 22 \\
3x_0 + 2x_1 + 4x_2 + 8x_3 + 8x_4 + 4x_5 & \leq & 24
\end{array}
$$

Example

Solution: items $\{0, 2, 5\}$

Solution value: $100 + 1200 + 2000 = 3300$

No constraints violated: solution is feasible

$$
\begin{array}{lcr|r}
8x_0 + 12x_1 + 13x_2 + 64x_3 + 22x_4 + 41x_5 & \leq & 80 & (62) \\
8x_0 + 12x_1 + 13x_2 + 75x_3 + 22x_4 + 41x_5 & \leq & 96 & (62) \\
3x_0 + 6x_1 + 4x_2 + 18x_3 + 6x_4 + 4x_5 & \leq & 20 & (11) \\
5x_0 + 10x_1 + 8x_2 + 32x_3 + 6x_4 + 12x_5 & \leq & 36 & (25) \\
5x_0 + 13x_1 + 8x_2 + 42x_3 + 6x_4 + 20x_5 & \leq & 44 & (33) \\
5x_0 + 13x_1 + 8x_2 + 48x_3 + 6x_4 + 20x_5 & \leq & 48 & (33) \\
0x_0 + 0x_1 + 0x_2 + 0x_3 + 8x_4 + 0x_5 & \leq & 10 & (0) \\
3x_0 + 0x_1 + 4x_2 + 0x_3 + 8x_4 + 0x_5 & \leq & 18 & (7) \\
3x_0 + 2x_1 + 4x_2 + 0x_3 + 8x_4 + 4x_5 & \leq & 22 & (11) \\
3x_0 + 2x_1 + 4x_2 + 8x_3 + 8x_4 + 4x_5 & \leq & 24 & (11)
\end{array}
$$

Example

Solution: items $\{0, 3\}$

Solution value: $100 + 2400 = 2500$

Some constraints violated: solution is not feasible

$8x_0 + 12x_1 + 13x_2 + 64x_3 + 22x_4 + 41x_5$	$\leq$	80	(72)
$8x_0 + 12x_1 + 13x_2 + 75x_3 + 22x_4 + 41x_5$	$\leq$	96	(83)
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$5x_0 + 13x_1 + 8x_2 + 42x_3 + 6x_4 + 20x_5$	$\leq$	44	(47)
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Item 3 is problematic here: takes a lot of resources for some constraints

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  1. First-improvement (FI)
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- Variable Neighbourhood Descent:
  1. based either on FI or BI (your choice)

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Don't implement 12 programs!

Reuse code and use command-line parameters

```
mkp --random --none
mkp --greedy --fi
...
```

Constructive heuristics for the MKP

- Random Insertion
- Greedy Insertion (deterministic)
- Insertion based on ratio profit/weight (Toyoda algorithm, deterministic)
- Will be used as initial solutions for FI, BI, VND
- All of them start from an empty solution, iteratively add items whilealways maintaining feasibility
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- Shuffle the items
- For each item in this “order”: add it if it does not violate any constraint

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Toyoda algorithm

- generic algorithm for binary mixed integer problems
- sorts the items according to their "convenience" (pseudo-utility)
- pseudo-utility defined as $\max_j p_j / V_j$, V_j aggregate consumption of resources
- several possible ways for defining V_j

For the 1-resource case ("classic" KP)

- simple: one weight per item $\Rightarrow p_j / w_j$
- gives a greedy approximation algorithm, $\rho = 2$
- not so clear how to extend it to the multidimensional case

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- consider $A = W/L$, that is, normalize in $[0, 1]$ each w_{ij} and store them in a new matrix A
- constraints now defined as: $\sum_{j=1}^{n} a_{ij} \cdot x_j \leq 1 \; \forall i$, $a_{ij} = w_{ij}/L_i$
- define U as the vector of resources used so far
- $U_i = \sum_{j=1}^{n} a_{ij} \cdot x_j$
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Constructive heuristics for the MKP

Toyoda algorithm

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Toyoda algorithm: example (1/6)

Consider our example instance with 6 items, 10 constraints

Object profits: 100 600 1200 2400 500 2000

Constraints:

$$
\begin{array}{lcr}
8x_0 + 12x_1 + 13x_2 + 64x_3 + 22x_4 + 41x_5 & \leq & 80 \\
8x_0 + 12x_1 + 13x_2 + 75x_3 + 22x_4 + 41x_5 & \leq & 96 \\
3x_0 + 6x_1 + 4x_2 + 18x_3 + 6x_4 + 4x_5 & \leq & 20 \\
5x_0 + 10x_1 + 8x_2 + 32x_3 + 6x_4 + 12x_5 & \leq & 36 \\
5x_0 + 13x_1 + 8x_2 + 42x_3 + 6x_4 + 20x_5 & \leq & 44 \\
5x_0 + 13x_1 + 8x_2 + 48x_3 + 6x_4 + 20x_5 & \leq & 48 \\
0x_0 + 0x_1 + 0x_2 + 0x_3 + 8x_4 + 0x_5 & \leq & 10 \\
3x_0 + 0x_1 + 4x_2 + 0x_3 + 8x_4 + 0x_5 & \leq & 18 \\
3x_0 + 2x_1 + 4x_2 + 0x_3 + 8x_4 + 4x_5 & \leq & 22 \\
3x_0 + 2x_1 + 4x_2 + 8x_3 + 8x_4 + 4x_5 & \leq & 24
\end{array}
$$

Toyoda algorithm: example (2/6)

Normalize the constraints: $\sum_{i=1}^{m} a_{ij} \cdot x_j \leq 1 \; \forall j, \; a_{ij} = w_{ij}/L_i$

Object profits: 100 600 1200 2400 500 2000

Constraints:

$$
\begin{array}{lcl}
0.1x_0 + 0.15x_1 + 0.1625x_2 + 0.8x_3 + 0.275x_4 + 0.5125x_5 & \leq & 1 \\
0.0833x_0 + 0.125x_1 + 0.1354x_2 + 0.78125x_3 + 0.2292x_4 + 0.4271x_5 & \leq & 1 \\
0.15x_0 + 0.3x_1 + 0.2x_2 + 0.9x_3 + 0.3x_4 + 0.2x_5 & \leq & 1 \\
0.1389x_0 + 0.2778x_1 + 0.2222x_2 + 0.8889x_3 + 0.1667x_4 + 0.3333x_5 & \leq & 1 \\
0.1136x_0 + 0.2955x_1 + 0.1818x_2 + 0.9546x_3 + 0.1364x_4 + 0.4545x_5 & \leq & 1 \\
0.1041x_0 + 0.2708x_1 + 0.1667x_2 + 1x_3 + 0.125x_4 + 0.41667x_5 & \leq & 1 \\
0x_0 + 0x_1 + 0x_2 + 0x_3 + 0.8x_4 + 0x_5 & \leq & 1 \\
0.1667x_0 + 0x_1 + 0.2222x_2 + 0x_3 + 0.4444x_4 + 0x_5 & \leq & 1 \\
0.1364x_0 + 0.0909x_1 + 0.1818x_2 + 0x_3 + 0.3636x_4 + 0.1818x_5 & \leq & 1 \\
0.125x_0 + 0.0833x_1 + 0.1667x_2 + 0.3333x_3 + 0.3333x_4 + 0.1667x_5 & \leq & 1
\end{array}
$$

$U = [0, 0, \ldots, 0]$

Toyoda algorithm: example (3/6)

We start from an empty solution, $U = [0, 0, \ldots, 0]$, $||U|| = 0$, hence in the first iteration we compute only $V_j = \sum_{i=1}^{m} a_{ij}$ (that is, the sum of each column)

$V_0 = 0.1 + 0.0833 + 0.15 + 0.1389 + 0.1136 + 0.1041 + 0 + 0.1667 + 0.1364 + 0.125 = 1.118$

$V_1 = 1.593 \qquad V_2 = 1.639$

$V_3 = 5.658 \qquad V_4 = 3.1736 \qquad V_5 = 2.6926$

$p_0/V_0 = 100/1.118 = 89.44 \qquad p_1/V_1 = 376.575$

$p_2/V_2 = 732.006 \qquad p_3/V_3 = 424.1768$

$p_4/V_4 = 157.549 \qquad p_5/V_5 = 742.772$

Item 5 has the highest pseudo-utility, and adding it to the solution does not violate any constraint $\Rightarrow s = \{5\}$,

$U = [0.5125, 0.4271, 0.2, 0.333, 0.4545, 0.4167, 0, 0, 0.1818, 0.167] \leq [1, \ldots, 1]$

Toyoda algorithm: example (3/6)

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Toyoda algorithm: example (4/6)

Second iteration: $s = \{5\}$, profit = 2000
$U = [0.5125, 0.4271, 0.2, 0.333, 0.4545, 0.4167, 0, 0, 0.1818, 0.167]$,
$||U|| = 1.018386$

From now on we have to consider U, $||U||$

$V_0 = (0.1*0.5125+0.0833*0.4271+\cdots+0.125*0.167)/1.018386 = 0.298$
$V_1 = 0.5503 \qquad V_2 = 0.4596$
$V_3 = 2.088 \qquad V_4 = 0.579$

$p_0/V_0 = 100/0.298 = 335.21$
$p_1/V_1 = 1090.379 \qquad p_2/V_2 = 2610.75$
$p_3/V_3 = 1149.67 \qquad p_4/V_4 = 862.93$

Item 2 has the highest pseudo-utility now, and adding it to the solution does not violate any constraint

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Toyoda algorithm: example (5/6)

Third iteration: $s = \{2, 5\}$, profit = 3200
$U = [0.675, 0.5625, 0.4, 0.555, 0.636, 0.583, 0, 0.2222, 0.3636, 0.33]$,
$||U|| = 1.509452$

$V_0 = (0.1 * 0.6750 + 0.0833 * 0.5625 + \cdots + 0.125 * 0.3337)/1.509452 = 0.3398$
$V_1 = 0.5649 \qquad V_3 = 2.0769 \qquad V_4 = 0.6816$

$p_0/V_0 = 100/0.3398 = 294.3 \qquad p_1/V_1 = 1062.13$
$p_3/V_3 = 1155.549 \qquad p_4/V_4 = 733.54$

Item 3 has the highest pseudo-utility now, but adding it would violate some constraints!
$U = [1.475, 1.344, 1.3, 1.44, 1.59, 1.58, 0, 0.222, 0.3636, 0.667]$
Next one is item 1, that does not violate any constraint

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Toyoda algorithm: example (6/6)

Fourth iteration: $s = \{1, 2, 5\}$, profit = 3800
$U = [0.825, 0.6875, 0.7, 0.833, 0.932, 0.854, 0, 0.222, 0.4545, 0.417]$,
$||U|| = 2.089237$

$V_0 = (0.1 * 0.825 + 0.0833 * 0.6875 + \cdots + 0.125 * 0.417)/2.089237 = 0.338$
$V_3 = 2.13 \qquad V_4 = 0.656$

$p_0/V_0 = 100/0.338 = 295.72 \qquad p_3/V_3 = 1126.7 \qquad p_4/V_4 = 762.5$

In order, we try to add items 3, 4, 0, but we cannot maintain feasibility. We're done.

Our final solution is $s = \{1, 2, 5\}$, with a profit of 3800.

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Toyoda algorithm: example (6/6)

Fourth iteration: $s = \{1, 2, 5\}$, profit = 3800
$U = [0.825, 0.6875, 0.7, 0.833, 0.932, 0.854, 0, 0.222, 0.4545, 0.417]$,
$||U|| = 2.089237$

$V_0 = (0.1 * 0.825 + 0.0833 * 0.6875 + \cdots + 0.125 * 0.417)/2.089237 = 0.338$
$V_3 = 2.13 \qquad V_4 = 0.656$

$p_0/V_0 = 100/0.338 = 295.72 \qquad p_3/V_3 = 1126.7 \qquad p_4/V_4 = 762.5$

In order, we try to add items 3, 4, 0, but we cannot maintain feasibility. We're done.

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Exercise 1.1: Heuristics for the MKP

Iterative Improvement

$s :=$ `GenerateInitialSolution()`

while s is not a local optimum **do**

  choose a neighbour $s' \in \mathcal{N}(s)$ such that $F(s') \geq F(s)$

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Which neighbour to choose? Pivoting rule

- **Best Improvement:** choose best from all neighbours of s
  - ✔ Better quality
  - ✘ Requires evaluation of all neighbours in each step
- **First improvement:** evaluate neighbours in fixed order and choose first improving neighbour.
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Variable Neighbourhood Descent

- Based on either FI or BI (your choice)
- Evaluates increasingly large neighbourhoods

Initial solution

- Random constructive heuristic
- Greedy constructive heuristic
- Toyoda algorithm

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Neighbourhood for the MKP

Many neighbourhoods are possible for the MKP:

- remove k items and add k different items
- remove k items and add as many different items as possible
- $\rightarrow$ in which order?
- add k items, make the solution non feasible, then remove k different items
- add k items, make the solution non feasible, then remove as many different items as needed
- $\rightarrow$ in which order?

For this project, the neighbourhood is given by the set of moves that remove k items and add as many different items as possible, according to the order specified as follows.

Every solution accepted has to be feasible.

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Neighbourhood for the MKP: example (1/2)

Consider our example instance, and an initial solution generated using the greedy criterion, and a BI rule: $s = \{3\}$, profit = 2400.

First iteration:

- shuffle the list of items in s
- shuffled list is [3]
- sort list of non-inserted items by profit: $[5, 2, 1, 4, 0]$
- remove first item from shuffled s: $\{\}$
- try to add the non-selected items, one by one, ordered by profit
- we can add 5, 2, 1, but not 4 or 0 (check feasibility). Hence: $s' = \{1, 2, 5\}$, profit = 3800, not inserted $\{0, 3, 4\}$
- we cannot remove any more items from the shuffled s, so we have evaluated the whole neighbourhood. s' is feasible and it improves over s, so we accept it.

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Neighbourhood for the MKP: example (2/2)

$s = \{1, 2, 5\}$, profit = 3800, not inserted $\{0, 3, 4\}$

Next iteration, shuffle s: $[2, 5, 1]$. The other items sorted by profit are $[3, 4, 0]$. Hence:

- Remove item 2 from s, $'s = \{1, 5\}$, try to add 3, then 4, then 0
- Remove item 5 from s, $'s = \{1, 2\}$, try to add 3, then 4, then 0
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Neighbourhood scan completed, no improving move found: stop.

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To recap:

Implement 3 constructive heuristics:

- Random
- Greedy
- Toyoda algorithm

Implement 3 FI heuristics:

- Random init. sol. + FI with random insertion
- Greedy init. sol. + FI with greedy insertion
- Toyoda init. sol. + FI with Toyoda insertion

Implement 3 BI heuristics:

- Random init. sol. + BI with random insertion
- Greedy init. sol. + BI with greedy insertion
- Toyoda init. sol. + BI with Toyoda insertion

Implement 3 VND heuristics (based either on FI or BI):

- Random init. sol. + VND with random insertion
- Greedy init. sol. + VND with greedy insertion
- Toyoda init. sol. + VND with Toyoda insertion

Material

Material provided on the course homepage:

- C code for
  - reading an MKP instance
  - computing the objective function value
- you can use the code as is, incapsulate it into C++ classes, rewrite it in Java
- 60 problem instances
  - + example instance for debugging
- list of best known values
- exercise sheet and this slide deck

Instance format

http://people.brunel.ac.uk/ mastjjb/jeb/orlib/mknapinfo.html

- in the first line: problem size
  - number of items n
  - number of constraints m
  - value of the optimal solution (0 if not known)
- the list of n profits p_j (on multiple lines)
- the list of $n \times m$ weights w_{ij} (row-wise, on multiple lines)
- the list of m capacities L_i (on multiple lines)

Exercise 1.1: Heuristics for the MKP

Instances

- Run the Random constructive heuristic and the II based on it 15 times for each instance, and average the results
- Run the Greedy and Toyoda constructive heuristics and the II based on them once per each instance

Experiments

For each algorithm k and instance i compute:

1. Relative percentage deviation $\Delta_{ki} = 100 \cdot \frac{\text{profit}_{ki} - \text{best-known}_i}{\text{best-known}_i}$
2. Computation time (t_{ki})
3. (using the average values when the Random constructive heuristic is involved)

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- Average relative percentage deviation
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Exercise 1.1: Heuristics for the MKP

Is there a statistically significant difference between the solution quality generated by the different algorithms?

Statistical test

- Paired t-test
- Wilcoxon signed-rank test
- compare the results across the same size

Is there a statistically significant difference between the solution quality generated by the different algorithms?

Background: Statistical hypothesis tests (1)

- *Statistical hypothesis tests* are used to assess the validity of statements about properties of or relations between sets of statistical data.
- The statement to be tested (or its negation) is called the *null hypothesis* (H_0) of the test.
  Example: For the Wilcoxon signed-rank test, the null hypothesis is that 'the median of the differences is zero'.
- The *significance level* (α) determines the maximum allowable probability of incorrectly rejecting the null hypothesis.
  Typical values of α are 0.05 or 0.01.

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Background: Statistical hypothesis tests (2)

- The application of a test to a given data set results in a *p-value*, which represents the probability that the null hypothesis is incorrectly rejected.
- The null hypothesis is rejected iff this p-value is smaller than the previously chosen significance level.
- Most common statistical hypothesis tests are already implemented in statistical software such as the *R software environment* (http://www.r-project.org/).

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Example in R

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best.known <- read.csv ("bestSolutions.txt")
a.cost <- read.table("ii-best-ex-rand.dat")$V1
a.cost <- 100 * (a.cost - best.known) / best.known$BS
b.cost <- read.table("ii-best-ins-rand.dat")$V1
b.cost <- 100 * (b.cost - best.known) / best.known$BS
t.test (a.cost, b.cost, paired=T)$p.value
[1] 0.8819112
wilcox.test (a.cost, b.cost, paired=T)$p.value
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- Instances and barebone code available at:
  http://iridia.ulb.ac.be/~stuetzle/Teaching/HO/
- Deadline is April 15 (23:59)
- Questions in the meantime?
  `alberto.franzin@ulb.ac.be`