Implementation exercises for the course
Heuristic Optimization

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March 14, 2018

1 Slides based on 2017 exercises by Federico Pagnozzi.
Implement constructive and perturbative local search algorithms for the MKP

1. Multidimensional 0-1 Knapsack Problem (MKP)
2. Constructive Heuristics
3. First-improvement and Best-improvement
4. Variable Neighbourhood Descent
5. Statistical Analysis
The 0-1 Knapsack Problem

Models many real-life allocation problems:

- a subset of item has to be chosen from a larger set
- each item has a *value*
- but items have also a *cost*.
- Resources are scarce: select the subset of items that give the highest cumulative value, while respecting the budget.
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The Multidimensional 0-1 Knapsack Problem

Multidimensional: several resources involved

- each item is associated to a value and a set of resources needed: cost, weight, volume, ...
- for every resource, the knapsack has a maximum capacity
- Goal: select the subset of items that give the highest profit, while respecting all the constraints.
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The Multidimensional 0-1 Knapsack Problem

More precisely:

- we have *n items* and *m resources*
- we have a knapsack that can contain some (but not all) our items
  - the knapsack has a maximum *capacity* for every resource
- each item has associated *profit* or *value* *p*, and consumes a certain amount *w_i* for every resource *i*
- profits and weights are non-negative (so-called knapsack constraints)
- Goal: select the subset of items that give the highest profit, while respecting all the constraints.
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The Multidimensional 0-1 Knapsack Problem

**Given**

A set of $n$ items and $m$ resources, where each item $j$ has a profit $p_j \geq 0$ and consumes an amount $w_{ij} \geq 0$ of each resource $i$, and each resource $i$ has a maximum capacity $L_i$ that cannot be exceeded.

**Objective**

Find the subset of items that maximizes

$$\sum_{j=1}^{n} p_j \cdot x_j$$

subject to

$$\sum_{j=1}^{n} w_{ij} \cdot x_j \leq L_i \forall i$$

$$x_j \in \{0, 1\}.$$
The Multidimensional 0-1 Knapsack Problem

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\]
\[
x_j \in \{0, 1\}.
\]
Example

6 items, 10 constraints

Object profits: 100 600 1200 2400 500 2000

Constraints:

\[
\begin{align*}
8x_0 + 12x_1 + 13x_2 + 64x_3 + 22x_4 + 41x_5 & \leq 80 \\
8x_0 + 12x_1 + 13x_2 + 75x_3 + 22x_4 + 41x_5 & \leq 96 \\
3x_0 + 6x_1 + 4x_2 + 18x_3 + 6x_4 + 4x_5 & \leq 20 \\
5x_0 + 10x_1 + 8x_2 + 32x_3 + 6x_4 + 12x_5 & \leq 36 \\
5x_0 + 13x_1 + 8x_2 + 42x_3 + 6x_4 + 20x_5 & \leq 44 \\
5x_0 + 13x_1 + 8x_2 + 48x_3 + 6x_4 + 20x_5 & \leq 48 \\
0x_0 + 0x_1 + 0x_2 + 0x_3 + 8x_4 + 0x_5 & \leq 10 \\
3x_0 + 0x_1 + 4x_2 + 0x_3 + 8x_4 + 0x_5 & \leq 18 \\
3x_0 + 2x_1 + 4x_2 + 0x_3 + 8x_4 + 4x_5 & \leq 22 \\
3x_0 + 2x_1 + 4x_2 + 8x_3 + 8x_4 + 4x_5 & \leq 24
\end{align*}
\]
Example

Solution: items $\{0, 2, 5\}$

Solution value: $100 + 1200 + 2000 = 3300$

No constraints violated: solution is feasible

\[
\begin{align*}
8x_0 + 12x_1 + 13x_2 + 64x_3 + 22x_4 + 41x_5 & \leq 80 & (62) \\
8x_0 + 12x_1 + 13x_2 + 75x_3 + 22x_4 + 41x_5 & \leq 96 & (62) \\
3x_0 + 6x_1 + 4x_2 + 18x_3 + 6x_4 + 4x_5 & \leq 20 & (11) \\
5x_0 + 10x_1 + 8x_2 + 32x_3 + 6x_4 + 12x_5 & \leq 36 & (25) \\
5x_0 + 13x_1 + 8x_2 + 42x_3 + 6x_4 + 20x_5 & \leq 44 & (33) \\
5x_0 + 13x_1 + 8x_2 + 48x_3 + 6x_4 + 20x_5 & \leq 48 & (33) \\
0x_0 + 0x_1 + 0x_2 + 0x_3 + 8x_4 + 0x_5 & \leq 10 & (0) \\
3x_0 + 0x_1 + 4x_2 + 0x_3 + 8x_4 + 0x_5 & \leq 18 & (7) \\
3x_0 + 2x_1 + 4x_2 + 0x_3 + 8x_4 + 4x_5 & \leq 22 & (11) \\
3x_0 + 2x_1 + 4x_2 + 8x_3 + 8x_4 + 4x_5 & \leq 24 & (11)
\end{align*}
\]
Solution: items \{0, 3\}

Solution value: \(100 + 2400 = 2500\)

Some constraints violated: solution is not feasible

\[
\begin{align*}
8x_0 + 12x_1 + 13x_2 + 64x_3 + 22x_4 + 41x_5 &\leq 80 \quad (72) \\
8x_0 + 12x_1 + 13x_2 + 75x_3 + 22x_4 + 41x_5 &\leq 96 \quad (83) \\
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0x_0 + 0x_1 + 0x_2 + 0x_3 + 8x_4 + 0x_5 &\leq 10 \quad (0) \\
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Item 3 is problematic here: takes a lot of resources for some constraints
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Implement 3 constructive heuristics and 9 iterative improvements algorithms for the MKP
Exercise 1.1: Heuristics for the MKP

Implement 3 constructive heuristics and 9 iterative improvements algorithms for the MKP

- **Constructive heuristics:**
  1. Random
  2. Greedy
  3. Toyoda (profit/weight ratio)

- **Iterative improvement with pivoting rules:**
  1. First-improvement (FI)
  2. Best-improvement (BI)

- **Variable Neighbourhood Descent:**
  1. based either on FI or BI (your choice)
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Don’t implement 12 programs! Reuse code and use command-line parameters

```
mkp --random --none
mkp --greedy --fi
...
```
Constructive heuristics for the MKP

- Random Insertion
  - Greedy Insertion (deterministic)
  - Insertion based on ratio profit/weight (Toyoda algorithm, deterministic)
- Will be used as initial solutions for FI, BI, VND
- All of them start from an empty solution, iteratively add items while always maintaining feasibility
- Alternative approach: start from / generate an infeasible solution, remove items until feasibility is reached (not considered in these exercises).
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Random Insertion
- Start from an empty solution
- Shuffle the items
- For each item in this “order”: add it if it does not violate any constraint

Greedy Insertion
- Start from an empty solution
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**Toyoda algorithm**
- generic algorithm for binary mixed integer problems
- sorts the items according to their “convenience” (pseudo-utility)
- pseudo-utility defined as $\max_j p_j / V_j$, $V_j$ aggregate consumption of resources
- several possible ways for defining $V_j$

**For the 1-resource case ("classic" KP)**
- simple: one weight per item $\Rightarrow p_j / w_j$
- gives a greedy approximation algorithm, $\rho = 2$
- not so clear how to extend it to the multidimensional case
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Toyoda algorithm

- consider $A = W/L$, that is, normalize in $[0, 1]$ each $w_{ij}$ and store them in a new matrix $A$
- constraints now defined as: $\sum_{j=1}^{n} a_{ij} \cdot x_j \leq 1 \forall i, a_{ij} = w_{ij}/L_i$
- define $U$ as the vector of resources used so far
  - $U_i = \sum_{j=1}^{n} a_{ij} \cdot x_j$
  - $V_j = \sum_{i=1}^{m} a_{ij} \cdot U_i / ||U||$ (where $||U||$ is the euclidean norm of $U$).
- careful in the first iteration: $0/0 \Rightarrow$ consider only $V_j^0 = \sum_{i=1}^{m} a_{ij}$
- sort the non-selected items according to the pseudo-utility $p_j/V_j$
- scan the list of items, until you can add one item
- when you add one item, stop the cycle, update and repeat the process until you cannot add any new item
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Consider $A = W/L$, that is, normalize in $[0, 1]$ each $w_{ij}$ and store them in a new matrix $A$.

Constraints now defined as: $\sum_{j=1}^{n} a_{ij} \cdot x_j \leq 1 \ \forall i, \ a_{ij} = w_{ij}/L_i$

Define $U$ as the vector of resources used so far:

$U_i = \sum_{j=1}^{n} a_{ij} \cdot x_j$

$V_j = \sum_{i=1}^{m} a_{ij} \cdot U_i / \|U\| \ (where \ \|U\| \ is \ the \ euclidean \ norm \ of \ U)$.  

Careful in the first iteration: $0/0 \Rightarrow$ consider only $V_j^0 = \sum_{i=1}^{m} a_{ij}$

Sort the non-selected items according to the pseudo-utility $p_j/V_j$

Scan the list of items, until you can add one item

When you add one item, stop the cycle, update and repeat the process until you cannot add any new item.
Constructive heuristics for the MKP

Toyoda algorithm

- consider $A = W/L$, that is, normalize in $[0, 1]$ each $w_{ij}$ and store them in a new matrix $A$
- constraints now defined as: $\sum_{j=1}^{n} a_{ij} \cdot x_j \leq 1 \ \forall i, a_{ij} = w_{ij}/L_i$
- define $U$ as the vector of resources used so far
  - $U_i = \sum_{j=1}^{n} a_{ij} \cdot x_j$
  - $V_j = \sum_{i=1}^{m} a_{ij} \cdot U_i / ||U||$ (where $||U||$ is the euclidean norm of $U$).
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  - when you add one item, stop the cycle, update and repeat the process until you cannot add any new item
Consider our example instance with 6 items, 10 constraints

Object profits: 100 600 1200 2400 500 2000

Constraints:

\[
8x_0 + 12x_1 + 13x_2 + 64x_3 + 22x_4 + 41x_5 \leq 80
\]

\[
8x_0 + 12x_1 + 13x_2 + 75x_3 + 22x_4 + 41x_5 \leq 96
\]

\[
3x_0 + 6x_1 + 4x_2 + 18x_3 + 6x_4 + 4x_5 \leq 20
\]

\[
5x_0 + 10x_1 + 8x_2 + 32x_3 + 6x_4 + 12x_5 \leq 36
\]

\[
5x_0 + 13x_1 + 8x_2 + 42x_3 + 6x_4 + 20x_5 \leq 44
\]

\[
5x_0 + 13x_1 + 8x_2 + 48x_3 + 6x_4 + 20x_5 \leq 48
\]

\[
0x_0 + 0x_1 + 0x_2 + 0x_3 + 8x_4 + 0x_5 \leq 10
\]

\[
3x_0 + 0x_1 + 4x_2 + 0x_3 + 8x_4 + 0x_5 \leq 18
\]

\[
3x_0 + 2x_1 + 4x_2 + 0x_3 + 8x_4 + 4x_5 \leq 22
\]

\[
3x_0 + 2x_1 + 4x_2 + 8x_3 + 8x_4 + 4x_5 \leq 24
\]
Normalize the constraints: \( \sum_{i=1}^{m} a_{ij} \cdot x_j \leq 1 \ \forall j, \ a_{ij} = w_{ij}/L_i \)

Object profits: 100 600 1200 2400 500 2000

Constraints:

\[
\begin{align*}
0.1x_0 + 0.15x_1 + 0.1625x_2 + 0.8x_3 + 0.275x_4 + 0.5125x_5 & \leq 1 \\
0.0833x_0 + 0.125x_1 + 0.1354x_2 + 0.78125x_3 + 0.2292x_4 + 0.4271x_5 & \leq 1 \\
0.15x_0 + 0.3x_1 + 0.2x_2 + 0.9x_3 + 0.3x_4 + 0.2x_5 & \leq 1 \\
0.1389x_0 + 0.2778x_1 + 0.2222x_2 + 0.8889x_3 + 0.1667x_4 + 0.3333x_5 & \leq 1 \\
0.1136x_0 + 0.2955x_1 + 0.1818x_2 + 0.9546x_3 + 0.1364x_4 + 0.4545x_5 & \leq 1 \\
0.1041x_0 + 0.2708x_1 + 0.1667x_2 + 1x_3 + 0.125x_4 + 0.41667x_5 & \leq 1 \\
x_0 + 0x_1 + 0x_2 + 0x_3 + 0.8x_4 + 0x_5 & \leq 1 \\
0.1667x_0 + 0x_1 + 0.2222x_2 + 0x_3 + 0.4444x_4 + 0x_5 & \leq 1 \\
0.1364x_0 + 0.0909x_1 + 0.1818x_2 + 0x_3 + 0.3636x_4 + 0.1818x_5 & \leq 1 \\
0.125x_0 + 0.0833x_1 + 0.1667x_2 + 0.3333x_3 + 0.3333x_4 + 0.1667x_5 & \leq 1 \\
\end{align*}
\]

\[U = [0, 0, \ldots, 0]\]
We start from an empty solution, $U = [0, 0, \ldots, 0]$, $\|U\| = 0$, hence in the first iteration we compute only $V_j = \sum_{i=1}^{m} a_{ij}$ (that is, the sum of each column)

\[
V_0 = 0.1 + 0.0833 + 0.15 + 0.1389 + 0.1136 + 0.1041 + 0 + 0.1667 + 0.1364 + 0.125 = 1.118 \\
V_1 = 1.593 \\
V_2 = 1.639 \\
V_3 = 5.658 \\
V_4 = 3.1736 \\
V_5 = 2.6926
\]

\[
p_0/V_0 = 100/1.118 = 89.44 \\
p_1/V_1 = 376.575 \\
p_2/V_2 = 732.006 \\
p_3/V_3 = 424.1768 \\
p_4/V_4 = 157.549 \\
p_5/V_5 = 742.772
\]

Item 5 has the highest pseudo-utility, and adding it to the solution does not violate any constraint $\Rightarrow s = \{5\}$,

$U = [0.5125, 0.4271, 0.2, 0.333, 0.4545, 0.4167, 0, 0, 0.1818, 0.167] \\
\leq [1, \ldots, 1]$
We start from an empty solution, $U = [0, 0, \ldots, 0]$, $\|U\| = 0$, hence in the first iteration we compute only $V_j = \sum_{i=1}^{m} a_{ij}$ (that is, the sum of each column)

$V_0 = 0.1 + 0.0833 + 0.15 + 0.1389 + 0.1136 + 0.1041 + 0 + 0.1667 + 0.1364 + 0.125 = 1.118$

$V_1 = 1.593 \quad V_2 = 1.639$

$V_3 = 5.658 \quad V_4 = 3.1736 \quad V_5 = 2.6926$

$p_0/V_0 = 100/1.118 = 89.44 \quad p_1/V_1 = 376.575$

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\[
U = [0.5125, 0.4271, 0.2, 0.333, 0.4545, 0.4167, 0, 0, 0.1818, 0.167] \leq [1, \ldots, 1]\]
Second iteration: \( s = \{5\} \), profit = 2000

\[
U = [0.5125, 0.4271, 0.2, 0.333, 0.4545, 0.4167, 0, 0, 0.1818, 0.167],
\]

\[||U|| = 1.018386\]

From now on we have to consider \( U, ||U|| \)

\[
V_0 = (0.1 \times 0.5125 + 0.0833 \times 0.4271 + \cdots + 0.125 \times 0.167) / 1.018386 = 0.298
\]

\[
V_1 = 0.5503 \quad V_2 = 0.4596
\]

\[
V_3 = 2.088 \quad V_4 = 0.579
\]

\[
p_0/V_0 = 100/0.298 = 335.21
\]

\[
p_1/V_1 = 1090.379 \quad p_2/V_2 = 2610.75
\]

\[
p_3/V_3 = 1149.67 \quad p_4/V_4 = 862.93
\]

Item 2 has the highest pseudo-utility now, and adding it to the solution does not violate any constraint
Second iteration: \( s = \{5\} \), profit = 2000
\[
U = [0.5125, 0.4271, 0.2, 0.333, 0.4545, 0.4167, 0, 0, 0.1818, 0.167],
\]
\[\|U\| = 1.018386\]

From now on we have to consider \( U, \|U\|\)

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\( \frac{p_0}{V_0} = 100/0.298 = 335.21 \)
\( \frac{p_1}{V_1} = 1090.379 \quad \frac{p_2}{V_2} = 2610.75 \)
\( \frac{p_3}{V_3} = 1149.67 \quad \frac{p_4}{V_4} = 862.93 \)

Item 2 has the highest pseudo-utility now, and adding it to the solution does not violate any constraint
Third iteration: $s = \{2, 5\}$, profit = 3200

$U = [0.675, 0.5625, 0.4, 0.555, 0.636, 0.583, 0, 0.2222, 0.3636, 0.33]$, $||U|| = 1.509452$

$V_0 = (0.1 \times 0.6750 + 0.0833 \times 0.5625 + \cdots + 0.125 \times 0.3337) / 1.509452 = 0.3398$

$V_1 = 0.5649 \quad V_3 = 2.0769 \quad V_4 = 0.6816$

$p_0/V_0 = 100/0.3398 = 294.3 \quad p_1/V_1 = 1062.13$
$p_3/V_3 = 1155.549 \quad p_4/V_4 = 733.54$

Item 3 has the highest pseudo-utility now, but adding it would violate some constraints!

$U = [1.475, 1.344, 1.3, 1.44, 1.59, 1.58, 0, 0.222, 0.3636, 0.667]$

Next one is item 1, that does not violate any constraint
Third iteration: $s = \{2, 5\}$, profit = 3200
$U = [0.675, 0.5625, 0.4, 0.555, 0.636, 0.583, 0, 0.2222, 0.3636, 0.33]$, 
$\|U\| = 1.509452$

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\[ U = [1.475, 1.344, 1.3, 1.44, 1.59, 1.58, 0, 0.222, 0.3636, 0.667] \]
Next one is item 1, that does not violate any constraint
Fourth iteration: \( s = \{1, 2, 5\} \), profit = 3800

\[ U = [0.825, 0.6875, 0.7, 0.833, 0.932, 0.854, 0, 0.222, 0.4545, 0.417] \],

\[ \|U\| = 2.089237 \]

\[ V_0 = \left( 0.1 \times 0.825 + 0.0833 \times 0.6875 + \cdots + 0.125 \times 0.417 \right) / 2.089237 = 0.338 \]
\[ V_3 = 2.13 \quad V_4 = 0.656 \]

\[ \rho_0 / V_0 = 100 / 0.338 = 295.72 \quad \rho_3 / V_3 = 1126.7 \quad \rho_4 / V_4 = 762.5 \]

In order, we try to add items 3, 4, 0, but we cannot maintain feasibility. We're done.

Our final solution is \( s = \{1, 2, 5\} \), with a profit of 3800.
Fourth iteration: \( s = \{1, 2, 5\} \), profit = 3800
\[
U = [0.825, 0.6875, 0.7, 0.833, 0.932, 0.854, 0, 0.222, 0.4545, 0.417], \\
||U|| = 2.089237
\]

\[
V_0 = (0.1 \times 0.825 + 0.0833 \times 0.6875 + \cdots + 0.125 \times 0.417)/2.089237 = 0.338
\]
\[
V_3 = 2.13 \quad V_4 = 0.656
\]

\[
\rho_0/V_0 = 100/0.338 = 295.72 \quad \rho_3/V_3 = 1126.7 \quad \rho_4/V_4 = 762.5
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$V_3 = 2.13 \quad V_4 = 0.656$

$p_0/V_0 = 100/0.338 = 295.72 \quad p_3/V_3 = 1126.7 \quad p_4/V_4 = 762.5$

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\( \|U\| = 2.089237 \)

\[
V_0 = \frac{(0.1 \times 0.825 + 0.0833 \times 0.6875 + \cdots + 0.125 \times 0.417)}{2.089237} = 0.338
\]
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\[
\rho_0/V_0 = 100/0.338 = 295.72 \quad \rho_3/V_3 = 1126.7 \quad \rho_4/V_4 = 762.5
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$V_3 = 2.13 \quad V_4 = 0.656$

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In order, we try to add items 3, 4, 0, but we cannot maintain feasibility. We're done.

Our final solution is $s = \{1, 2, 5\}$, with a profit of 3800.
Iterative Improvement

\[
\begin{align*}
  s &:= \text{GenerateInitialSolution}() \\
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Exercise 1.1: Heuristics for the MKP

Iterative Improvement

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Which neighbour to choose? Pivoting rule

- **Best Improvement**: choose best from all neighbours of \( s \)
  - ✔ Better quality
  - ✗ Requires evaluation of all neighbours in each step

- **First improvement**: evaluate neighbours in fixed order and choose first improving neighbour.
  - ✔ More efficient
  - ✗ Order of evaluation may impact quality / performance
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Exercise 1.1: Heuristics for the MKP

**Variable Neighbourhood Descent**
- Based on either FI or BI (your choice)
- Evaluates increasingly large neighbourhoods

**Initial solution**
- Random constructive heuristic
- Greedy constructive heuristic
- Toyoda algorithm
Exercise 1.1: Heuristics for the MKP

Variable Neighbourhood Descent
- Based on either FI or BI (your choice)
- Evaluates increasingly large neighbourhoods

Initial solution
- Random constructive heuristic
- Greedy constructive heuristic
- Toyoda algorithm
Many neighbourhoods are possible for the MKP:

- remove $k$ items and add $k$ different items
- remove $k$ items and add as many different items as possible

→ in which order?

- add $k$ items, make the solution non feasible, then remove $k$ different items
- add $k$ items, make the solution non feasible, then remove as many different items as needed

→ in which order?

For this project, the neighbourhood is given by the set of moves that remove $k$ items and add as many different items as possible, according to the order specified as follows.

Every solution accepted has to be feasible.
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- use delta evaluation for both obj. function and constraint checking
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- use delta evaluation for both obj. function and constraint checking
Consider our example instance, and an initial solution generated using the greedy criterion, and a BI rule: $s = \{3\}$, profit = 2400.

First iteration:

- shuffle the list of items in $s$
- shuffled list is [3]
- sort list of non-inserted items by profit: [5, 2, 1, 4, 0]
- remove first item from shuffled $s$: {}
- try to add the non-selected items, one by one, ordered by profit
  - we can add 5, 2, 1, but not 4 or 0 (check feasibility). Hence: $s' = \{1, 2, 5\}$, profit = 3800, not inserted \{0, 3, 4\}
  - we cannot remove any more items from the shuffled $s$, so we have evaluated the whole neighbourhood. $s'$ is feasible and it improves over $s$, so we accept it.
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Neighbourhood for the MKP: example (2/2)

\[ s = \{1, 2, 5\}, \text{ profit } = 3800, \text{ not inserted } \{0, 3, 4\} \]

Next iteration, shuffle \( s \): \([2, 5, 1]\). The other items sorted by profit are \([3, 4, 0]\). Hence:

- Remove item 2 from \( s \), \( s' = \{1, 5\} \), try to add 3, then 4, then 0
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Neighbourhood scan completed, no improving move found: stop.
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Neighbourhood scan completed, no improving move found: stop.
To recap:

Implement 3 constructive heuristics:
- Random
- Greedy
- Toyoda algorithm

Implement 3 FI heuristics:
- Random init. sol. + FI with random insertion
- Greedy init. sol. + FI with greedy insertion
- Toyoda init. sol. + FI with Toyoda insertion

Implement 3 BI heuristics:
- Random init. sol. + BI with random insertion
- Greedy init. sol. + BI with greedy insertion
- Toyoda init. sol. + BI with Toyoda insertion

Implement 3 VND heuristics (based either on FI or BI):
- Random init. sol. + VND with random insertion
- Greedy init. sol. + VND with greedy insertion
- Toyoda init. sol. + VND with Toyoda insertion
Material provided on the course homepage:

- C code for
  - reading an MKP instance
  - computing the objective function value
- you can use the code as is, incapsulate it into C++ classes, rewrite it in Java
- 60 problem instances
  - + example instance for debugging
- list of best known values
- exercise sheet and this slide deck
http://people.brunel.ac.uk/~mastjjb/jeb/orlib/mknapsackinfo.html

- in the first line: problem size
  - number of items $n$
  - number of constraints $m$
  - value of the optimal solution (0 if not known)

- the list of $n$ profits $p_j$ (on multiple lines)

- the list of $n \times m$ weights $w_{ij}$ (row-wise, on multiple lines)

- the list of $m$ capacities $L_i$ (on multiple lines)
Exercise 1.1: Heuristics for the MKP

Instances
- Run the Random constructive heuristic and the II based on it 15 times for each instance, and average the results
- Run the Greedy and Toyoda constructive heuristics and the II based on them once per each instance

Experiments
For each algorithm \( k \) and instance \( i \) compute:

1. Relative percentage deviation \( \Delta_{ki} = 100 \cdot \frac{\text{profit}_{ki} - \text{best-known}_i}{\text{best-known}_i} \)
2. Computation time (\( t_{ki} \))
3. (using the average values when the Random constructive heuristic is involved)
Exercise 1.1: Heuristics for the MKP

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Experiments

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1. Relative percentage deviation $\Delta_{ki} = 100 \cdot \frac{\text{profit}_{ki} - \text{best-known}_i}{\text{best-known}_i}$

2. Computation time ($t_{ki}$)

3. (using the average values when the Random constructive heuristic is involved)

Report for each algorithm $k$

- Average relative percentage deviation
- Sum of computation time across all instances of a same size
Exercise 1.1: Heuristics for the MKP

Experiments

For each algorithm $k$ and instance $i$ compute:

1. Relative percentage deviation $\Delta_{ki} = 100 \cdot \frac{\text{profit}_{ki} - \text{best-known}_{i}}{\text{best-known}_{i}}$

2. Computation time ($t_{ki}$)

3. (using the average values when the Random constructive heuristic is involved)

Report for each algorithm $k$

- Average relative percentage deviation
- Sum of computation time across all instances of a same size
Exercise 1.1: Heuristics for the MKP

Is there a statistically significant difference between the solution quality generated by the different algorithms?

Statistical test
- Paired t-test
- Wilcoxon signed-rank test
- compare the results across the same size
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Background: Statistical hypothesis tests (1)

- **Statistical hypothesis tests** are used to assess the validity of statements about properties of or relations between sets of statistical data.

  The statement to be tested (or its negation) is called the null hypothesis ($H_0$) of the test.

  **Example:** For the Wilcoxon signed-rank test, the null hypothesis is that ‘the median of the differences is zero’.

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**Background: Statistical hypothesis tests (2)**

- The application of a test to a given data set results in a *p-value*, which represents the probability that the null hypothesis is incorrectly rejected.

- The null hypothesis is rejected iff this p-value is smaller than the previously chosen significance level.

- Most common statistical hypothesis tests are already implemented in statistical software such as the R software environment (http://www.r-project.org/).
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Example in R

```r
best.known <- read.csv("bestSolutions.txt")
a.cost <- read.table("ii-best-ex-rand.dat")$V1
a.cost <- 100 * (a.cost - best.known) / best.known$BS
b.cost <- read.table("ii-best-ins-rand.dat")$V1
b.cost <- 100 * (b.cost - best.known) / best.known$BS
t.test (a.cost, b.cost, paired=T)$p.value
[1] 0.8819112
wilcox.test (a.cost, b.cost, paired=T)$p.value
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Instances and barebone code available at:
http://iridia.ulb.ac.be/~stuetzle/Teaching/HO/

Deadline is April 15 (23:59)

Questions in the meantime?
alberto.franzin@ulb.ac.be