Independence Concepts for Belief Functions

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Abstract

In this paper, we try to study the independence concept for belief functions theory, as applied to one interpretation of this theory called the *transferable belief model* (TBM). In this context, two new results are given in this paper : first, the concept of belief function independence has different intuitive meaning which are non-interactivity, irrelevance and doxastic independence, second, the concepts of non-interactivity and independence are identical under a new property called *irrelevance preservation under Dempster's rule of combination*.

Keywords: Independence, Irrelevance, Non-Interactivity, Belief functions, Transferable Belief Model.

1 Introduction

For managing uncertainty reasoning systems, a main requirement is to specify the conditions under which one item of information is considered dependent (or independent) from another, given what we already know, and to represent knowledge in structures that display these conditions. In probabilistic framework, these conditions are identified with the notion of *independence*, also called *irrelevance* or *informational irrelevance* [13].

In addition to the obvious theoretical reasons for the study of independence, there are also practical interest. Indeed, using independence, we can modularize a complex problem (for example, knowledge base) into simpler components in such a way that we only treat the smaller sub models (for example, pieces of information having relevance to the question we are interested in). So, in order to get an efficient performance, reasoning systems must take into account independence considerations.

There are two main approaches to define independence:

1. Irrelevance approach : Two variables are said to be *independent* if no piece of information that can be learn about one of them can change our state of knowledge about the other. This form of independence can be expressed by experts.

2. Decomposition approach : Two variables are said to be *independent* if the global information about the two variables can be expressed as a combination of two pieces of knowledge, one for each variable. This form of independence permits to work in efficient way (local computations without loosing any information).

In the case of probability theory, both approaches are equivalent. However, these approaches do not have identical meaning for belief functions theory.

In this paper, we try to clarify the notion of independence between variables (or subsets of variables) when uncertainty is expressed by belief functions. In other words, our main purpose is to find a common sense meaningful definition of independence. Indeed, this concept, related to one interpretation of the belief functions theory called the transferable belief model [16], has different intuitive meaning, but unlike probability theory, this concept has not received a complete treatment in the literature.

The rest of this paper is organized as follows: in Section 2, we present some useful notations needed for belief functions. Next, in Section 3, we briefly review the basic elements of belief functions theory based on the transferable belief model. The different definitions of the concept of belief function independence have been discussed in Section 4, making clear the links between them. Finally, in Section 5, we give a brief summary.

2 Notations

In this section, we give some notations which are convenient when belief functions are used.

Let U = $\{X, Y, Z, ...\}$ be the set of all variables, $\Theta_X = \{x_1, ..., x_n\}$ be the domain relative to the variable X (with a cardinality n), and x represents any instance of X. Let Ω be a frame of discernment [14] composed of the Cartesian product of all domains of variables in U (finite domain on which beliefs are held), where, for example, $\Theta_X \times \Theta_Y$ represents the product space of the variables X and Y, and when there is no ambiguity, it is simply denoted by XY.

Given a background knowledge, denoted by BK, representing conditioning event, we propose the following notations:

- $bel^{\Omega}[BK]$ denotes the belief function on Ω when BK holds. It can be seen as a vector in a $2^{|\Omega|}$ dimensional space. Classically, it was denoted as $bel^{\Omega}(. \mid BK)$.
- $bel^{\Omega}[BK](A)$ denotes the value of the belief function at A (A $\subseteq \Omega$) given BK.

- $bel^{\Omega\downarrow X}$ is the marginal of bel^{Ω} on X. The Ω superscript will not be mentioned when there is no risk of confusion.

3 Belief Function Theory and Transferable Belief Model

The theory of belief functions, also known as Dempster-Shafer theory and theory of evidence, aims to model someone's degree of belief. It is regarded as a generalization of the Bayesian approach. Since this theory was developed by Shafer [14], many interpretations have been proposed. Among them, we can distinguish:

- **a lower probability model** where beliefs are represented by families of probability functions. This model is considered as a special case of imprecise probabilities.
- **Dempster's model** derived from probability theory and represented by hints theory [10].
- **The transferable belief model (TBM)** unrelated to probability theory where beliefs are represented by belief functions. This model is introduced by Smets [16] in order to justify the use of belief functions.

In this paper, we are only concerned with the TBM, so we will use the concepts based on this model.

Definition 1 *Let Ω be the frame of discernment. The mapping $bel : 2^{\Omega} \rightarrow [0,1]$ is an (unnormalized) belief function if and only if there exists a basic belief assignment (bba) $m : 2^{\Omega} \rightarrow [0,1]$ such that:*
(i) $\sum m(A) = 1$ *for* $A \subseteq \Omega$
(ii) $bel(A) = \sum m(B)$ *for* $B \subseteq A, B \neq \emptyset$
(iii) $bel(\emptyset) = 0$

The value $m(A)$ represents the degree of belief that is exactly committed to A. Due to the lack of information, $m(A)$ cannot support any more specific event. The value $bel(A)$ quantifies the strength of the belief that the event A occurs. A subset A such that $m(A) > 0$ is called a *focal element* of bel. bel is *vacuous* if the only focal element is Ω. In TBM context, we accept that none of the elements could be true, so $m(\emptyset)$ can be positive (open-world assumptions) [16].

Given a belief function bel, we can define a *plausibility function* pl: $2^{\Omega} \rightarrow [0,1]$ and a *commonality function* q: $2^{\Omega} \rightarrow [0,1]$ as follows: for $A \subseteq \Omega$,

$$pl(A) = bel(\Omega) - bel(\overline{A}) \quad \text{and} \quad pl(\emptyset) = 0$$
$$= \sum m(B) \quad \text{for } B \cap A \neq \emptyset$$

$$q(A) = \sum m(B) \quad \text{for } A \subseteq B \subseteq \Omega$$

The value $pl(A)$ quantifies the maximum amount of potential specific support that could be given to A. The commonality function $q(A)$ is useful for simplifying some computations. It is proved that m, bel, pl and q are in one-to-one correspondence with each other [14].

4 Belief Function Independence

The notion of informational irrelevance has been extensively studied in probability theory [6], [7], [12], [13], where it is identified with independence or more specifically conditional independence. The concept of independence has also been studied in non-probabilistic frameworks such that Spohn's theory of ordinal conditional functions [17], Zadeh's possibility theory [1], [3], [8], [9], [19], [20], upper and lower probabilities theory [2], [4], [5], and in abstract framework that unifies different calculi called valuation-based system [15]. However, the concept of independence for variables has not been widely treated in belief functions theory.

The aim of this section is to investigate some ways to define independence relationships between variables when uncertainty is expressed by belief functions. Some other researches studying this topic are [2], [14], and [18]. We concentrate on the intuitive meaning on each definition and we discuss the possible links between them.

In this section, we consider two variables X and Y. The frame of discernment Ω is the Cartesian product of X and Y. Formally, $\Omega = \Theta_X \times \Theta_Y$ (simply noted as $\Omega = XY$) where Ω is the minimal refinement of X and Y ([14], page 123).

Previously, we recall the definition of probabilistic independence. We say that two random variables X and Y are (*marginally*) *independent* under a distribution P on the space XY, denoted by $X \amalg_P Y$, if and only if one of the following conditions is satisfied: $\forall x \subseteq X$, $\forall y \subseteq Y$

- $P^{XY}(x,y) = P^{XY\downarrow X}(x)P^{XY\downarrow Y}(y)$

  where $P^{XY\downarrow X}$ and $P^{XY\downarrow Y}$ are the marginal probabilities of P on X and Y, respectively.

- $P^{XY}[y]^{\downarrow X}(x) = P^{XY\downarrow X}(x)$

  where $P^{XY}[y]^{\downarrow X}$ is the conditional probability on X given y.

Remark. This notation is more cumbersome than the usual one (i.e. such as in [7], [13]), but it helps when belief functions are involved as it avoids confusion.

The first definition of independence is presented in terms of the *factorization* of the joint probability distribution through its marginal distributions on X and Y, respectively (a mathematical property). However, the second can be interpreted in terms of *(ir)relevance* of information, it means that any information about Y is irrelevant to the uncertainty about X (an epistemic property). In probability context, there is no distinction between irrelevance and independence.

To extend these definitions of independence to the case of belief functions, Shafer ([14], page 147 *et seq.*) proposes two definitions of independence. After recalling these definitions, we introduce our definitions of marginal non-interactivity, irrelevance and doxastic independence for variables.

4.1 Cognitive Independence : Weak Independence

Following Shafer [14], two variables are "*cognitively independent*" with respect to a belief function if new evidence that bears on only one of them does not change the degrees of belief for propositions discerned by the other. This notion of "cognitive independence" is also called *weak independence* in [11]. The formal definition of "cognitive independence" is the following :

Definition 2 *[14] : The variables X and Y are cognitively independent with respect to m^{XY} if and only if: for all $x \subseteq X$, all $y \subseteq Y$,*

$$pl^{XY}(x,y) = pl^{XY\downarrow X}(x)\; pl^{XY\downarrow Y}(y)$$

4.2 Evidential Independence : Strong Independence

The definition of "cognitive independence" is very weak. Shafer [14] proposed another notion of independence called "*evidential independence*" [1]: two variables are "*evidentially independent*" if their joint belief function is represented by the combination of their marginals using Dempster's rule of combination. The formal definition of evidential independence is as follows :

Definition 3 *[14] : The variables X and Y are evidentially independent with respect to m^{XY} if and only if: for all $x \subseteq X$, all $y \subseteq Y$,*
(i) $pl^{XY}(x,y) = pl^{XY\downarrow X}(x)\; pl^{XY\downarrow Y}(y)$
(ii) $bel^{XY}(x,y) = bel^{XY\downarrow X}(x)\; bel^{XY\downarrow Y}(y)$

Based on the definition of evidential independence, let us state the following theorems:

Theorem 1 *The variables X and Y are "evidentially independent" with respect to m^{XY} if and only if:*

$$\begin{aligned} m^{XY}(w) &= m^{XY\downarrow X}(x)m^{XY\downarrow Y}(y),\ \textit{if } w=(x,y) \\ &= 0,\ \textit{otherwise}. \end{aligned}$$

where x is the projection of w on X, and y is the projection of w on Y.

Theorem 2 *The variables X and Y are "evidentially independent" relative to m^{XY} if and only if:*

$$q^{XY}(w) = q^{XY\downarrow X}(x)\; q^{XY\downarrow Y}(y),\ \forall w \subseteq XY$$

where x is the projection of w on X, and y is the projection of w on Y.

In fact, "cognitive independence" is a weaker condition than "evidential independence" : if two variables are "evidentially independent" with respect to a belief function, then they will be "cognitively independent" with respect to it. Indeed, "evidential independence" requires constraints on *bel* and on *pl* whereas "cognitive independence" requires only constraints on *pl*.

Remark. Cognitive independence may hold whereas evidential independence fails. In addition, neither (1) nor (2) implies the other [14]. This may be shown in the following example.

Example. Let G and S be two variables representing Gender and Smoking and taking their values in $\Theta_G = \{\text{Male, Female}\}$ and $\Theta_S = \{\text{Yes, No}\}$, respectively. Let $\Omega = \{(\text{Male,Yes}), (\text{Female,Yes}), (\text{Male,No}), (\text{Female,No})\}$ be the frame of discernment representing the product space $\Theta_G \times \Theta_S$.

[1] It is also called *strong independence* in [11].

Define the refining $\omega_1 : 2^{\Theta_G} \to 2^{\Omega}$ by:
ω_1({Male}) = {(Male,Yes), (Male,No)} ≡ M,
ω_1({Female}) = {(Female,Yes), (Female,No)} ≡ F,
and define the refining $\omega_2 : 2^{\Theta_S} \to 2^{\Omega}$ by:
ω_2({Yes}) = {(Male,Yes), (Female,Yes)} ≡ Y,
ω_2({No}) = {(Male,No), (Female,No)} ≡ N,

Suppose that we have the following basic belief assignment (bba) over the product space Ω:
m{(Male,Yes)} = 0.5
m{(Male,Yes), (Female,Yes), (Male,No)} = 0.25
m{Ω} = 0.25

The corresponding plausibility functions are:
pl(M) = pl({(Male,Yes), (Male,No)}) = 1
pl(F) = pl({(Female,Yes), (Female,No)}) = 0.5
pl(Y) = pl({(Male,Yes), (Female,Yes)}) = 1
pl(N) = pl({(Male,No), (Female,No)}) = 0.5

Then, we can easily verify that :
pl({(Male,Yes)} = pl(M,Y) = pl(M) pl(Y) = 1
pl({(Female,Yes)} = pl(F,Y) = pl(F) pl(Y) = 0.5
pl({(Male,No)} = pl(M,N) = pl(M) pl(N) = 0.5
pl({(Female,No)} = pl(F,N) = pl(F) pl(N) = 0.25

But bel({(Male,Yes)} = bel(M,Y) = 0.5 is not equal to bel(M) . bel(Y) = 0.5 × 0.5 = 0.25. So, we conclude that G and S are cognitively independent. But, they are not evidentially independent.

4.3 Belief function Non-Interactivity

In this section, we propose the definition of decompositional belief function based on the basic belief assignment functions. In possibility theory, there is an analogous definition introduced by Zadeh [20] where the decompositional independence between two variables is represented by the **non-interactivity** relation. The non-interactivity is a mathematical property useful for computation considerations. In our work, we use the same terminology.

Intuitively, the *non-interactivity* of two variables X and Y with respect to m^{XY} means that the joint mass can be reconstructed from its marginals. The formal definition of non-interactivity is as follows :

Definition 4 *Given two variables X and Y, and $m = m^{XY}$ on XY. X and Y are non-interactive with respect to m , denoted by $X \perp_m Y$, if and only if :*

$$m^{XY} = m^{XY\downarrow X} \oplus m^{XY\downarrow Y}$$

¿From theorem 1, non-interactivity and Shafer's evidential independence are equivalent.

The straightforward propositions are:

Proposition 1 *The product of m implies the product of plausibility, but NOT the reverse.*

Proposition 2 *The product of m implies the product of belief function, but NOT the reverse.*

4.4 Belief Function Irrelevance

In probability theory, the notion of independence can be defined in term of **irrelevance**. This kind of independence is based on *conditioning*. The intuitive meaning of irrelevance is that knowing the value y of Y does not affect belief on X. In belief functions theory, the formal definition of irrelevance is the following:

Definition 5 *Given two variables X and Y, and $m = m^{XY}$ on XY. Y is irrelevant to X with respect to m, denoted by $IR_m(X,Y)$, if and only if: $\forall y \subseteq Y$ such that $pl^{XY}(y) > 0$*

$$m^{XY}[y]^{\downarrow X}(x) \quad \propto \quad m^{XY\downarrow X}(x),\ \forall x \subseteq X, x \neq \emptyset$$

and $\forall y \subseteq Y$ *such that* $pl^{XY}(y) = 0$

$$m^{XY}[y]^{\downarrow X}(x) = 0,\ \forall x \subseteq X, x \neq \emptyset,\ \text{and } m^{XY}[y]^{\downarrow X}(\emptyset) = 1.$$

We need $\propto$ because in the TBM context we don't normalize when applying Dempster's rule of conditioning. Under normalization, proportionality becomes equality.
Based on the definition of irrelevance, we can deduce the following consequences, where the second item of the proposition 3 implies that IR is rquivalent to Shafer's cognitive independence. Nevertheless, we will show that IR is not equivalent to non-interactivity.

Proposition 3 *Y is irrelevant to X with respect to m, if and only if :*

1. $$pl^{XY}[y_1]^{\downarrow X} = \alpha_y pl^{XY}[y_2]^{\downarrow X} \tag{1}$$

   where $\alpha_y = \frac{pl^{XY}(Xy_1)}{pl^{XY}(Xy_2)}$ *(α_y independent of x)*

2. $$pl^{XY}(xy) = \frac{pl^{XY}(xY)\ pl^{XY}(Xy)}{pl^{XY}(XY)} \tag{2}$$

3. $$IR_m(X,Y) = IR_m(Y,X) \tag{3}$$

In the following example, we show that irrelevance does not imply non-interactivity between variables.
Example. Suppose $X = \{x_1, x_2\}$ and $Y = \{y_1, y_2\}$ and let $\Omega = X \times Y = \{a, b, c, d\}$ where $a = (x_1, y_1), b = (x_2, y_1), c = (x_1, y_2), d = (x_2, y_2)$. We present in table 1 a bba m^Ω such that $IR_{m^\Omega}(X,Y)$ even though we do not have $X \perp_{m^\Omega} Y$.

We start with m^Ω, compute its related pl^Ω (table 1). Then we present in table 2 (3) the values of the bba and their related plausibility functions on X (Y) after conditioning on y_1, y_2, and Y (x_1, x_2 and X).

Using the marginals on X and on Y, that is $m^{\Omega\downarrow X}$ and $m^{\Omega\downarrow Y}$, which are given in the rightmost columns of the tables 2 and 3, we compute $m^{\Omega\downarrow X} \oplus m^{\Omega\downarrow Y}$ which result is given in table 1, fourth column. It can be seen that $m^\Omega \neq m^{\Omega\downarrow X} \oplus m^{\Omega\downarrow Y}$.

Nevertheless X and Y are irrelevant to each other with respect to m^Ω. Indeed, we can see that the conditional bba's on X (Y) table 2 (3) satisfy the

Ω	m^Ω	pl^Ω	M	PL	Required
$\emptyset$	.0000	0	0	.0000	$\surd$
a	.1275	.3000	.13	.3000	$\surd$
b	.17	.3500	.17	.3500	$\surd$
ab	.1025	.5000	.10	.5000	$\surd$
c	.16	.3600	.16	.3600	$\surd$
ac	.0125	.6000	.01	.6000	$\surd$
bc	0	.6550	.00	.6800	
abc	.0075	.7925	.01	.8000	
d	.2075	.4200	.21	.4200	$\surd$
ad	.01	.6700	.01	.6900	
bd	.0225	.7000	.02	.7000	$\surd$
abd	0	.8400	.00	.8500	
cd	.1325	.6000	.13	.6000	$\surd$
acd	0	.8300	.00	.8400	
bcd	.0075	.8725	.01	.8800	
abcd	.04	1.0000	.04	1.0000	$\surd$

Table 1: For each subset of $\Omega = X \times Y$, listed in column 1, the columns 2 and 3 present the value of m^Ω and of its related pl^Ω. The column 4 presents the values of M $= m^X \oplus m^Y$ where $m^X = m^{\Omega\downarrow X}$ and $m^Y = m^{\Omega\downarrow Y}$. Column 5 presents the plausibility function PL $= pl^{\Omega\downarrow X} \oplus pl^{\Omega\downarrow Y}$ related to the bba of column 4. Column 6 indicates by $\surd$ those subsets of Ω where the equality for the plausibility functions is required.

proportionality requirement of the IR definition. Identically, we show that pl^Ω and $pl^{\Omega\downarrow X} \oplus pl^{\Omega\downarrow Y}$ are equal on those subsets of Ω where such equality is required by the IR definition, that is those indicated by a $\surd$ symbol in the 'Required' column of table 1.

X	$m^\Omega[ab]^{\downarrow X}$	$pl^\Omega[ab]^{\downarrow X}$	X	$m^\Omega[cd]^{\downarrow X}$	$pl^\Omega[cd]^{\downarrow X}$
$\emptyset$	.50	0	$\emptyset$	.40	0
a	.15	.30	c	.18	.36
b	.20	.35	d	.24	.42
ab	.15	.50	cd	.18	.60

X	$m^{\Omega\downarrow X}$	$pl^{\Omega\downarrow X}$
$\emptyset$	0	0
ac	.30	.60
bd	.40	.70
abcd	.30	1.00

Table 2: Bba and plausibility after marginalization on X of the bba obtained by the conditioning of m^Ω on the values of Y.

But irrelevance seems to weak and does not imply what we feel should be the definition of "independence". In particular, our definition of irrelevance does not imply "non-interactivity" as shown in the last example. Nevertheless, we feel the next property should also be satisfied by irrelevance, in which case non-interactivity and irrelevance become equal.

Y	$m^{\Omega}[ac]^{\downarrow Y}$	$pl^{\Omega}[ac]^{\downarrow Y}$	Y	$m^{\Omega}[bd]^{\downarrow Y}$	$pl^{\Omega}[bd]^{\downarrow Y}$
$\emptyset$	.40	0	$\emptyset$	.30	0
a	.24	.30	b	.28	.36
c	.30	.36	d	.35	.42
ac	.06	.60	bd	.07	.60

Y	$m^{\Omega\downarrow Y}$	$pl^{\Omega\downarrow Y}$
$\emptyset$	0	0
ab	.40	.50
cd	.50	.60
abcd	.10	1.00

Table 3: Bba and plausibility after marginalization on Y of the bba obtained by the conditioning of m^{Ω} on the values of X.

Let A_1 and A_2 denote two agents whose beliefs are considered. The idea is when the first agent A_1 claim that Y is irrelevant to X and produce his beliefs and the second A_2 tell me that Y is irrelevant to X and produce his own beliefs, then I want that Y is still irrelevant to X for me and my belief will be equal to the combination of agents' beliefs (by application of Dempster's rule of combination).

This idea can be explicitly formulate by the next property called **Irrelevance Preservation under Dempster' rule of combination**, denoted by IRP$\oplus$.

Definition 6 *: Irrelevance Preservation under Dempster' rule of combination*
If $IR_{m_1}(X,Y)$ and $IR_{m_2}(X,Y)$ then $IR_{m_1 \oplus m_2}(X,Y)$

Remark. This property is not describeed in probability theory as the concept of combination and the $\oplus$ operation are hardly considered.

Now, we can state one main theorem of this work:

Theorem 3 *Let $\Omega = XY$ and $m = m^{XY}$. If $IR_m(X,Y)$ and if for all m' defined on XY such that $IR_{m'}(X,Y)$, we have $IR_{m \oplus m'}(X,Y)$, then $X \perp_m Y$.*

This theorem means that when Y is irrelevant to X under m and this irrelevance is preserved under Dempster' rule of combination then X and Y are non-interactive under m.

4.5 Doxastic Independence

The most obvious difference between probabilistic independence and belief function independence is that irrelevance and independence have not identical meaning in the belief function framework. This distinction is not commonly considered in probabilistic framework where both Pearl [13] and Dawid [7] use the words irrelevance and independence interchangeably.

In order to enhance this distinction, we use the expression **doxastic independence** for belief function independence. In Greek, 'doxein' means to believe. The formal definition of doxastic independence is as follows :

Definition 7 *Given two variables X and Y, and m on XY. X and Y are doxastically independent with respect to m, denoted by $X \amalg_m Y$, if and only if m satisfies:*

- $IR_m(X, Y)$
- $\forall m_0$ *on* XY : $IR_{m_0}(X, Y) \Rightarrow IR_{m \oplus m_0}(X, Y)$

The intuitive meaning of this definition is that two variables are considered as doxastically independent only when they are irrelevant and this irrelevance is preserved under Dempster's rule of combination.

Theorem 4 *Doxastic independence preservation under* $\oplus$.
If $X \amalg_{m_1} Y$ *and* $X \amalg_{m_2} Y$ *then* $X \amalg_{m_1 \oplus m_2} Y$

The link between doxastic independence and non- interactivity is given by the next theorem :

Theorem 5 *Given two variables X and Y, and m on XY. X and Y are doxastically independent with respect to m if and only if X and Y are non-interactive with respect to m.*

5 Conclusion

In this paper, we have studied different concepts of independence for belief functions. Of special interest for us is to clarify the relationships between the concepts of non-interactivity, irrelevance and doxastic independence when uncertainty is expressed under the form of belief functions. These concepts of marginal independence for belief functions can be extended to conditional case which successfully depict our intuition about how dependencies should update in response to new pieces of information. In fact, the study of *conditional independence* in the framework of belief functions theory was not sufficiently developed. More detailed research of conditional belief function independence is under way. It will be usefull for the practical use of belief functions in Artificial Intelligence.

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