Target Identification Using Belief Functions and Implication Rules

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Abstract

The paper presents the theoretical basis of data fusion for the purpose of target identification using the belief function theory. The key feature of this paper is that we allow the knowledge sources to supply their information in the form of the implication rules. The paper describes how these rules can be elegantly handled within the framework of the belief function theory. A small scale practical example for target identification is worked out in details in order to clarify the theory for future users.

Keywords: Target identification, belief function theory, evidential theory, implication rules.

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1 Introduction

Target identification is an important component of data fusion systems for air surveillance, with a task to combine the information collected over time from multiple sources in order to refine our knowledge of target allegiance (e.g. friend, neutral, hostile), target class (e.g. commercial jet, bomber, fighter plane, missile, etc) and platform type (for example for a class of fighter planes, the possible platform types could be F/A-18, Su-35, Tornado Typhoon, etc) [1].

Target identification is based on a set of features which can up to a certain degree distinguish between the targets. Three groups of target features are typically used for this purpose: the features based on target shape, kinematic behaviour and electro-magnetic (EM) emissions. Radar can provide the kinematic features (maximum observed speed, acceleration, target specific Doppler signatures) and some shape features (range profile, RCS). Kinematic information can be used to determine the originating airfield or to determine the level of agreement with the approved flight plan, mission route or a flight corridor. An IR sensor can provide additional shape features, such as the target spatial distribution, or target area. By analysing target EM emissions, an ESM sensor can supply the transmitted frequency, the pulse repetition interval and the pulse width of the target radar, which can then be related to the emitter type or even the platform type. The IFF provides a high confidence positive identification of friendly aircraft. Target features supplied by information sources are typically uncertain due to randomness, unspecificity or fuzziness [2].

The belief function theory as used in the transferable belief model (TBM) [3], provides a flexible and accurate framework for target identification at least for two reasons. First, it solves the problem of ill known or ill defined prior probabilities or feature likelihoods required in the Bayesian probabilistic framework. Using such probabilities as the true ones is potentially dangerous because it may result in inadequate decisive decisions based on "arbitrary" information. The TBM provides the means to handle these issues. The design and performance of a TBM based cautious classifier are described in [4].

The second reason to use the TBM for target classification is relevant when the sensors/sources of target attributes are unspecific. An example of an unspecific sensor report is a declaration that the target is "small", because there could be at least several possible candidates that satisfy this description (note that this declaration is also fuzzy, but this aspect is ignored in this paper). Unspecificity is not adequately represented by probability functions, as illustrated in [5] in the context of data association in multi-object classification. The belief function theory, on the contrary, handles unspecifity in a correct manner, thus being able to fuse sensor reports at different levels of granularity [6].

Prior knowledge at disposal for target identification sometimes also includes one or more implication rules. An example of such a rule could be as follows: if the target is a friendly aircraft, then its platform type must be F/A-18, Tornado or B-52. The rules can in general be assigned some belief mass, taking value in the range between 0 and 1. In this paper we describe the methodology for incorporation of implication rules into the belief function theory framework for target identification. The subject has been ignored so far in the literature on target identification using the belief functions [7],[8],[9, Ch.8]. The paper works out in detail a small scale problem for target identification, in order to clarify the theory for the future users.

2 A review of the belief function theory

We adopt the framework and terminology of the belief function theory [10] as interpreted by the transferable belief model (TBM) [11, 3, 12]. The theory is originally developed for a discrete set of elementary events related to a given problem [10, 3]. This set is referred to as the frame of discernment (or frame):

$$\Theta = \{\theta_1, \theta_2, \ldots, \theta_N\} \tag{1}$$

and it has a finite cardinality $N = |\Theta|$. Beliefs are expressed on the subsets of Θ. The power set of Θ, denoted as 2^Θ, is a set containing all the possible subsets of Θ, i.e. $2^\Theta = \{A : A \subseteq \Theta\}$. The cardinality of the power set equals 2^N. The belief is represented by a so-called *basic belief assignment* (bba) $m : 2^\Theta \to [0, 1]$, that satisfies $\sum_{A\subseteq\Theta} m(A) = 1$. The bba assigned to the empty set $m(\emptyset)$ is interpreted as the amount of conflict or as the result of the possibility that Θ is not exhaustive. The subsets A with a property $m(A) > 0$ are referred to as focal sets of the bba. The union of all focal sets of a bba is called its *core*. The state of complete ignorance is represented by a *vacuous* bba defined as $m(A) = 1$ if $A = \Theta$ and zero otherwise.

Belief and plausibility. The belief function $bel : 2^\Theta \to [0, 1]$ and plausibility function $pl : 2^\Theta \to [0, 1]$ are associated to a bba m, and introduced as a convenient interpretation of belief:

$$bel(A) = \sum_{\emptyset\neq B\subseteq A} m(B) \quad \forall A \subseteq \Theta, \tag{2}$$

$$pl(A) = \sum_{A\cap B\neq\emptyset} m(B) \quad \forall A \subseteq \Theta. \tag{3}$$

Both bel and pl are in one-to-one correspondence with their associated m: $bel(A)$ represents the total belief that is committed to A without also being committed to its complement $\bar{A}$; $pl(A)$ corresponds to the total belief which does not contradict A.

Refinement and vacuous extension. Suppose a bba $m^{\Theta'}$ is defined on a frame of discernment[1] Θ'. Let Θ be a refinement ρ of Θ', that is, every element $\theta' \in \Theta'$ is mapped by ρ into one or more elements of Θ, and the image of θ' under ρ on Θ is an element of a partition of Θ. The bba $m^{\Theta'}$ can be extended on Θ in an information content preserving manner via the vacuous extension, denoted as $m^{\Theta'\uparrow\Theta}$. The values of $m^{\Theta'\uparrow\Theta}$ are given by:

$$m^{\Theta'\uparrow\Theta}(\theta) = \begin{cases} m^{\Theta'}(\theta'), & \text{if } \theta = \rho(\theta') \\ 0, & \text{otherwise,} \end{cases} \tag{4}$$

where $\rho(\theta')$ is the image of θ' under ρ. For example, let $\Theta' = \{a, b\}$ and $m^{\Theta'}$ specified as $m^{\Theta'}(\{a\}) = 0.6$, $m^{\Theta'}(\Theta') = 0.4$. Let $\Theta = \{a_1, a_2, b_1, b_2, b_3\}$ be a refinement of Θ' such that $\rho(a) = \{a_1, a_2\}$ and $\rho(b) = \{b_1, b_2, b_3\}$. The vacuous extension of $m^{\Theta'}$ is a bba with two focal sets, $m^{\Theta'\uparrow\Theta}(\{a_1, a_2\}) = 0.6$ and $m^{\Theta'\uparrow\Theta}(\Theta) = 0.4$.

Coarsening. Suppose a bba m^{Θ} is defined on Θ. Let Θ' be a coarsening of Θ such that Θ is a refinement ρ of Θ'. The bba induced on Θ' by m^{Θ} is denoted by $m^{\Theta\downarrow\Theta'}$, and the values of its related $bel^{\Theta\downarrow\Theta'}$ are:

$$bel^{\Theta\downarrow\Theta'}(\theta') = bel^{\Theta}(\rho(\theta')) \quad \forall\theta' \in \Theta'.$$

Vacuous extension and the product space. Given a bba m^X, its vacuous extension on space $X \times Y$, denoted $m^{X\uparrow X\times Y}$ is given by

$$m^{X\uparrow X\times Y}(C) = \begin{cases} m^X(A) & \text{if } C = A \times Y, A \subseteq X \\ 0 & \text{otherwise.} \end{cases} \tag{5}$$

Conjunctive rule of combination. Let m_1 and m_2 be two bba's defined on the same frame Θ. Suppose that the two bba's are induced by two distinct[2] pieces of evidence. Then the joint impact of

[1] The superscript in our notation will always denote the domain, that is the frame of discernment on which the bba and its associated functions is defined.

[2] The notion of "distinctness", discussed in [3], is often called independence, although these two concepts are subtly different.

the two pieces of evidence can be expressed by the conjunctive rule of combination which results in the bba:

$$
\begin{aligned}
m_{12}(A) &= (m_1 \textcircled{\cap} m_2)(A) \\
&= \sum_{B,C \subseteq \Theta : B \cap C = A} m_1(B) \cdot m_2(C)
\end{aligned} \tag{6}
$$

Note that the conjunctive rule is both commutative and associative.

Given two bba's m_1^X and m_2^Y, their conjunctive combination on $X \times Y$ can be obtained by combining their vacuous extensions on $X \times Y$ using (6). Formally:

$$
m_1^X \textcircled{\cap} m_2^Y = m_1^{X \uparrow X \times Y} \textcircled{\cap} m_2^{Y \uparrow X \times Y}. \tag{7}
$$

We thus obtain

$$
(m_1^X \textcircled{\cap} m_2^Y)(C) = \begin{cases} m_1^X(A)\, m_2^Y(B) & \text{iff } C = A \times B, A \subseteq X, B \subseteq Y, \\ 0 & \text{otherwise.} \end{cases} \tag{8}
$$

Marginalisation (a special case of coarsening) represents a projection of a bba defined on $X \times Y$ into a bba on X or Y.

Ballooning extension. Let $m^{X'}$ be a bba defined on a frame of discernment X'. We would like to build a bba on a larger frame X with $X' \subseteq X$ in such a way that the subsets of X do not receive more support than justified. The least committed bba [3] on X, such that its conditioning on X' is $m^{X'}$, is given by the so called "ballooning" extension, denoted $m^{X' \Uparrow X}$ and defined as:

$$
m^{X' \Uparrow X}(A) = \begin{cases} m^{X'}(A'), & \text{if } (A' \subseteq X'), A = A' \cup \bar{X}' \\ 0 & \text{otherwise.} \end{cases} \tag{9}
$$

Here $\bar{A}$ denotes the complement of A in the frame X. The ballooning extension plays a key role in the implementation of implication rules within the TBM framework and therefore we give a simple example to illustrate its concept. Let $X' = \{x_1, x_2\}$ and $m^{X'}(\{x_1\}) = 0.7$, $m^{X'}(\{x_1, x_2\}) = 0.3$. The ballooning extension of $m^{X'}$ to $X = \{x_1, x_2, x_3, x_4\}$ results in a bba m^X with two focal elements: $m^X(\{x_1, x_3, x_4\}) = 0.7$ and $m^X(\{x_1, x_2, x_3, x_4\}) = 0.3$.

Pignistic probability. The pignistic probability is the result of mapping a belief measure to a probability measure. For the frame of eq.(1) and its singletons $\theta_i \in \Theta$, the pignistic probability is defined as:

$$BetP(\theta_i) = \sum_{\theta_i \in A \subseteq \Theta} \frac{1}{|A|} \frac{m(A)}{1 - m(\emptyset)}, \tag{10}$$

and for $A \subseteq \Theta$, $BetP(A) = \sum_{\theta_i \in A} BetP(\theta_i)$. The pignistic transformation (10) is linear and has some other useful properties [3], such as that for all $A \subseteq \Theta$, $bel(A) \leq BetP(A) \leq pl(A)$, if $m(\emptyset) = 0$. $BetP$ is the probability measure that we use for decision making (betting) and hence its name[3].

3 Implication rules within the TBM framework

Handling implication rules within the TBM is conceptually simple, which is not the case with probability theory. Assuming that the probability of the implication 'if A then B' is equal to the conditional probability of B given A leads to the trivialization results of Lewis, i.e., the probability can then only take three values [13]. Under this assumption, probability theory degenerates into a useless theory.

In the TBM framework, on the contrary, one can assume the equality between the implication and the conditional belief and still avoid the trivialization. Suppose an information source tells you that a rule 'If A then B' is supported to a certain degree α which takes a value from the interval $[0, 1]$. The rule 'If A then B' is logically equivalent to the rule 'not A or B' (by the definition of implication). The bba related to this rule is given by:

$$\begin{aligned} m(\text{not } A \text{ or } B) &= \alpha \\ m(\top) &= 1 - \alpha \end{aligned} \tag{11}$$

where $\top$ denotes the logical tautology.

We assume that the rule is informative, i.e. B does not imply 'not A' [14]. In that case $bel(\text{not } A) = 0$. By Dempster's rule of conditioning [12], one gets $bel[A](B) = bel(\text{not } A \text{ or } B) = \alpha$, where $bel[A](B)$ is the conditional belief of B given A. Thus the belief of B given A is equal to the belief that the implication holds. This last property explains why handling the implication rules is straightforward within the TBM.

[3] *Pignus* means a bet or a wage in Latin.

In practice, a rule 'If A then B' with a belief mass α, is translated into a bba given by (11). This bba is also what the ballooning extension would produce if one starts with a conditional belief function and extends it on the underlying space as explained in more details below.

Suppose there are two frames, $\mathbb{A}$ and $\mathbb{B}$. An implication rule is an expression of the form

$$R: \quad x \in A \subseteq \mathbb{A} \Rightarrow x \in B \subseteq \mathbb{B}$$

where x is the actual (true) value. In a shortened notation we write simply $A \Rightarrow B$. A rule is assigned some belief mass α.

Based on the principle of least commitment, the implication rule $A \Rightarrow B$ in the TBM framework is represented by a belief function on a product space $\mathbb{A} \times \mathbb{B}$ with two focal sets, defined as:

$$m_R^{\mathbb{A}\times\mathbb{B}}(C) = \begin{cases} \alpha, & \text{if } C \in (A \times B) \cup (\bar{A} \times \mathbb{B}) \\ 1-\alpha, & \text{if } C \in \mathbb{A} \times \mathbb{B}. \end{cases} \tag{12}$$

This belief function is in effect the result of the ballooning extension of belief function $m^{\mathbb{B}}$ defined as:

$$m^{\mathbb{B}}(C) = \begin{cases} \alpha, & C = B \\ 1-\alpha, & C = \mathbb{B} \end{cases}$$

that is,

$$m_R^{\mathbb{A}\times\mathbb{B}} \equiv m^{\mathbb{B}\Uparrow(\mathbb{A}\times\mathbb{B})}.$$

This bba is graphically shown in Figure 1, where the white areas represent the subsets of the product space $\mathbb{A}\times\mathbb{B}$ which are assigned the belief mass of α. For example, let $\mathbb{A} = \{a_1, a_2, a_3\}$, $\mathbb{B} = \{b_1, b_2, b_3\}$, $A = \{a_1, a_2\}$ and $B = \{b_2\}$. The bba corresponding to the rule $A \Rightarrow B$ with the belief mass α is shown graphically in Figure 2. As before, the white areas are assigned belief mass α.

In order to illustrate the theory we next consider a small scale data fusion problem where we combine the available sensor reports and implication rules in order to determine the identification (ID) of a target in the air surveillance context.

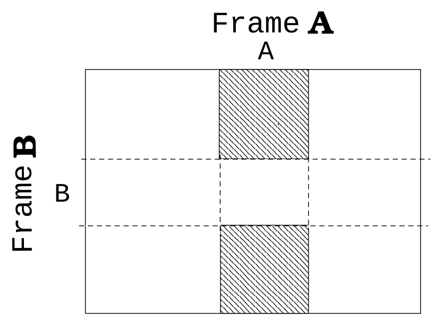

Figure 1: The belief function corresponding to rule $A \Rightarrow B$ with belief mass α. The white areas are assigned mass α.

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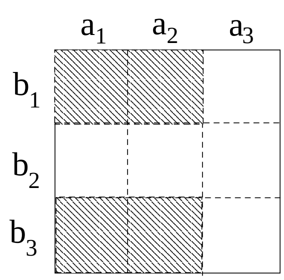

Figure 2: The belief function corresponding to rule $A \Rightarrow B$, where $\mathbb{A} = \{a_1, a_2, a_3\}$, $\mathbb{B} = \{b_1, b_2, b_3\}$, $A = \{a_1, a_2\}$ and $B = \{b_2\}$.

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4 Target ID: a case study

4.1 Problem description

Suppose that

$$\mathbb{A} = \{f, n, h\} \tag{13}$$

is the frame of discernment for target allegiance (e.g. f = friend, n= neutral, h = hostile) and let

$$\mathbb{B}' = \{B_1, B_2, B_3\} \tag{14}$$

be the frame of discernment for target basic class (e.g. B_1 are commercial planes, B_2 are fighter planes, B_3 are bombers). Each basic class usually consists of several platform types. Let us assume that:

$$B_1 = \{b_{11}, b_{12}\} \tag{15}$$

$$B_2 = \{b_{21}, b_{22}, b_{23}\} \tag{16}$$

$$B_3 = \{b_{31}, b_{32}\}. \tag{17}$$

If B_1 is a class of commercial aircraft, b_{11} and b_{12} could be, for example, Airbus-320 and Boeing-737, respectively. The frame of discernment

$$\mathbb{B} = B_1 \cup B_2 \cup B_3 = \{b_{11}, b_{12}, b_{21}, b_{22}, b_{23}, b_{31}, b_{32}\} \tag{18}$$

is a refinement of $\mathbb{B}'$ (that is $\mathbb{B}'$ is a coarsening of $\mathbb{B}$). Target identification will be carried out over the product space $\mathbb{A} \times \mathbb{B}$.

Now suppose the following implication rules have been provided (expert advice):

$$\begin{array}{lll} R_a: & \text{if } x \in B_1 & \text{then } x \in \{n\}, \text{with belief mass } 0.9; \\ R_b: & \text{if } x \in \{b_{22}\} & \text{then } x \in \{h\}, \text{ with belief mass } 0.8; \\ R_c: & \text{if } x \in \{f\} & \text{then } x \in \{b_{21}, b_{22}, b_{31}\}, \text{ with belief mass } 1. \end{array}$$

where x is the true target identification (allegiance, platform type). Rules R_a and R_b essentially state $x \in B \subseteq \mathbb{B} \Rightarrow x \in A \subseteq \mathbb{A}$. Rule R_c has a different direction, i.e. it states $x \in A \subseteq \mathbb{A} \Rightarrow x \in B \subseteq \mathbb{B}$. The expert advice expressed by rules R_a and R_b is not fully reliable and hence the belief masses associated to these two rules are less than 1.0. The rule R_c is also referred to a categorical rule as the associated belief mass is 1.0.

Suppose the following sensor reports are available for target identification:

- Based on maneuvering capability of the target, a bba over $\mathbb{B}'$ is reported as follows:

$$\begin{aligned} m_1^{\mathbb{B}'}(\{B_2\}) &= 0.7 \\ m_1^{\mathbb{B}'}(\{B_2, B_3\}) &= 0.2 \\ m_1^{\mathbb{B}'}(\mathbb{B}') &= 0.1 \end{aligned}$$

Table 1: *The bba* $m_{12}^{\mathbb{B}}$

focal set	belief mass
$\{b_{22}\}$	0.50
$\{b_{23}\}$	0.30
$\{b_{21}, b_{22}, b_{23}\}$	0.14
$\{b_{21}, b_{22}, b_{23}, b_{31}, b_{32}\}$	0.04
$\mathbb{B}$	0.02

- Based on target EM emissions, an ESM sensor reported a bba over $\mathbb{B}$ as follows:

$$\begin{aligned} m_2^{\mathbb{B}}(\{b_{22}\}) &= 0.5 \\ m_2^{\mathbb{B}}(\{b_{23}\}) &= 0.3 \\ m_2^{\mathbb{B}}(\mathbb{B}) &= 0.2 \end{aligned}$$

- Based on the IFF response, we have a high confidence that the target is friendly, which is expressed by a bba over $\mathbb{A}$ as follows:

$$\begin{aligned} m_3^{\mathbb{A}}(\{f\}) &= 0.8 \\ m_3^{\mathbb{A}}(\mathbb{A}) &= 0.2 \end{aligned}$$

4.2 Solution

The available sensor reports ($m_1^{\mathbb{B}'}$, $m_2^{\mathbb{B}}$ and $m_3^{\mathbb{A}}$) and implication rules (R_a, R_b and R_c) can be fused for the purpose of target identification in any order, since the combination is performed by the conjunctive rule (which is both commutative and associative).

Let us first combine $m_1^{\mathbb{B}'}$ with $m_2^{\mathbb{B}}$:

$$m_{12}^{\mathbb{B}} = m_1^{\mathbb{B}'\uparrow\mathbb{B}} \textcircled{\cap} m_2^{\mathbb{B}}$$

There are 5 focal sets in $m_{12}^{\mathbb{B}}$ as shown in Table 1.

The combination of $m_{12}^{\mathbb{B}}$ and $m_3^{\mathbb{A}}$ is done using (7), that is:

$$m_{123}^{\mathbb{A}\times\mathbb{B}} = m_{12}^{\mathbb{B}\uparrow\mathbb{A}\times\mathbb{B}} \textcircled{\cap} m_3^{\mathbb{A}\uparrow\mathbb{A}\times\mathbb{B}}$$

Table 2: *The bba* $m_{123}^{\mathbb{A}\times\mathbb{B}}$

focal set	belief mass
$\{(f, b_{22})\}$	0.400
$\{(f, b_{23})\}$	0.240
$\{(f, b_{21}), (f, b_{22}), (f, b_{23})\}$	0.112
$\{(f, b_{22}), (n, b_{22}), (h, b_{22})\}$	0.100
$\{(f, b_{23}), (n, b_{23}), (h, b_{23})\}$	0.060

The product space $\mathbb{A} \times \mathbb{B}$ has $3 \times 7 = 21$ elements, and they are pairs such as: (f, b_{11}), (f, b_{12}), etc. The bba $m_{123}^{\mathbb{A}\times\mathbb{B}}$ has 10 focal sets - five of them, with highest belief masses, are listed in Table 2.

The decision about the target ID could be made based on sensor reports by application of the pignistic transform (10) to $m_{123}^{\mathbb{A}\times\mathbb{B}}$. The highest values of pignistic probabilities corresponding to $m_{123}^{\mathbb{A}\times\mathbb{B}}$ are obtained for the following singletons of $\mathbb{A} \times \mathbb{B}$:

$$\begin{aligned} BetP(f, b_{22}) &= 0.4832 \\ BetP(f, b_{23}) &= 0.3099 \\ BetP(f, b_{21}) &= 0.0499 \\ BetP(n, b_{22}) &= 0.0372 \\ BetP(h, b_{22}) &= 0.0372. \end{aligned} \tag{19}$$

Next we represent the rule R_a with a bba $m_a^{\mathbb{A}\times\mathbb{B}}$. The rule can be expressed as $x \in B \subseteq \mathbb{B} \Rightarrow x \in A \subseteq \mathbb{A}$, where $B = \{b_{11}, b_{12}\}$ and $A = \{n\}$. According to Section 3, the ballooning extension results in a bba with two focal sets, which in a shorten notation can be expressed as follows: $(A \times B) \cup (\mathbb{A} \times \bar{B})$ is the first focal set with belief mass 0.9 and $(\mathbb{A} \times \mathbb{B})$ is the second focal set (the entire frame on the product space) with the belief mass 0.1. For the convenience, we write the explicit description of the first focal set:

$$\begin{aligned}(A \times B) \cup (\mathbb{A} \times \bar{B}) =& \{(f, b_{21}), (f, b_{22}), (f, b_{23}), (f, b_{31}), \\ & (f, b_{32}), (n, b_{11}), (n, b_{12}), (n, b_{21}), \\ & (n, b_{22}), (n, b_{23}), (n, b_{31}), (n, b_{32}), \\ & (h, b_{21}), (h, b_{22}), (h, b_{23}), (h, b_{31}), \\ & (h, b_{32})\}.\end{aligned}$$

The conjunctive combination of $m_a^{\mathbb{A}\times\mathbb{B}}$ with $m_{123}^{\mathbb{A}\times\mathbb{B}}$ leaves the pignistic probabilities in (19) almost unchanged, meaning that this rule has almost no effect on the decision about the target ID in this example.

The rule R_b, however, will have a significant impact on ID decision. The conjunctive combination of $m_b^{\mathbb{A}\times\mathbb{B}}$, $m_a^{\mathbb{A}\times\mathbb{B}}$ and $m_{123}^{\mathbb{A}\times\mathbb{B}}$ results in a bba $m_{123ab}^{\mathbb{A}\times\mathbb{B}}$. The pignistic transform of this bba results in the following probabilities (only 5 highest values are listed):

$$\begin{aligned} BetP(f, b_{23}) &= 0.5163 \\ BetP(h, b_{22}) &= 0.1533 \\ BetP(f, b_{21}) &= 0.1100 \\ BetP(n, b_{23}) &= 0.0387 \\ BetP(h, b_{23}) &= 0.0387 \end{aligned} \tag{20}$$

Comparing (20) with (19), we observe that the two highest ranked alternatives have swaped the order.

The rule R_c is represented by a categorical belief function, with all belief mass given to one focal set: $(A \times B) \cup (\bar{A} \times \mathbb{B})$ where $A = \{f\}$ and $B = \{b_{21}, b_{23}, b_{31}\}$. The final bba which results from the fusion of sensor reports and implication rules is then:

$$m_{123abc}^{\mathbb{A}\times\mathbb{B}} = m_{123}^{\mathbb{A}\times\mathbb{B}} \textcircled{\cap} m_a^{\mathbb{A}\times\mathbb{B}} \textcircled{\cap} m_b^{\mathbb{A}\times\mathbb{B}} \textcircled{\cap} m_c^{\mathbb{A}\times\mathbb{B}}$$

The pignistic transform of this bba results in the following probabilities (again only 5 highest values listed):

$$\begin{aligned} BetP(f, b_{23}) &= 0.5615 \\ BetP(h, b_{22}) &= 0.1665 \\ BetP(f, b_{21}) &= 0.1281 \\ BetP(n, b_{23}) &= 0.0415 \\ BetP(h, b_{23}) &= 0.0415. \end{aligned} \tag{21}$$

Comparing (20) with (21) we observe that the effect of rule R_c was only to slightly increase the values of pignistic probabilities for the same list of singleton pairs. This increase results from the fact that rule R_c forces $BetP(f, b_{11})$, $BetP(f, b_{12})$, $BetP(f, b_{22})$, and $BetP(f, b_{23})$ to zero.

Finally we can marginalise bba $m_{123abc}^{\mathbb{A}\times\mathbb{B}}$:

$$m_{123abc}^{\mathbb{A}} = m_{123abc}^{\mathbb{A}\times\mathbb{B}\downarrow\mathbb{A}} \tag{22}$$

$$m_{123abc}^{\mathbb{B}} = m_{123abc}^{\mathbb{A}\times\mathbb{B}\downarrow\mathbb{B}} \tag{23}$$

The pignistic transform of $m_{123abc}^{\mathbb{A}}$ results in the following pignistic probabilities of target allegiance:

$$\begin{aligned} BetP(f) &= 0.7179 \\ BetP(n) &= 0.0627 \\ BetP(h) &= 0.2194. \end{aligned} \tag{24}$$

Similarly, the pignistic transform of $m_{123abc}^{\mathbb{B}}$ results in the following probabilities of platform types:

$$\begin{aligned} BetP(b_{11}) &= 0.0005 \\ BetP(b_{12}) &= 0.0005 \\ BetP(b_{21}) &= 0.1444 \\ BetP(b_{22}) &= 0.1755 \\ BetP(b_{23}) &= 0.6444 \\ BetP(b_{31}) &= 0.0314 \\ BetP(b_{32}) &= 0.0031. \end{aligned} \tag{25}$$

5 Summary

The paper presents an application of the belief function theory to the problem of data fusion of sensor reports and rules for the purpose of target identification in the context of air surveillance. The elegance of the proposed framework for handling the implication rules is illustrated by a small scale practical example.

References

[1] E. Waltz and J. Llinas. *Multisensor Data Fusion*. Artech house, 1990.

[2] R. Kruse, E. Schwecke, and J. Heinsohn. *Uncertanty and vagueness in knowledge based systems*. Springer-Verlag, Berlin, 1991.

[3] Ph. Smets. The transferable belief model for quantified belief representation. In D. M. Gabbay and Ph. Smets, editors, *Handbook of Defeasible Reasoning and Uncertainty Management Systems*, volume 1, pages 267–301. Kluwer, Doordrecht, The Netherlands, 1998.

[4] B. Ristic and Ph. Smets. An approach to target classification based on the belief function theory. *IEEE Trans. Aerospace and Electronic Systems*, 2004. In review.

[5] B. Ristic and Ph. Smets. Solving the data association poblem for the purpose of classification of multiple objects. *Information fusion*, 2004. In review.

[6] F. Delmotte and P. Smets. Target identification based on the transferable belief model interpretation of Dempster-Shafer model. *IEEE Trans. Systems, Man and Cybernetics - Part A*, 2004. (Accepted for publication).

[7] S. S. Blackman. *Multiple Target Tracking with Radar Applications*. Artech House, 1986.

[8] A. Bosse and M.-A. Simard. Managing evidential reasoning for identity information fusion. *Opt. Eng.*, 37 (2):391–399, 1998.

[9] S. Blackman and R. Popoli. *Design and Analysis of Modern Tracking Systems*. Artech House, 1999.

[10] G. Shafer. *A mathematic theory of evidence*. Princeton University Press, 1976.

[11] Ph. Smets and R. Kennes. The transferable belief model. *Artificial Intelligence*, 66:191–234, 1994.

[12] Ph. Smets. Data fusion in the transferable belief model. In *Proc. 3rd Intern. Conf. Inforation Fusion (FUSION 2000)*, pages PS21–33, Paris, France, 2000.

[13] D. Lewis. Probabilities of conditionals and conditional probabilities. *Philosophical Review*, 85:297–315, 1976.

[14] J. D. McCawley. *Everything that linguists have always wanted to know about logic but were ashamed to ask*. Basil Blackwell Publishers, 1981.