Paper # 17

# 1 Introduction

Suppose a remote sensor $RS$ that detects the presence or absence of a given object and communicates this information to a coordination center $CC$. Consider an enemy, a deceiver agent $DA$, who has the ability to interfere with the communication. $DA$ could fool $CC$ by either interrupting the communication or by communicating a tampered message, like communicating an absence of a given object when the object is present and a presence of a given object when the object is absent. Interrupting the communication is maybe not the best strategy as $CC$ might conclude from the interruption that $DA$ has interfered with the communication, which is already an information. A best fooling strategy might be the second one, i.e., to send the negation of the original message.

Now you are working at $CC$ and collect a message from $RS$. How to handle it, specially if the problem is not as easy as just described as when there are many possible messages, added noise and other sources of unreliability.

Classically tampered messages are not considered, and probability theory is used to handle noisy and unreliable messages. Recently the belief function theory has been suggested as an alternative to the probability approach. We have defended the use of the transferable belief model (TBM), an interpretation of the belief function theory well adapted, among others, for object detection and classification (Ristic & Smets, 2004). We show here that the TBM can also handle tampered messages. Our presentation is focussed on a classification issue, where $RS$ sends to $CC$ a report about the class to which the observed object belongs, and this report is under the form of a belief function expressing $RS$'s opinion about what is the actual class of the object. The issue studied here is the possibility that the message is tampered in that $DA$ tries to deceive $CC$ by sending 'false' data. We show that this can be achieved for instance by sending to $CC$ the negation of the belief function sent by $RS$.

The purpose of this paper is not to solve the general problem of tampered messages, but to show what the principle of negated belief functions can offer to that problem. Decoding tampered messages is an enormous issue that we only skim over. Our paper considers only one form of tampering, a form where negated belief functions can be helpful.

Our presentation is organized as follows. In section 2, we provide the needed background material about belief functions. In section 3, we present the concept of the negation of a belief function and the de Morgan's laws that apply to it. In section 4 we explain why using negated belief functions is a good strategy for what concerns $DA$ . In section 5, we present an illustrative example of our method. We conclude in section 6.

# 2 The TBM background

In the transferable belief model (TBM), we consider the actual value $\omega_0$ of a variable which finite domain is a given set $\Omega$. We represent the uncertainty about the value of $\omega_0$ by a basic belief assignment (bba) denoted $m^\Omega$ where $m^\Omega(A)$ is the basic belief mass (bbm) given to $A \subseteq \Omega$, and $m^\Omega$ maps $2^\Omega$, the power set of $\Omega$, on $[0, 1]$ with the constraint that $\sum_{A\subseteq\Omega} m^\Omega(A) = 1$. The mass $m^\Omega(A)$ represents the part of belief that supports that the actual value $\omega_0$ belongs to $A$ and nothing more specific. Classical material about the TBM and belief function theory can be found in (Shafer, 1976; Smets & Kennes, 1994; Smets & Kruse, 1997; Smets, 1998).

Several useful functions related to $m^\Omega$ have been described.

*Belief function.* A belief function is defined as:

$$bel^\Omega(A) = \sum_{B:\emptyset\neq B\subseteq A} m^\Omega(B).$$

The value $bel^\Omega(A)$ represents the total amount of belief that supports that $\omega_0$ is in $A$ without supporting that it is in $\overline{A}$.

*Plausibility function.* A plausibility function is defined as:

$$pl^\Omega(A) = \sum_{B:A\cap B\neq\emptyset} m^\Omega(B) = bel^\Omega(\Omega) - bel^\Omega(\overline{A}).$$

The value $pl^\Omega(A)$ represents the total amount of belief that supports that $\omega_0$ might be in $A$ without supporting that it might be in $\overline{A}$.

*Commonality function.* A commonality function is defined as:

$$q^\Omega(A) = \sum_{B:A\subseteq B} m^\Omega(B).$$

The value $q^\Omega(A)$ represents the conditional amount of uncertainty should we accept that $\omega_0$ is in $A$.

*Implicability function.* An implicability function is defined as:

$$b^\Omega(A) = \sum_{B:B\subseteq A} m^\Omega(B) = bel^\Omega(A) + m^\Omega(\emptyset).$$

The value $b^\Omega(A)$ represents the total amount of belief that supports that $\omega_0$ is in $A$.

Some particular belief functions are used in this paper.

**Definition 2.1 Categorical belief function.** *A categorical belief function on $\Omega$ focussed on $A^* \subseteq \Omega$, is a belief function which related bba $m^\Omega$ satisfies:*

$$m^\Omega(A) = \begin{cases} 1 & \text{if } A = A^* \\ 0 & \text{otherwise.} \end{cases} \qquad (1)$$

When all bbas are categorical, the TBM becomes equivalent to classical propositional logic. Two limiting cases of categorical bbas have received special names.

**Definition 2.2 Vacuous belief function.** *The vacuous belief function on $\Omega$ is a categorical belief function focused on $\Omega$. It is denoted by $VBF$.*

**Definition 2.3 Contradictory belief function.** *A contradictory belief function on $\Omega$ is a categorical belief function focused on $\emptyset$. It is denoted by $CBF$.*

### 2.1 Decision making

In the TBM, decision are made using the pignistic probabilities derived from the bba by the pignistic transformation. (Smets, 1990, 2002, 2005).

**Definition 2.4 The pignistic transformation.** *The pignistic transformation maps bbas to so called pignistic probability functions. The pignistic transformation of $m^\Omega$ is given by:*

$$BetP^\Omega(A) = \sum_{B\subseteq\Omega} \frac{|A\cap B|}{|B|}\frac{m^\Omega(B)}{1-m^\Omega(\emptyset)}, \ \forall A\subseteq\Omega$$

*where $|A|$ is the number of elements of $\Omega$ in $A$.*

$BetP^\Omega$ is a probability measure.

### 2.2 Notation

We use the notation $m^\Omega_{Ag,t}[Ev]$ to express the bba held by an agent $Ag$ at time $t$ about the actual value $\omega_0$ of a variable which possible values belong to the finite set $\Omega$, where the bba is conditioned on $Ev$ i.e., what the agent $Ag$ accepts as true at time $t$. Indexes are neglected when the context makes them obvious.

### 2.3 Types of combinations

Given the distinct pieces of evidence $Ev_i$ and their induced $m^\Omega[Ev_i], i=1,2$, the belief holder $Ag$ can build combinations based on the conjunction (AND, denoted $\wedge$) or the disjunction (OR, denoted $\vee$) of the two pieces of evidence.

- $m^\Omega[Ev_1\wedge Ev_2]$: the belief that results from the conjunctive combination of the two pieces of evidence. It applies when $Ag$ considers that the two $Ev_i$'s hold.
- $m^\Omega[Ev_1\vee Ev_2]$: the belief that results from the disjunctive combination of the two pieces of evidence. It applies when $Ag$ considers that at least one of the two $Ev_i$'s holds but $Ag$ does not know which one.

In the TBM, these combinations are achieved by the next rules. We provide their formal definition based on the bba itself, and the simple relations based on the commonality functions and the implicability functions. For all $A\subseteq\Omega$:

conjunctive combination rule

$$m^\Omega[Ev_1\wedge Ev_2](A) = \sum_{X\cap Y=A} m^\Omega[Ev_1](X)m^\Omega[Ev_2](Y)$$
$$q^\Omega[Ev_1\wedge Ev_2](A) = q^\Omega[Ev_1](A)q^\Omega[Ev_2](A)$$

disjunctive combination rule

$$m^\Omega[Ev_1\vee Ev_2](A) = \sum_{X\cup Y=A} m^\Omega[Ev_1](X)m^\Omega[Ev_2](Y)$$
$$b^\Omega[Ev_1\vee Ev_2](A) = b^\Omega[Ev_1](A)b^\Omega[Ev_2](A)$$

## 3 The negation of a belief function

Suppose a bba $m^\Omega$. Dubois and Prade (1986) have defined the negation of $m^\Omega$.

**Definition 3.1 Negation of a bba.** *The negation of $m^\Omega$ is the bba $\overline{m}^\Omega$ that satisfies:*

$$\overline{m}^\Omega(A) = m^\Omega(\overline{A}), \qquad \forall A\subseteq\Omega \tag{2}$$

They also show that for all $A\subseteq\Omega$:

**Theorem 3.1** *For any bba $m^\Omega$, one has:*

$$\overline{b}^\Omega(A) = q^\Omega(\overline{A})$$
$$\overline{q}^\Omega(A) = b^\Omega(\overline{A}).$$

De Morgan's laws apply also.

**Theorem 3.2** *For any pair of bbas $m^\Omega_1$ and $m^\Omega_2$ on $\Omega$, one has:*

$$\overline{m}^\Omega_1 \textcircled{\cap} \overline{m}^\Omega_2 = m^\Omega_1 \textcircled{\cup} m^\Omega_2$$
$$\overline{m}^\Omega_1 \textcircled{\cup} \overline{m}^\Omega_2 = m^\Omega_1 \textcircled{\cap} m^\Omega_2.$$

The meaning of this negated belief function was detailed in (Smets, 1997) when studying the $\alpha$-junction operators, among which one finds the conjunctive combination rule and the disjunctive combination rule. Suppose two distinct pieces of evidence $Ev_1$ and $Ev_2$ that bear on $\Omega$. We can write the combination rules as:

$$m^\Omega[Ev_1\wedge Ev_2] = m^\Omega[Ev_1]\textcircled{\cap} m^\Omega[Ev_2]$$
$$m^\Omega[Ev_1\vee Ev_2] = m^\Omega[Ev_1]\textcircled{\cup} m^\Omega[Ev_2]$$

We define $\sim Ev$ as the piece of evidence that induces the bba $\overline{m}^\Omega[Ev]$, the negation of $m^\Omega[Ev]$. The de Morgan's laws are satisfied and can then be written as:

$$m^\Omega[\sim(Ev_1\wedge Ev_2)] = m^\Omega[\sim Ev_1 \vee \sim Ev_2]$$
$$m^\Omega[\sim(Ev_1\vee Ev_2)] = m^\Omega[\sim Ev_1 \wedge \sim Ev_2].$$

## 4 Negated belief functions and deceitful reports

### 4.1 Possible behavior from $DA$

Let $m^{\Omega}_{RS}$ be the bba sent by $RS$. What is the 'best' attitude $DA$ could take to fool $CC$. $DA$ will build a bba $m^{\Omega}_{DA}$ and will communicate it to $CC$. Let $m^{\Omega}_{Col}$ be the bba collected by $CC$. We assume $m^{\Omega}_{Col} = m^{\Omega}_{DA}$, thus there is no further interference between $DA$ and $CC$. What bba $DA$ should send to $CC$ in order to get $CC$ as 'confused'or deceived as possible.

1. **Stopping communication.** $DA$ could stop the communication line, in which case $CC$ would become aware of $DA$ presence, something we feel $DA$ would try to avoid.

2. **Sending a vacuous belief function.** Sending a vacuous belief function might also trigger some suspicion from $CC$ that there is a $DA$ along the communication line, something we feel $DA$ would try to avoid.

3. **Smart permutation of the elements.** $DA$ could collect $m^{\Omega}_{RS}$ and compute its pignistic transformation $BetP^{\Omega}_{RS}$. Suppose $BetP^{\Omega}_{RS}(\omega_{i_1}) > BetP^{\Omega}_{RS}(\omega_{i_2}) \ldots BetP^{\Omega}_{RS}(\omega_{i_n})$ with $n = |\Omega|$ (equality are handled trivially). $DA$ permutes the elements such that the most probable becomes the least probable.... It means replacing $\omega_{i_k}$ by $\omega_{i_{n+1-k}}$ for all $k = 1, \ldots, n$. E.g., suppose $n = 4$ and the probability ordered elements are $(2, 4, 3, 1)$ (hence $BetP^{\Omega}_{RS}(\omega_2)$ is the largest). After transformation, the set $A = \{\omega_2\}$ becomes the set $\{\omega_1\}$, and the set $B = \{\omega_1, \omega_3\}$ becomes the set $\{\omega_2, \omega_4\}$, etc...

   This is surely not a bad attitude, but requires that $DA$ understands what the bba $m^{\Omega}_{RS}$ represents. The permutation depends indeed on $m^{\Omega}_{RS}$. This solution requires maybe too much knowledge from $DA$.

4. **Random bba.** $DA$ could also send a random bba to $CC$. $CC$ would be partially confused, but usually not fooled as much as when receiving the negated bba considered below.

5. **Negated bba.** Finally $DA$ might send the negated bba, i.e., $m^{\Omega}_{DA} = \overline{m}^{\Omega}_{RS}$. A nice property is that the transformation does not depend on $m^{\Omega}_{RS}$ and can be applied by any $DA$, even a moronic robot. Does such an attitude fits with common sense?

   (a) Suppose $m^{\Omega}_{RS}$ is a categorical bba focussed on $A \subset \Omega$. It seems perfect to send to $CC$ another categorical bba $m^{\Omega}_{DA}$ focussed on $\overline{A}$.

   (b) Suppose $m^{\Omega}_{RS}$ is a vacuous bba. Leaving it so will not alter the $CC$ opinions, whereas sending a contradictory bba could seriously disturbed $CC$ as such a bba could lead $CC$ to conclude anything and its contrary. $CC$ will have to spend energy handling $RS$ and the communication line to find out the origin of the contradictory bba.

   (c) Suppose $m^{\Omega}_{RS}$ is a contradictory bba. It is obvious $CC$ should spend some energy to repair $RS$ and/or the communication line. Sending a vacuous bba to $CC$ will stop performing this task that should have been done.

   In these three cases, using the negation of the bba seems thus an adequate strategy. For the general case, common sense can hardly dictate the 'good' behavior, but using the negated bba seems still an adequate strategy for $DA$.

### 4.2 Reaction by $CC$

How could $CC$ react when collecting the bba $m^{\Omega}_{DA}$? For that we must consider $CC$'s beliefs about the fact that the message might have been tampered or not. Consider a space $\mathcal{L} = \{U, T\}$ where $U$ = Untampered, and $T$ = Tampered, where the two states mean that:

1. Untampered: the message has not been altered by some $DA$, so $m^{\Omega}_{Col} = m^{\Omega}_{RS}$. In that case, $CC$ should accept the incoming bba.

2. Tampered: the message has been altered by some $DA$, so $m^{\Omega}_{Col} = \overline{m}^{\Omega}_{RS}$. In that case, $CC$ should accept the negation $\overline{m}^{\Omega}_{Col}$ of the incoming bba (as negation is involutive, i.e., $\overline{\overline{m}} = m$).

Should $CC$ be certain about which of the two contexts prevails, $CC$ just accepts the incoming message or its negation, accordingly, and feed it into $CC$'s multisensor data fusion system.

#### *4.2.1 Prior on the nature of the collected bba*

Such an ideal situation seems hardly expectable as too unrealistic. In practice we could accept that $CC$ has some beliefs over $\mathcal{L}$. Let $m^{\mathcal{L}}$ be the belief held by $CC$ about the fact the message has been tampered of not.

To compute the belief over $\Omega$ held by $CC$ based on $\mathcal{L}$, we need to derive the bba induced on $\Omega$ when $CC$ is ignorant about the value of $\mathcal{L}$. Due to the particular nature of the two involved bba's (which are not distinct of course), it can be shown that $m^{\Omega}_{CC}[\{U, T\}] = VBF$, where $VBF$ is the vacuous belief function, a quite natural result.

Given $\mathcal{L}$ and $m^{\Omega}_{DA}$, one gets:

$$m^{\Omega}_{CC} = m^{\mathcal{L}}(\emptyset)CBF + m^{\mathcal{L}}(U)m^{\Omega}_{DA} + m^{\mathcal{L}}(T)\overline{m}^{\Omega}_{DA} + m^{\mathcal{L}}(\mathcal{L})VBF. \quad (3)$$

This bba can then be fed into $CC$'s multisensor data fusion system. This solution is correct as it results from a direct application of the TBM, but it may be too demanding as $CC$ might be unable to assess $m^{\mathcal{L}}$.

#### 4.2.2 Absence of prior on $\mathcal{L}$ by CC

In many cases, we feel $CC$ does not have any prior beliefs about the presence or absence of $DA$, thus about $\mathcal{L}$. A nice procedure would be to derive a method so that $m^{\mathcal{L}}$ is derived from the collected data themselves, provided the $CC$ has some prior beliefs about $\Omega$.

If the $CC$ has neither prior beliefs on $\Omega$ nor on $\mathcal{L}$, there is no hope to find out if $m^{\Omega}_{Col}$ represents what $RS$ has send or not.

Let us suppose $CC$ had some prior on $\Omega$ represented by $m^{\Omega}_{CC,0}$ and none on $\mathcal{L}$, what is probably the most classical context to be encountered. $m^{\Omega}_{CC,0}$ will probably be collected from $CC$'s multisensor data fusion system.

We can compute the conflict $m^{\Omega}_{CC}[U](\emptyset)$ between $m^{\Omega}_{Col}$ and $m^{\Omega}_{CC,0}$ and the conflict $m^{\Omega}_{CC}[T](\emptyset)$ between $\overline{m}^{\Omega}_{Col}$ and $m^{\Omega}_{CC,0}$.

Suppose two bbas $m^{\Omega}_1$ and $m^{\Omega}_2$ both on $\Omega$. The plausibility that they both concern the same variable has been shown by Denoeux (2004) to be equal to

$$\begin{aligned} pl(var(m^{\Omega}_1) = var(m^{\Omega}_2)) &= \sum_{A\cap B\neq\emptyset} m^{\Omega}_1(A)m^{\Omega}_2(B) \\ &= 1 - m^{\Omega}_{1\cap\!\!\!\bigcirc 2}(\emptyset) \end{aligned}$$

where $var(m)$ represents the variable considered by $m$ (see also (Smets, 2004)).

Consider the bba $m^{\Omega}_{Col}$ collected by $CC$. To say that $m^{\Omega}_{Col}$ is untampered ($U$) is the same thing as stating $var(m^{\Omega}_{CC,0}) = var(m^{\Omega}_{Col})$. To say that $m^{\Omega}_{Col}$ is tampered ($T$) is the same thing as stating $var(m^{\Omega}_{CC,0}) = var(\overline{m}^{\Omega}_{Col})$. Finally to say that $m^{\Omega}_{Col}$ is untampered or tampered ($U, T$) is the same thing as stating $var(m^{\Omega}_{CC,0}) = var(m^{\Omega}_{Col})$ or $var(m^{\Omega}_{CC,0}) = var(\overline{m}^{\Omega}_{Col})$ in which case every mass in $m^{\Omega}_{CC,0}$ is compatible with those of either $m^{\Omega}_{Col}$ of $\overline{m}^{\Omega}_{Col}$ or both, the only one incompatible with any of these bbs is the mass given by $m^{\Omega}_{CC,0}$ to the empty set.

We have

$$\begin{aligned} pl^{\mathcal{L}}_{CC}(U) &= 1 - (m^{\Omega}_{CC,0} \cap\!\!\!\!\bigcirc m^{\Omega}_{Col})(\emptyset) \\ pl^{\mathcal{L}}_{CC}(T) &= 1 - (m^{\Omega}_{CC,0} \cap\!\!\!\!\bigcirc \overline{m}^{\Omega}_{Col})(\emptyset) \\ pl^{\mathcal{L}}_{CC}(U,T) &= 1 - m^{\Omega}_{CC,0}(\emptyset) \end{aligned}$$

The corresponding bba is given by:

$$\begin{aligned} m^{\mathcal{L}}_{CC}(\emptyset) &= m^{\Omega}_{CC,0}(\emptyset) \\ m^{\mathcal{L}}_{CC}(U) &= (m^{\Omega}_{CC,0} \cap\!\!\!\!\bigcirc \overline{m}^{\Omega}_{Col})(\emptyset) - m^{\Omega}_{CC,0}(\emptyset) \\ m^{\mathcal{L}}_{CC}(T) &= (m^{\Omega}_{CC,0} \cap\!\!\!\!\bigcirc m^{\Omega}_{Col})(\emptyset) - m^{\Omega}_{CC,0}(\emptyset) \\ m^{\mathcal{L}}_{CC}(U,T) &= 1 - (m^{\Omega}_{CC,0} \cap\!\!\!\!\bigcirc \overline{m}^{\Omega}_{Col})(\emptyset)... \\ &\quad - (m^{\Omega}_{CC,0} \cap\!\!\!\!\bigcirc m^{\Omega}_{Col})(\emptyset) + m^{\Omega}_{CC,0}(\emptyset) \end{aligned}$$

We have thus been able to build a prior belief on $\mathcal{L}$ based on the data themselves. We apply then the relation 3 and get the bba $m^{\Omega}_{new}$ that represents the beliefs held by $CC$ about the nature of the class to which the target belongs to. This bba can then be fed into $CC$'s multisensor data fusion system.

A nice property of this solution is that $m^{\Omega}_{new}$ is the same should $CC$ collect $m^{\Omega}_{RS}$ or $\overline{m}^{\Omega}_{RS}$. The action of $DA$ is annihilated. The cost of such an approach is that $m^{\Omega}_{new}$ is more 'cautious' as the bba we would have used if we had known for sure that the message had been tampered or not.

Another property is that if $m^{\Omega}_{CC,0}$ corresponds to a vacuous belief function, then both $m^{\mathcal{L}}_{CC}(U) = 0$ and $m^{\mathcal{L}}_{CC}(T) = 0$ in which case $m^{\Omega}_{new}$ corresponds also to a vacuous belief function. This corresponds to what we had mentioned in the second paragraph of section 4.2.2 using just common sense.

The strategy here described is just one among all those one could consider. It is more or less ad hoc as the assumptions of distinctness that underlie the combination rules are not fully satisfied. Other methods based on cautious combination rules could be considered. Still as shown in the next examples, the method developed here behaves nicely.

## 5 Illustrative use

### 5.1 A static combination

Suppose the frame $\Omega = \{\omega_1, \omega_2, \omega_3\}$. $RS$ observes an $\omega_1$ object. Its bba $m^{\Omega}_{RS}$ is the convex sum of two components: the first $m_1$ is a 'coherent' bba where every focal set contains $\omega_1$ and the second $m_2$ is a purely random bba where random masses are allocated to every possible subsets of $\Omega$. The bba $m_2$ can be seen as a noise. Let $\pi$ be the weight given to the 'noise' bba. $\pi$ can be seen as the 'noise' level. The bba sent by $RS$ at time 1 is given by:

$$m^{\Omega}_{RS} = (1-\pi)m_1 + \pi m_2.$$

The prior bba $m^{\Omega}_{CC,0}$ held by $CC$ at time 0 is built identically. We present in table 1 the bbas used to build $m^{\Omega}_{CC,0}$ and $m^{\Omega}_{RS}$ where $\pi = 0.4$ for $m^{\Omega}_{CC,0}$ and $\pi = 0.1$ for $m^{\Omega}_{RS}$.

Once $CC$ considers that the message might have been tampered, thus that $DA$ might be acting, the computation will not depend on the fact that $DA$ has or not negated the transferred bba. Suppose the bba collected by $CC$ at time 1 is in fact $m^{\Omega}_{RS}$, thus $DA$ did not negated the message. In table 2, we present successively $m^{\Omega}_{CC,0}$ that represents $CC$ beliefs at time 0, $m^{\Omega}_{Col} = m^{\Omega}_{RS}$ the bba communicated by $RS$ at time 1 and its negation $\overline{m}^{\Omega}_{Col}$. The conflict between $m^{\Omega}_{CC,0}$ and $m^{\Omega}_{Col}$ is given by $(m^{\Omega}_{CC,0} \cap\!\!\!\!\bigcirc m^{\Omega}_{Col})(\emptyset) = 0.09$ and the conflict between $m^{\Omega}_{CC,0}$ and $\overline{m}^{\Omega}_{Col}$ is given by $(m^{\Omega}_{CC,0} \cap\!\!\!\!\bigcirc \overline{m}^{\Omega}_{Col})(\emptyset) = 0.58$. These conflicts point to the fact that the collected bba seems not to have been tampered.

The bba induced on $\mathcal{L}$ by these conflicts is given by: $m^{\mathcal{L}}(\emptyset) = 0.04$, $m^{\mathcal{L}}(U) = 0.54$, $m^{\mathcal{L}}(T) = 0.06$ and $m^{\mathcal{L}}(U, T) = 0.36$.

| $\Omega$ | $m^{\Omega}_{CC,0}$ | | | $m^{\Omega}_{RS}$ | | |
|---|---|---|---|---|---|---|
| $\Omega$ | $m_1$ | $m_2$ | bba | $m_1$ | $m_2$ | bba |
| $\emptyset$ | 0.00 | 0.09 | 0.04 | 0.00 | 0.05 | 0.01 |
| $\omega_1$ | 0.19 | 0.01 | 0.12 | 0.09 | 0.02 | 0.09 |
| $\omega_2$ | 0.00 | 0.16 | 0.06 | 0.00 | 0.07 | 0.01 |
| $\omega_1, \omega_2$ | 0.28 | 0.22 | 0.25 | 0.42 | 0.33 | 0.41 |
| $\omega_3$ | 0.00 | 0.14 | 0.06 | 0.00 | 0.03 | 0.00 |
| $\omega_1, \omega_3$ | 0.23 | 0.33 | 0.27 | 0.21 | 0.10 | 0.20 |
| $\omega_2, \omega_3$ | 0.00 | 0.05 | 0.02 | 0.00 | 0.16 | 0.02 |
| $\omega_1, \omega_2, \omega_3$ | 0.30 | 0.01 | 0.19 | 0.28 | 0.24 | 0.27 |

Table 1: With $\Omega = \{\omega_1, \omega_2, \omega_3\}$, computation of $m^{\Omega}_{CC,0}$, the prior bba held by $CC$ at time 0 on $\Omega$ and $m^{\Omega}_{RS}$, the bba communicated by $RS$ at time 1. We present the coherent bbas $m_1$, the random bbas $m_2$, and their convex combination with a noise level of 40% for $m^{\Omega}_{CC,0}$ and 10% for $m^{\Omega}_{RS}$.

We then computed the bba $m^{\Omega}_{new}$ indued by $m^{\mathcal{L}}$ and $m^{\Omega}_{Col}$ using

$$m^{\Omega}_{new} = 0.04CBF + 0.54m^{\Omega}_{Col} + 0.06\overline{m}^{\Omega}_{Col} + 0.36VBF. \quad (4)$$

Table 2 presents this new bba $m^{\Omega}_{new}$ and the final bba $m^{\Omega}_{CC,1}$ that results from the conjunctive combination of $m^{\Omega}_{new}$ with the past beliefs $m^{\Omega}_{CC,0}$, i.e., $m^{\Omega}_{CC,1} = m^{\Omega}_{new} \textcircled{\cap} m^{\Omega}_{CC,0}$.

At the bottom of table 2, we provide the pignistic probabilities for each of the bba listed.

Initially $\omega_1$ was the most probable class ($BetP(\omega_1) = 0.46$). The hypothesis that the message is untampered is the most supported, and the final pignistic probability gives 0.54 to $\omega_1$.

## 5.2 A dynamic combination

Imagine a dynamic context where $CC$ starts at time 0 with some prior bba on $\Omega = \{\omega_1, \omega_2, \omega_3\}$ collected from a non tampered sensor, and then collects new bba at time 1 to 10 from a new sensor $RS$. The bbas send by $RS$ are negated by $DA$ at time 1, 2, 5, 7, 10. The data collected at time 0 is very noisy (noise weight of 0.8) whereas those from $RS$ are much better (noise weight of 0.2), but are sometimes tampered. We compute the pignistic probabilities according to relation 3. Figure 1 presents the three pignistic probability functions averaged over 30 simulations. The solid line indicates $BetP(\omega_1)$, the probabilities for the actual class. Not too bad at time 0, it increases more or less regularly with new data.

For comparison purpose we show in figure 2 what would be the equivalent curves if $CC$ knew which bbas had been negated, in which case $CC$ would had negated the collected bba to recover the bba as send by $RS$ and use this de-tampered bba for the data fusion process. Results are quite satisfactory as $BetP(\omega_1)$ grows monotonically to 1.

Finally we repeat the combination using the data as collected, thus what $CC$ would have computed if $CC$

| $\Omega$ | $m^{\Omega}_{CC,0}$ | $m^{\Omega}_{Col}$ | $\overline{m}^{\Omega}_{Col}$ | $m^{\Omega}_{new}$ | $m^{\Omega}_{CC,1}$ |
|---|---|---|---|---|---|
| $\emptyset$ | 0.04 | 0.01 | 0.27 | 0.05 | 0.13 |
| $\omega_1$ | 0.12 | 0.09 | 0.02 | 0.05 | 0.23 |
| $\omega_2$ | 0.06 | 0.01 | 0.20 | 0.02 | 0.06 |
| $\omega_1, \omega_2$ | 0.25 | 0.41 | 0.00 | 0.22 | 0.23 |
| $\omega_3$ | 0.06 | 0.00 | 0.41 | 0.03 | 0.05 |
| $\omega_1, \omega_3$ | 0.27 | 0.20 | 0.01 | 0.11 | 0.19 |
| $\omega_2, \omega_3$ | 0.02 | 0.02 | 0.09 | 0.01 | 0.01 |
| $\omega_1, \omega_2, \omega_3$ | 0.19 | 0.27 | 0.01 | 0.51 | 0.10 |
| $\omega_1$ | 0.46 | 0.48 | 0.03 | 0.41 | 0.54 |
| $\omega_2$ | 0.27 | 0.31 | 0.34 | 0.32 | 0.25 |
| $\omega_3$ | 0.27 | 0.20 | 0.63 | 0.27 | 0.21 |

Table 2: We present successively 1) $m^{\Omega}_{CC,0}$, the prior bba held by $CC$ at time 0 on $\Omega$, 2) $m^{\Omega}_{Col}$, the bba collected by $C$ at time 1 and that happens to be equal to $m^{\Omega}_{RS}$, the bba communicated by $RS$ at time 1, 3) its negation $\overline{m}^{\Omega}_{Col}$, 4) the bba $m^{\Omega}_{new}$ computed from the last two using relation (4), 5) the bba $m^{\Omega}_{CC,1}$ that represents $CC$ beliefs at time 1 and results from the conjunctive combination of $m^{\Omega}_{CC,0}$ with $m^{\Omega}_{new}$. The bottom part of the table present the pignistic probabilities computed from each bba.

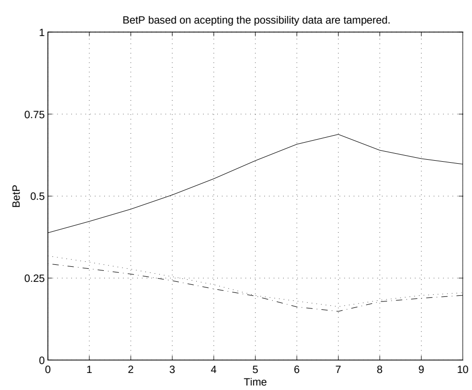


Fig. 1: Pignistic probabilities for the three classes: solid line for the actual class. Bbas updated by relation 3, thus accepting the possibility that the message has been tampered.

neglects the possibility that bbas may have been tampered. Results given in figure 3 show how much $CC$ would have been deceived. In fact $BetP(\omega_1)$ goes to 0 whereas the two erroneous alternatives are equally supported. $DA$ would have been quite efficient in deceiving $CC$.

## 6 Conclusions

We consider that messages between a remote sensor and a coordination center may be tampered by some deceiving agent. The messages send by the remote sensor are basic belief assignments (bba) over some finite frame $\Omega$. We discuss deceiving strategies that would fool the coordination center? We enhance the interest for the deceiving agent to transfer the negation of the bba that was sent by the remote sensor. A nice property is that the negation tampering method does not

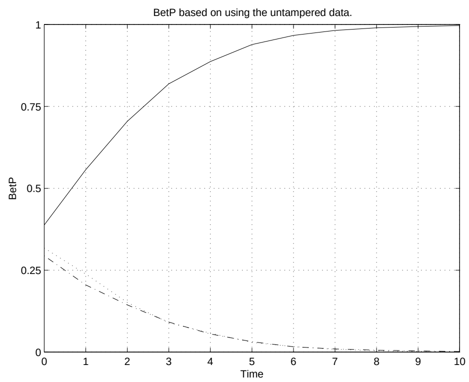


Fig. 2: Pignistic probabilities for the three classes: solid line for the actual class. Bbas updated using the $RS$ data, thus what $CC$ would compute is $CC$ knew which messages had been tampered.

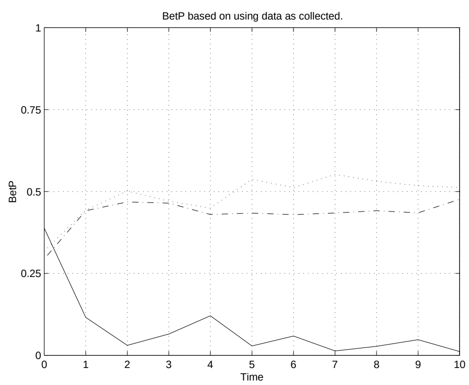


Fig. 3: Pignistic probabilities for the three classes: solid line for the actual class. Bbas updated using the collected data, thus neglecting that messages might have been tampered.

depend on the decoding of the message itself, as the transformation operator does not depend on the bba sent by the remote sensor.

If the coordination center accepts the bba as collected, the updated bbas will be inadequate and misleading.

If the coordination center knew which messages had been tampered, it would be immediate to recover the original message but such a context is not supposed to prevail in reality.

If the coordination center has some prior beliefs about the presence of the deceiving agent, the application of the TBM allows to construct the bba that takes in account both the collected bba and these prior beliefs.

If the coordination center has no prior belief about the presence of the deceiving agent, we describe a strategy that permits the construction of a prior belief about the presence of the deceiving agent, that can be used as in the previous case. A nice property of this method is its robustness in that the final beliefs will be the same, had the deceiving agent act or not.

We illustrate that last method, and show that it seems to be adequate and helpful. The method is somehow ad hoc as the distinctness assumptions underlying the use of the combination rules is not exactly satisfied. Other methods can be imagined to handle such inter evidential correlations.

We think we have been able to show that the TBM is somehow well adapted to cope with some kinds of tampered data, and that it might be interesting in real life contexts. Its applicability will of course depend on the nature of the sensors and of the possible behaviors of the deceiving agents.