

Advanced ACO Algorithms

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The Ant System Algorithm

```
For #iterations
  For each ant
    Choose a random starting city
    While tour is not complete
      Select next city using random
      proportional transition rule
    End While
  End for
  Update pheromone information
End for
```

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Successors and Extensions of the AS

- MAX-MIN Ant System (MMAS)
 - Only iteration best or best-so-far ants update pheromone
 - Pheromone trails have explicit upper and lower limits
 - Pheromone trails initialized to upper limit
 - Pheromone trails are re-initialized when stagnated
- Ant Colony System (ACS)
 - Pheromone is also updated while building solution
 - Only iteration best or best-so-far ants update pheromone

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MAX-MIN Ant System (MMAS)

- Only iteration best or best-so-far ants update pheromone

$$\begin{aligned}\tau_{ij} &\leftarrow (1 - \rho) \cdot \tau_{ij} & \forall (i, j) \\ \tau_{ij} &\leftarrow \tau_{ij} + \Delta\tau_{ij}^{best} & \forall (i, j) \\ \Delta\tau_{ij}^{best} &= \begin{cases} \frac{1}{L'} & \text{if } \text{arc}(i, j) \in T^{best} \\ 0 & \text{otherwise} \end{cases}\end{aligned}$$

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MAX-MIN Ant System (MMAS)

- Pheromone trail values are subject to bounds

$$\tau_{min} \leq \tau_{ij} \leq \tau_{max}$$

$$\tau_{max} = \frac{1}{\rho \cdot L^{opt}} \quad \hat{\tau}_{max} = \frac{1}{\rho \cdot L^{bs}}$$

$$\tau_{min} = \frac{\hat{\tau}_{max}}{a} \quad \tau_0 = \infty$$

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MAX-MIN Ant System (MMAS)

- Pheromone trails are reinitialized:
 - When the algorithm converges;
 - When no improved solutions has been generated for a certain number of consecutive iterations.

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Ant Colony System (ACS)

- Three main ideas:
 - Different *state transition* rule;
 - Different *global pheromone update* rule;
 - New *local pheromone update* rule

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ACS - State Transition Rule

- A.k.a. pseudo-random proportional rule:
 - with probability q_0 , exploitation ;
 - with probability $1 - q_0$, biased exploration

$$j = \begin{cases} \arg \max_{j \in \mathcal{N}_i^k} (\tau_{ij} \cdot \eta_{ij}^\beta) & \text{if } q \leq q_0 \text{ (Exploitation)} \\ J & \text{otherwise (Exploration)} \end{cases}$$

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ACS - Global Update Rule

- Pheromone modified only on edges of the best tour so far

$$\begin{aligned}\tau_{ij} &\leftarrow (1 - \rho) \cdot \tau_{ij} & \forall (i, j) \in T^{bs} \\ \tau_{ij} &\leftarrow \tau_{ij} + \rho \cdot \Delta\tau_{ij}^{bs} & \forall (i, j) \in T^{bs} \\ \text{where } \Delta\tau_{ij}^{bs} &= \frac{1}{L^{bs}}\end{aligned}$$

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ACS - Local Update Rule

- While building a solution each ant updates pheromone on visited edges:

$$\tau_{ij} \leftarrow (1 - \rho) \cdot \tau_{ij} + \rho \cdot \tau_0$$

- This update rule introduces diversification

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ACS - Pseudocode

Loop

Randomly select starting cities

For step=1 to #cities

For k=1 to #ants

 Apply the **state transition rule**

 Apply the **online trail updating rule**

End-for

End-for

Apply the **offline trail updating rule**

Until End_condition

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