Swarm Intelligence

Class Exercises 4

March 19, 2020

1. What is the inertia coefficient, and how is it related to exploration and exploitation?
   Answer: The value of the inertia coefficient modifies the magnitude of the velocity vector, which allows the algorithm to control the influence that the current search direction will have when particles update their position in the search space. That is, for large inertia values, particles will behave more exploratory as they can move through the search space with larger step size, while for small inertia values, particles will behave more exploitative and focus on the most promising regions.

2. What happens if we set $\psi_1 = 0$ and $\psi_2 = 1$? What happens if we instead set $\psi_1 = 1$ and $\psi_2 = 0$?
   Answer: For a parameter settings where $\psi_1 = 0$ and $\psi_2 = 1$, the cognitive influence is neglected, that is, particles will not take into account their personal best position. Conversely, for $\psi_1 = 1$ and $\psi_2 = 0$, the social component is not taken into account, and thus particles will moved through the search space based only on their own previous experience.

3. What is the purpose of the U_1 and U_2 matrices?
   Answer: The purpose of the random diagonal matrices U_1 and U_2 is to perform random changes to the magnitude and direction of the personal and social components $(p_i - x_i(t))$ and $(p_i - g_i(t))$, in order to provide diversity to particles' movement.

4. What is the role of topologies? What happens if a particle can communicate with all the rest of the swarm? What happens if it can instead communicate with only few other particles?
   Answer: The topology determines from which particles in the swarm each particle i will be able to communicate (that is, the neighbors of i). Both the topology and the model of influence play a key role in determining how rapidly information will be spread among particles, which may result in a more exploratory or exploitative behavior. In a fully-connected topology, the information of the best-so-far solution will always be communicated to the entire swarm, which may be useful to enhance the exploitation of the search space, while in a ring topology this communication will be slower, and therefore, the behavior of the swarm will be more exploratory.

5. Assume you want to minimize the Rosenbrock function in $N = 4$ dimensions
$$\sum_{i=1}^{N-1} 100 \times (x_{i+1} - x_i^2)^2 + (1 - x_i)^2 \tag{1}$$

when $\omega = 1$, $\psi_1 = 1$ and $\psi_2 = 2$. At the current iteration, particle P_1 has position $[0.43, 1.25, -3.2, 2.4]$ with value 8546.17, and velocity $[-0.8, 0.2, -1.2, 0.1]$; the personal best is $[1.2, 2.5, -2.1, 2.2]$ with value 7584.92 and the global best known by P_1 in its neighbourhood is $[1.4, 1.1, -0.4, 3.1]$ with value 1199.66. The diagonals of the matrices U_1 and U_2 are respectively $[0.21, 0.43, 0.12, 0.84]$ and $[0.63, 0.12, 0.92, 0.43]$. Update velocity and position of P_1. What is the value of the solution now?

Answer: Using $V(t+1) =$ Inertia + Congnitive influence + Social influence, and $x(t+1) = x(t) + V(t+1)$, we get:

$$\begin{aligned}
\text{Inertia} &= [-0.8, 0.2, -1.2, 0.1] \\
\\
\text{Congnitive influence} &= [0.21 * (1.2 - (0.43)), \\
&\quad 0.43 * (2.5 - (1.25)), \\
&\quad 0.12 * (-2.1 - (-3.2)), \\
&\quad 0.84 * (2.2 - (2.4))] \\
&= [0.1617, 0.5375, 0.132, -0.168] \\
\\
\text{Social influence} &= [2 * 0.63 * (1.4 - (0.43)), \\
&\quad 2 * 0.12 * (1.1 - (1.25)), \\
&\quad 2 * 0.92 * (-0.4 - (-3.2)), \\
&\quad 2 * 0.43 * (3.1 - (2.4))] \\
&= [1.2222, -0.036, 5.152, 0.602] \\
\\
V(t+1) &= [-0.8 + 0.1617 + 1.2222, \\
&\quad 0.2 + 0.5375 + (-0.036), \\
&\quad -1.2 + 0.132 + 5.152, \\
&\quad 0.1 + (-0.168) + 0.602] \\
&= [0.5839, 0.7015, 4.084, 0.534] \\
\\
x(t+1) &= [0.43 + 0.5839, \\
&\quad 1.25 + 0.7015, \\
&\quad -3.2 + 4.084, \\
&\quad 2.4 + 0.534] \\
&= [1.0139, 1.9515, 0.884, 2.934] \\
\\
f_{\text{Rosenbrock}}(x(t+1)) &= ((1.9515 - (1.0139)^2)^2 + (1 - 1.0139)^2) + \\
&\quad ((0.884 - (1.9515)^2)^2 + (1 - 1.9515)^2) + \\
&\quad ((2.934 - (0.884)^2)^2 + (1 - 0.884)^2) \\
&= 0.853058001 + 9.457188332 + 4.646901672 \\
&= 14.957148005
\end{aligned}$$