Swarm Intelligence
Extensions of Ant System

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Outline

1. Implementation exercise
2. Review of AS
3. MAX-MIN Ant System (MMAS)
4. Ant Colony System
5. Elitist Ant System
6. Rank-based Ant System
7. Best-worst Ant System
8. ACOTSP
Implementation exercise 1

AS solution quality $\alpha, \beta=1, \rho=0.5$

Mean solution quality

$m1$ $m2$ $m5$ $m7$ $m10$ $m20$ $m50$ $m100$
Compare results statistical tests

- Is there a **statistically significant** difference between the solution quality generated by the different algorithms?

- **Null hypothesis**: The statement to be tested.
  - Example: For the Wilcoxon signed-rank test, the null hypothesis is that ‘the median of the differences is zero’

- The **significance level** (α) determines the maximum allowable probability of incorrectly rejecting the null hypothesis.

- The null hypothesis is rejected if this p-value is smaller than the previously chosen significance level.
Implementation exercise 1

- Wilcoxon test p-values:  
  - m1 vs. m2: 1.907e-06  
  - m2 vs. m5: 0.003654  
  - m5 vs. m7: 0.6676  
  Corrected (Bonferroni)  
  - m1 vs. m2: 0.00001  
  - m2 vs. m5: 0.011  
  - m5 vs. m7: 1.000
Implementations exercise 1

AS solution quality $m=10 \rho=0.5$

AS convergence

Mean solution quality

Mean solution quality

Tours
Implementation exercise 1

Wilcoxon test p-values

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Implementation exercise 1

- More examples of parameter analysis:
Implementation exercise 1

- More examples of parameter analysis:

```
AS solution quality $\beta$, $\alpha=1$ $\rho=0.2$ $m=10$
```

![Box plot showing mean solution quality for different values of $b$.](image)
Implementation exercise 1

- We can also analyse interactions:
Extensions of AS

- MAX-MIN Ant System (MMAS)
  - Only iteration best or best-so-far ants update pheromone
  - Pheromone trails have explicit upper and lower limits
  - Pheromone trails initialized to upper limit
  - Pheromone trails are re-initialized when stagnated

- Ant Colony System (ACS)
  - Pheromone is updated also while building the solution
  - Only iteration best or best-so-far ants update pheromone
MAX-MIN Ant System (MMAS)

- Only iteration best or best-so-far ants update pheromone

\[ \tau_{ij}(t) = (1 - \rho) \cdot \tau_{ij}(t - 1) + \Delta \tau_{ij}^{best} \]

\[ \Delta \tau_{ij}^{best} = \frac{1}{L_{best}}, \text{ if } arc(i, j) \in \text{ best tour} \]

- \( L_{best} \): length of the shortest tour found
MAX-MIN Ant System (MMAS)

- Pheromone trail values are subject to bounds

\[ \tau_{\text{min}} \leq \tau_{ij} \leq \tau_{\text{max}} \]

\[ \tau_{\text{max}} = \frac{1}{\rho \cdot L^{\text{opt}}} \]

\[ \tau_{\text{min}} = \frac{\tau_{\text{max}}}{a} \]

\[ \tau'_{\text{max}} = \frac{1}{\rho \cdot L^{bs}} \]

\[ \tau'_{\text{min}} = \frac{\tau_{\text{max}}}{2 \cdot n} \]

\[ \tau_0 = \infty \]
MAX-MIN Ant System (MMAS)

- Pheromone trails are re-initialized:
  - When the algorithm converges
  - When no improving solution has been generated for a certain number of consecutive iterations
Ant Colony System (ACS)

- Three main ideas:
  - Different state transition rule
  - Different global pheromone update rule
  - New local pheromone update rule

- Goal is: better control on exploration/exploitation
Ant Colony System (ACS)

- State transition (pseudo-random proportional) rule, which is biased towards:
  - exploitation with probability $q_0$
    
    $$j = \arg\max_{j \in N_i^k} (\tau_{ij} \cdot \eta_{ij}^\beta) \text{ if } q \leq q_0$$

  - exploration with probability $1 - q_0$
    
    $j$ is chosen according to the usual proportional transition rule.
Ant Colony System (ACS)

- **Local update rule** (to introduce diversification): while building a solution, each ant updates pheromone on visited edges

\[ \tau_{ij} = (1 - \rho) \cdot \tau_{ij} + \rho \cdot \tau_0 \]
Ant Colony System (ACS)

- **Global update rule**: pheromone updated **only on edges of the best tour** found so far

\[
\tau_{ij} = (1 - \rho) \cdot \tau_{ij} + \rho \cdot \Delta \tau_{ij}^{bs} \quad \forall (i, j) \in T^{bs}
\]

\[
\Delta \tau_{ij}^{bs} = \frac{1}{L^{bs}}
\]
Ant Colony System for TSP
Simple pseudo code

1 While !termination()
2 For each ant Do
3      select random initial starting city
4      While tour is not complete
5          select next city using state transition rule
6          apply local pheromone update rule
7      EndWhile
8 EndFor
9    Apply global pheromone update rule
10 EndWhile
Elitist Ant System

- Elitism refers to favour best individuals to guide the search. → intensification
- After each iteration the **global best ant** deposit pheromone along with the others.
- Introduce a new parameter $e$ that controls the contribution of the global best ant to the pheromone update.

$$
\tau_{ij}(t) = (1 - \rho) \cdot \tau_{ij}(t-1) + \sum_{k=1}^{m} \Delta \tau_{ij}^k + e \Delta \tau_{ij}^{bs}
$$
Rank-based Ant System

- A number of the best ants are allowed to update pheromone.
- All the ants are ranked regarding their tour quality and the best $\omega - 1$ are selected.
- They deposit pheromone according to their rank. So the best ones contribute more.
- Parameter $\omega$ controls the number of ants allowed to deposit pheromone (usually 25%) and also controls the amount of pheromone contributed by each ant.
- The global best ant deposit pheromone with the others.

$$\tau_{ij}(t) = (1 - \rho) \cdot \tau_{ij}(t-1) + \sum_{r=1}^{\omega-1} (\omega - r) \Delta \tau_{ij}^r + \omega \Delta \tau_{ij}^{bs}$$
Best-worst Ant System

- Transition rule and pheromone evaporation as in Ant System
- Pheromone update after each iteration:
  - The global best ant contributes positively to the pheromone update
  - The worst ant contributes negatively to the pheromone update (additional evaporation)
    - This is only applied in the edges present in the worst ant and absent in the global best ant.
- Pheromone trails \textbf{mutation} $\rightarrow$ \textbf{diversification}
- Restart of the search when stagnation ($\tau_0$)
ACOTSP

- ACOTSP developed by Thomas Stutzle, provides the implementation of a set of ACO algorithms to solve TSP.
- Which algorithms are implemented?
  - Ant System
  - Elitist Ant System
  - Max-min Ant System
  - Rank based Ant System
  - Best-worst Ant System
  - Ant Colony System
ACOTSP
Options: Algorithms

- How to specify the algorithm?
  - **--as**: Ant System
  - **--eas**: Elitist Ant System
  - **--ras**: Rank-based version of Ant System
  - **--mmas**: MAX-MIN ant system
  - **--bwas**: Best-worst ant system
  - **--acs**: Ant colony system

- Look for other parameters using **./acotsp --help**

- Related parameters:
  - **--q0**: prob. of best choice in tour construction (ACS)
  - **elitistants**: number of elitist ants (MMAS)
  - **rasranks**: number of ranks in rank-based Ant System (RAS)
ACOTSP
Options: Other

- Other general parameters
  - \textit{--tries}: number of independent trials (runs)
  - \textit{--tours}: number of steps in each trial (max tours evaluated per trial)
  - \textit{--time}: maximum time for each trial (seconds)
  - \textit{--seed}: seed for the random number generator
  - \textit{--optimum}: to stop if tour better or equal optimum is found
  - \textit{--ants}: number of ants
  - \textit{--nnants}: nearest neighbours in tour construction
    - To use of candidate list to construct solutions
  - \textit{--alpha}: alpha (influence of pheromone trails)
  - \textit{--beta}: beta (influence of heuristic information)
  - \textit{--rho}: rho (pheromone trail evaporation)
  - \textit{--localsearch}: 0: no local search 1: 2-opt 2: 2.5-opt 3: 3-opt
ACOTSP
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ACOTSP
Options: Local search

• **Local search** starts from a solution already constructed and moves through the search space from one neighbour to other.

• ACOTSP offers the possibility to apply a local search procedure to improve the tours found.

• The options are:
  - 2-opt
  - 2.5-opt
  - 3-opt
ACOTSP
Options: Local search

• 2-opt
  – Heuristic: Select two edges and exchange them (2-exchange)
  – Repeat this process for all the edges combinations looking for improvement

• 3-opt follows the same idea using 3 edges, also 2-opt moves are evaluated.

• 2.5-opt: Evaluates the insertion of a node coming from edge (A-B) between the nodes of other edge (C-D). Ex. A-C-B-D