Swarm deadlock for symmetric choices over three or more options

Andreagiovanni Reina, James A. R. Marshall, and Thomas Bose
University of Sheffield, Sheffield S1 4DP, UK, a.reina@sheffield.ac.uk

Abstract. We consider a model of decentralised value-sensitive consensus decision-making, based on observations of house-hunting honeybees and proposed for swarm robotics applications. This model has been shown to adaptively maintain or break deadlock between equal options in binary choices, as a function of their quality, under the control of a single distributed decision parameter. We show that this model cannot break deadlock between three equal alternatives, as currently formulated.

Model. We study a model of decentralised decision-making [1] inspired by the observation of cross-inhibitory stop-signalling behaviour in swarms of house-hunting honeybees choosing between multiple potential nest-sites [2].

The general model for \( N \) options is:

\[
\begin{align*}
\frac{dx_i}{dt} &= \gamma_i x_u - \alpha_i x_i + \rho_i x_u x_i - \sum_{j=1}^{N} x_i \tilde{\beta}_{ij} x_j, & i \in \{1, \ldots, N\}, \\
x_u &= 1 - \sum_{i=1}^{N} x_i
\end{align*}
\]

where \( x_i \) represents the subpopulation committed to option \( i \) and \( x_u \) the uncommitted subpopulation. \( \gamma_i \) represents the discovery rate for option \( i \), \( \alpha_i \) the abandonment rate for option \( i \), \( \rho_i \) the recruitment rate for option \( i \) and \( \beta_{ij} \) the cross-inhibition rate from subpopulation \( j \) to subpopulation \( i \). By defining

\[
\gamma_i = kv_i, \quad \alpha_i = kv_i^{-1}, \quad \rho_i = hv_i, \quad \tilde{\beta}_{ij} = \tilde{\beta}, \quad \frac{\tilde{\beta}}{k} = \beta
\]

and applying (2) to (1), we obtain:

\[
\begin{align*}
\frac{dx_i}{d\tau} &= v_i x_u - \frac{x_i}{v_i} + r v_i x_u x_i - \beta \sum_{j=1}^{N} x_i x_j, & i \in \{1, \ldots, N\}, \\
x_u &= 1 - \sum_{i=1}^{N} x_i
\end{align*}
\]

where \( r = h/k \) is the ratio of interaction over spontaneous transitions, and \( \tau = k t \) is the dimensionless time. The parameterisation of (2) is a generalisation of that in [1], since, using \( r = 1 \), the system (1) reduces to the original, and thus displays the same dynamics.

Due to its simplicity and its adaptive decision-making characteristics, this model is particularly interesting for the design of large-scale decentralised systems (e.g.,...
robot swarms or wireless sensor networks) able to make consensus decisions [3]. A recent work has implemented the model of [1] on a swarm of 150 kilobot robots [4].

Results. A bifurcation analysis of (3) shows that for $r \leq 1$ there is no value of $\beta$ that breaks the decision deadlock in the case of $N = 3$ same-quality options (see Fig. 1). A formal proof for $N = 3$ and $r = 1$ is provided in [5]. This result motivates the change of parameterisation with respect to previous work [1]. A full analysis of the resulting collective decision dynamics in both symmetric decisions, with $N$ equal options, and best-of-$N$ decisions, with one best option and $N - 1$ inferior distractors, is provided in [5].

Our results emphasise that $r$, the ratio of interactions over spontaneous behaviour, is the key parameter that allows, or prevents, the swarm to make a decision. Scarce communication hampers the attainment of consensus within the swarm, while frequent signalling between peers provides them a constant feedback from others that results in a coordinated collective response, in our case a consensus decision. We believe this finding may both help the better understanding of natural swarms and the design of large-scale decentralised systems.

References


