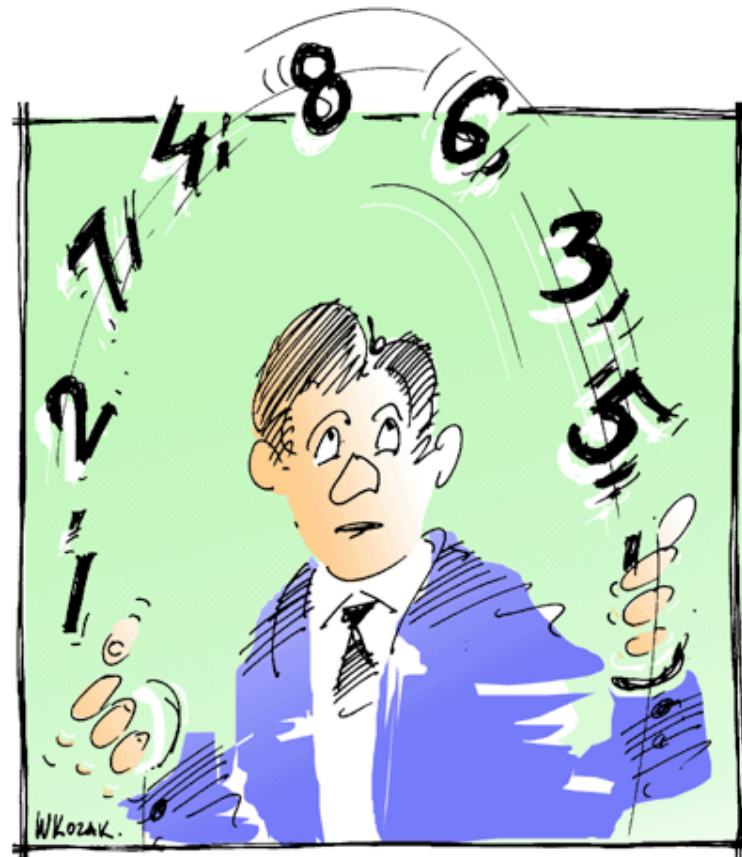
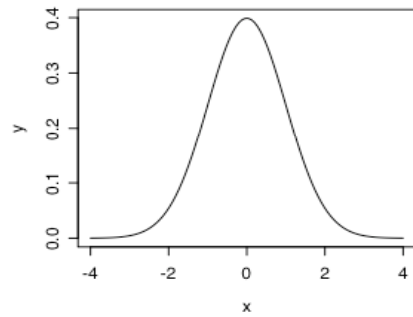


# Chapter 5

## Computer-Intensive Statistical Methods



# Last Week....

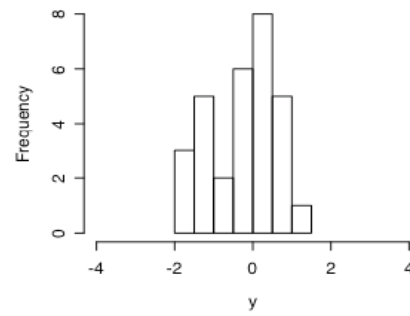


Assume Gaussian



Experiment  
(take sample)

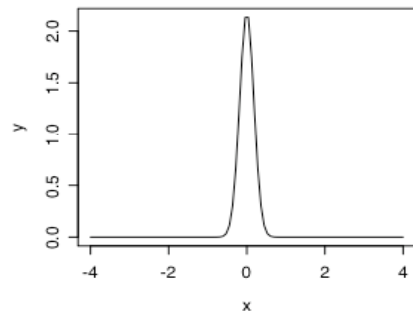
Histogram of y



Assume  $\sim$  Population  
Compute std. deviation,  $s$



Central Limit Theorem



Assume Gaussian  
Compute std. error  
( $s/\sqrt{n}$ )  
Make inferences

# Motivations and Goals

You might be interested in things that

- aren't normal
- aren't mathematically computable in convenient form
- come from data with unknown distribution
- statistics of interest is not mean
- to test hypotheses only on samples!

Construct sampling distribution empirically with computers!

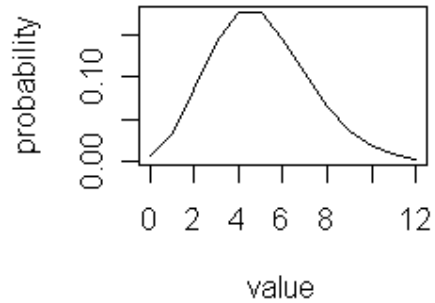
# Monte Carlo Test

- The distribution of the population can be described parametrically e.g Binomial, Poisson
- The statistic of interest can be anything

# Monte Carlo Test

## population

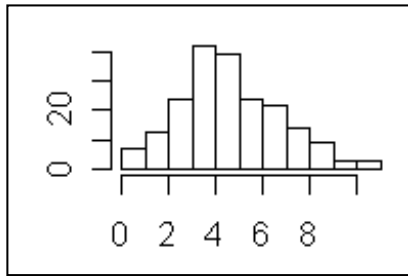
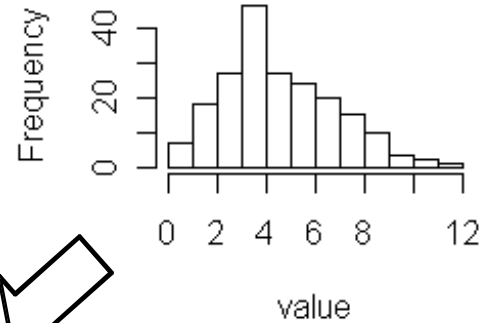
Assume of known form



Experiment  
(take data  
sample)

## sample

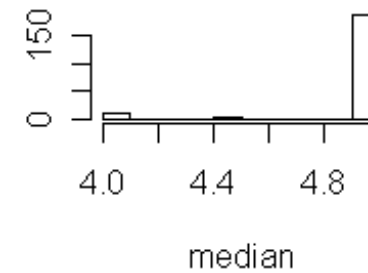
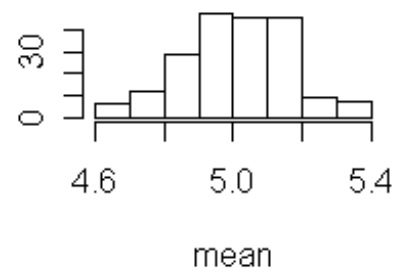
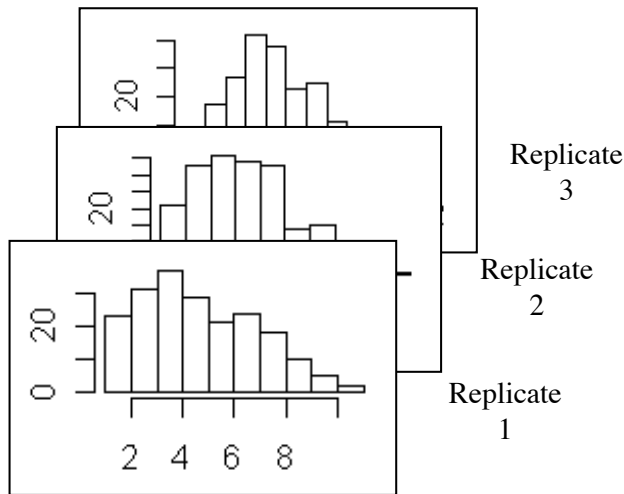
Assume ~ Population



Replicate  
r

Calculate distribution  
parameters and  
generate r resampled  
data sets

Compute  
statistic of  
interest

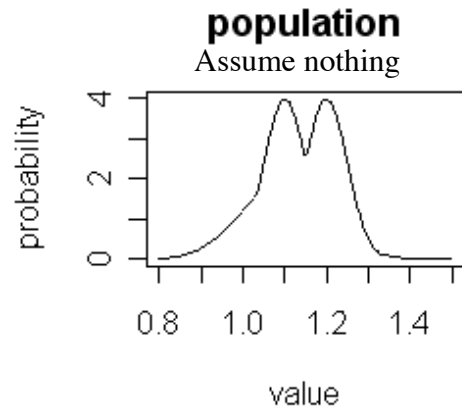


# Bootstrap Method

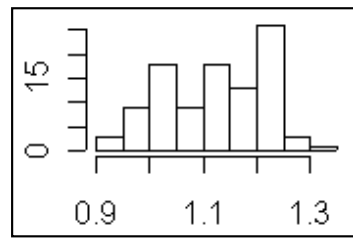
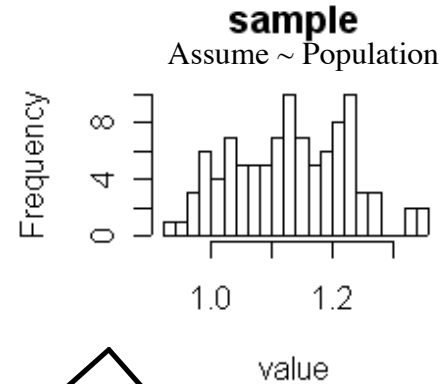
- A special case of Monte Carlo test
- The distribution need not be described parametrically
- The statistics of interest can be anything
- Assumption: the data be your best estimate of the distribution of population
- Draw samples with **replacement** from your data many times



# Bootstrap Method



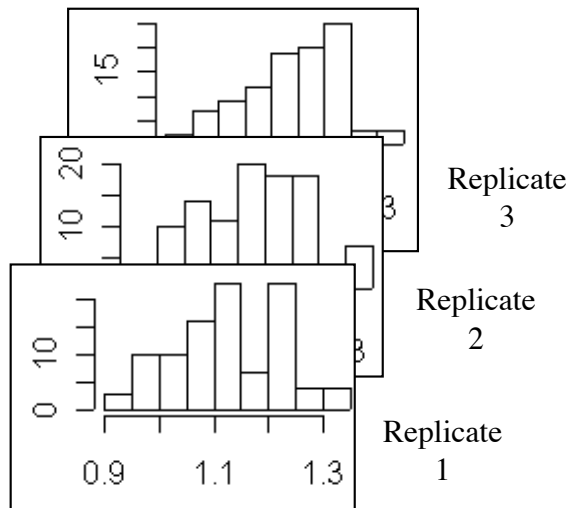
Experiment  
(take data  
sample)



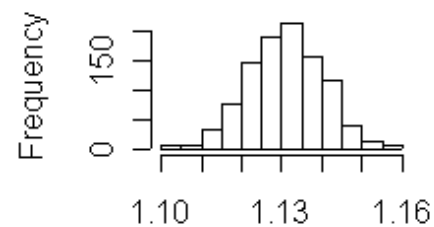
Replicate  
r

Generate r resampled  
data sets

...



Compute  
statistic of  
interest



# Bootstrap Sampling Distribution

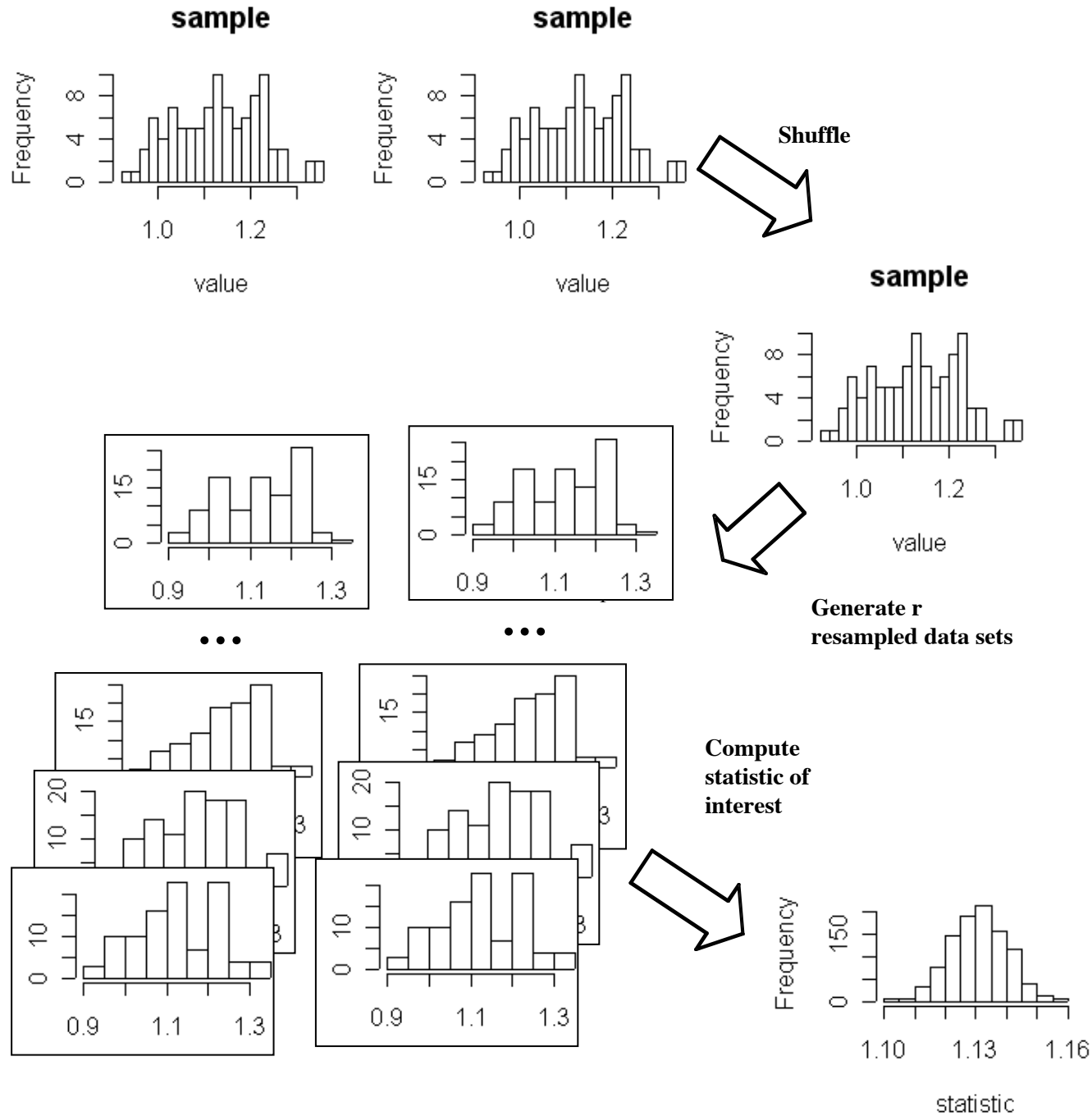
- Shift Method:
  - bootstrap and null hypothesis sampling distributions are different but they have the same shape
- Normal Approximation Method
  - null hypothesis sampling distribution is normal
  - bootstrap sampling distribution - to estimate the variance of the normal distribution
  - Z-test to estimate the p value



# Randomization Test

- Resampling without replacement
- To test hypothesis about samples without drawing any conclusion about population
- Mainly to test if the samples are unrelated

# Randomization Test



# Nonparametric Test

- Cohen's view - rather pessimistic!
- Number and nature of the parameters is flexible and not fixed in advance - distribution free
- Less power than the appropriate parametric tests, but are more robust
- Sampling distributions are usually exact/analytical

# Wilcoxon Signed-Rank Test

- compute absolute difference  $|X_a - X_b|$  for each pair;
- omit cases where  $|X_a - X_b| = 0$ ;
- rank the remaining absolute differences, from smallest to largest, employing tied ranks where appropriate;
- "+" sign when  $X_a - X_b > 0$  and "-" sign when  $X_a - X_b < 0$
- sum the signed ranks  $T_+, T_-$  and choose the minimum

$$\mu = n(n + 1)/4$$

$$\sigma^2 = n(n + 1)(2n + 1)/4$$

$$z = (T - \mu) / \sigma$$

# Take Home Message

