

# Computational results for an automatically tuned CMA-ES with increasing population size on the CEC'05 benchmark set: Supplementary material

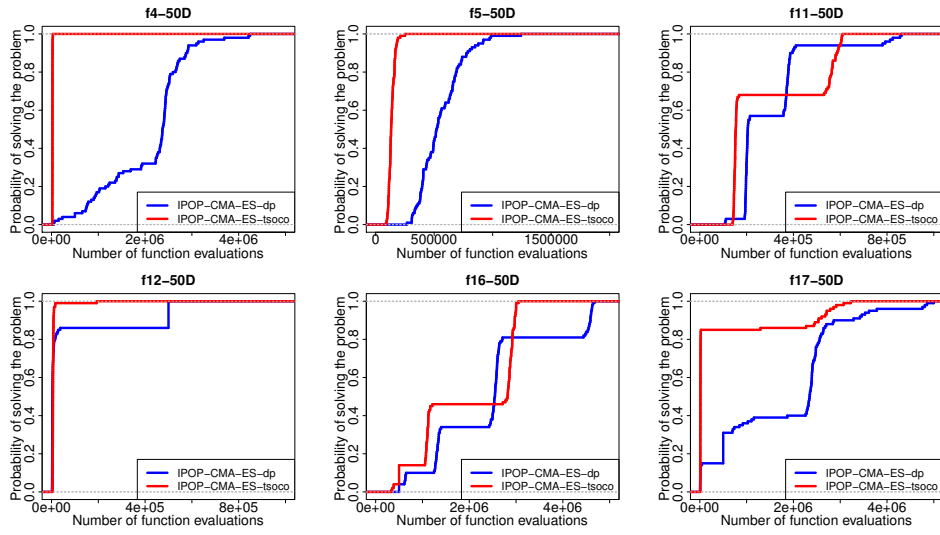
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**Abstract.** This document is the supplementary material for the article: Computational results for an automatically tuned CMA-ES with increasing population size on the CEC'05 benchmark set.

1. The qualified run-length distributions (RLDs, for short) over 100 independent runs obtained by iCMA-ES-dp and iCMA-ES-tsc on the 50 dimensional versions of functions  $f_{cec4}$ ,  $f_{cec5}$ ,  $f_{cec11}$ ,  $f_{cec12}$ ,  $f_{cec16}$  and  $f_{cec17}$ .
2. The average errors obtained by Sep-iCMA-ES-tsc and iCMA-ES-tsc over 25 independent runs for CEC'05 functions.
3. The data obtained by different variations of the tuning setups over 25 independent runs for CEC'05 functions of dimension 50.



**Fig. 1.** The qualified run-length distributions (RLDs, for short) over 100 independent runs obtained by iCMA-ES-dp and iCMA-ES-tsc on the 50 dimensional versions of functions  $f_{cec4}$ ,  $f_{cec5}$ ,  $f_{cec11}$ ,  $f_{cec12}$ ,  $f_{cec16}$  and  $f_{cec17}$ . The required solution qualities are  $1.00E+03$ ,  $1.00E-08$ ,  $1.00E-08$ ,  $1.00E+05$ ,  $1.00E+00$  and  $1.00E+02$ , respectively.

**Table 1.** The average errors obtained by Sep-iCMA-ES-tsc and iCMA-ES-tsc over 25 independent runs for CEC'05 functions. The numbers in parenthesis represent the times of  $<$ ,  $=$ , and  $>$ , respectively, when the corresponding algorithms are compared with iCMA-ES-tsc on the average errors.

| $f_{cec}$ | 10 dimensions           |             | 30 dimensions           |             | 50 dimensions           |             |
|-----------|-------------------------|-------------|-------------------------|-------------|-------------------------|-------------|
|           | Mean Errors             |             | Mean Errors             |             | Mean Errors             |             |
|           | Sep-iCMA-ES-tsc         | iCMA-ES-tsc | Sep-iCMA-ES-tsc         | iCMA-ES-tsc | Sep-iCMA-ES-tsc         | iCMA-ES-tsc |
| $f_1$     | 1.00E-08                | 1.00E-08    | 1.00E-08                | 1.00E-08    | 1.00E-08                | 1.00E-08    |
| $f_2$     | 1.00E-08                | 1.00E-08    | 1.00E-08                | 1.00E-08    | 1.00E-08                | 1.00E-08    |
| $f_3$     | 3.45E+04                | 1.00E-08    | 1.07E+05                | 1.00E-08    | 1.17E+05                | 1.00E-08    |
| $f_4$     | 1.00E-08                | 1.00E-08    | 1.40E+03                | 1.00E-08    | 1.54E+04                | 1.00E-08    |
| $f_5$     | 1.00E-08                | 1.00E-08    | 1.03E+03                | 1.00E-08    | 2.80E+03                | 1.00E-08    |
| $f_6$     | 2.62E-06                | 1.00E-08    | 4.78E-01                | 1.00E-08    | 7.97E-01                | 1.00E-08    |
| $f_7$     | 5.81E-03                | 1.00E-08    | 1.00E-08                | 1.00E-08    | 1.00E-08                | 1.00E-08    |
| $f_8$     | 2.00E+01                | 2.02E+01    | 2.00E+01                | 2.08E+01    | 2.00E+01                | 2.10E+01    |
| $f_9$     | 2.78E-07                | 4.81E-02    | 5.57E-01                | 1.99E+00    | 2.23E+00                | 4.18E+00    |
| $f_{10}$  | 1.81E-05                | 3.73E-03    | 1.27E+00                | 1.59E+00    | 1.31E+00                | 2.71E+00    |
| $f_{11}$  | 2.22E+00                | 1.00E-08    | 3.55E+00                | 5.09E-05    | 3.82E+00                | 6.03E-02    |
| $f_{12}$  | 4.46E+02                | 1.00E-08    | 5.40E+03                | 4.22E+02    | 1.54E+04                | 4.69E+03    |
| $f_{13}$  | 6.95E-01                | 7.14E-01    | 2.72E+00                | 2.53E+00    | 4.95E+00                | 4.70E+00    |
| $f_{14}$  | 3.47E+00                | 2.03E+00    | 1.25E+01                | 1.10E+01    | 2.16E+01                | 2.09E+01    |
| $f_{15}$  | 2.04E+02                | 3.32E+02    | 2.16E+02                | 2.00E+02    | 2.12E+02                | 2.00E+02    |
| $f_{16}$  | 1.07E+02                | 8.86E+01    | 5.17E+01                | 1.11E+01    | 4.15E+01                | 5.34E+00    |
| $f_{17}$  | 2.31E+02                | 9.34E+01    | 3.06E+02                | 2.08E+02    | 2.61E+02                | 6.36E+01    |
| $f_{18}$  | 7.11E+02                | 3.60E+02    | 9.06E+02                | 9.04E+02    | 9.19E+02                | 9.13E+02    |
| $f_{19}$  | 7.19E+02                | 3.20E+02    | 9.06E+02                | 9.04E+02    | 9.19E+02                | 9.13E+02    |
| $f_{20}$  | 7.18E+02                | 3.40E+02    | 9.06E+02                | 9.04E+02    | 9.19E+02                | 9.13E+02    |
| $f_{21}$  | 9.59E+02                | 5.00E+02    | 5.00E+02                | 5.00E+02    | 9.05E+02                | 7.05E+02    |
| $f_{22}$  | 7.84E+02                | 7.28E+02    | 8.40E+02                | 8.17E+02    | 8.61E+02                | 8.19E+02    |
| $f_{23}$  | 5.92E+02                | 5.59E+02    | 5.34E+02                | 5.34E+02    | 8.99E+02                | 7.30E+02    |
| $f_{24}$  | 2.43E+02                | 2.00E+02    | 2.00E+02                | 2.00E+02    | 2.00E+02                | 2.00E+02    |
| $f_{25}$  | 4.06E+02                | 4.03E+02    | 2.11E+02                | 2.09E+02    | 2.16E+02                | 2.13E+02    |
| V.S.      | (5, 4, 16) <sup>†</sup> |             | (3, 6, 16) <sup>†</sup> |             | (3, 4, 18) <sup>†</sup> |             |

<sup>†</sup> denotes there is a significant difference over the distribution of average errors between Sep-iCMA-ES-tsc with iCMA-ES-tsc by a two-sided Wilcoxon matched-pairs signed-ranks test at 0,05  $\alpha$ -level.

**Table 2.** The average errors obtained by iCMA-ES- $\oplus$ , iCMA-ES-uni,multi, iCMA-ES-uni and iCMA-ES-multi for each CEC'05 function. Error values lower than  $10^{-8}$  are approximated to  $10^{-8}$ . The lowest average errors values are highlighted.

| $f_{cec}$ | iCMA-ES- $\oplus$ | iCMA-ES-uni,multi | iCMA-ES-uni | iCMA-ES-multi |
|-----------|-------------------|-------------------|-------------|---------------|
| $f_1$     | 1.00E-08          | 1.00E-08          | 1.00E-08    | 1.00E-08      |
| $f_2$     | 1.00E-08          | 1.00E-08          | 1.00E-08    | 1.00E-08      |
| $f_3$     | 1.00E-08          | 1.00E-08          | 1.00E-08    | 1.00E-08      |
| $f_4$     | 1.27E-08          | 1.82E+03          | 1.00E-08    | 2.05E+04      |
| $f_5$     | 4.88E-07          | 1.00E-08          | 1.00E-08    | 4.51E+01      |
| $f_6$     | 1.79E-01          | 1.00E-08          | 1.00E-08    | 1.59E-01      |
| $f_7$     | 1.00E-08          | 1.00E-08          | 1.00E-08    | 1.00E-08      |
| $f_8$     | 2.08E+01          | 2.08E+01          | 2.09E+01    | 2.07E+01      |
| $f_9$     | 3.66E+00          | 6.92E+00          | 3.38E+00    | 7.80E+00      |
| $f_{10}$  | 2.19E+00          | 5.57E+00          | 1.59E+00    | 7.28E+00      |
| $f_{11}$  | 4.45E-04          | 3.47E-05          | 8.50E-02    | 8.91E-04      |
| $f_{12}$  | 3.45E+03          | 6.03E+03          | 3.58E+03    | 2.94E+04      |
| $f_{13}$  | 4.72E+00          | 4.44E+00          | 4.23E+00    | 4.47E+00      |
| $f_{14}$  | 1.79E+01          | 2.10E+01          | 2.00E+01    | 2.26E+01      |
| $f_{15}$  | 2.00E+02          | 2.00E+02          | 2.00E+02    | 2.00E+02      |
| $f_{16}$  | 5.12E+00          | 8.36E+00          | 5.23E+00    | 1.81E+01      |
| $f_{17}$  | 1.36E+02          | 2.30E+02          | 1.24E+02    | 2.55E+02      |
| $f_{18}$  | 9.13E+02          | 9.15E+02          | 9.15E+02    | 9.12E+02      |
| $f_{19}$  | 9.13E+02          | 9.14E+02          | 9.16E+02    | 9.16E+02      |
| $f_{20}$  | 9.13E+02          | 9.15E+02          | 9.16E+02    | 9.17E+02      |
| $f_{21}$  | 7.25E+02          | 9.67E+02          | 8.57E+02    | 8.44E+02      |
| $f_{22}$  | 8.21E+02          | 8.64E+02          | 8.49E+02    | 8.44E+02      |
| $f_{23}$  | 7.49E+02          | 9.23E+02          | 8.78E+02    | 9.26E+02      |
| $f_{24}$  | 2.00E+02          | 2.00E+02          | 2.00E+02    | 2.00E+02      |
| $f_{25}$  | 2.13E+02          | 2.13E+02          | 2.13E+02    | 2.15E+02      |

**Table 3.** The average errors obtained by iCMA-ES-uni+, iCMA-ES-multi+ and iCMA-ES-tsc++ for each CEC'05 function. Error values lower than  $10^{-8}$  are approximated to  $10^{-8}$ . The lowest average errors values are highlighted.

| $f_{cec}$ | iCMA-ES-uni+ | iCMA-ES-multi+ | iCMA-ES-tsc++ |
|-----------|--------------|----------------|---------------|
| $f_1$     | 1.00E-08     | 1.00E-08       | 1.00E-08      |
| $f_2$     | 1.00E-08     | 1.00E-08       | 1.00E-08      |
| $f_3$     | 1.00E-08     | 1.00E-08       | 1.00E-08      |
| $f_4$     | 1.00E-08     | 1.70E+04       | 1.00E-08      |
| $f_5$     | 1.01E-08     | 9.30E-01       | 1.00E-08      |
| $f_6$     | 1.00E-08     | 1.00E-08       | 1.00E-08      |
| $f_7$     | 1.00E-08     | 1.00E-08       | 1.00E-08      |
| $f_8$     | 2.10E+01     | 2.06E+01       | 2.10E+01      |
| $f_9$     | 7.00E+00     | 5.33E+00       | 3.70E+00      |
| $f_{10}$  | 5.01E+00     | 4.78E+00       | 1.79E+00      |
| $f_{11}$  | 4.35E-02     | 1.00E-08       | 3.63E-03      |
| $f_{12}$  | 5.86E+03     | 1.46E+04       | 1.44E+04      |
| $f_{13}$  | 4.30E+00     | 4.68E+00       | 4.51E+00      |
| $f_{14}$  | 2.06E+01     | 2.28E+01       | 2.15E+01      |
| $f_{15}$  | 2.00E+02     | 2.00E+02       | 2.00E+02      |
| $f_{16}$  | 1.55E+01     | 8.75E+00       | 3.14E+00      |
| $f_{17}$  | 1.48E+02     | 2.34E+02       | 7.77E+01      |
| $f_{18}$  | 9.16E+02     | 9.20E+02       | 9.13E+02      |
| $f_{19}$  | 9.15E+02     | 9.22E+02       | 9.13E+02      |
| $f_{20}$  | 9.15E+02     | 9.20E+02       | 9.13E+02      |
| $f_{21}$  | 9.66E+02     | 5.41E+02       | 7.06E+02      |
| $f_{22}$  | 8.18E+02     | 8.71E+02       | 8.55E+02      |
| $f_{23}$  | 9.05E+02     | 6.21E+02       | 8.68E+02      |
| $f_{24}$  | 2.00E+02     | 2.00E+02       | 2.00E+02      |
| $f_{25}$  | 2.13E+02     | 2.14E+02       | 2.13E+02      |

**Table 4.** The average and median errors obtained by tuning setup (i) t-test and (ii) 25000 runs over 25 independent runs for CEC'05 functions of dimension 50. The numbers in parenthesis represent the times of <, =, and >, respectively, when the iCMA-ES-dp are compared with iCMA-ES-(i), iCMA-ES-(ii) on the average and median errors.

| $f_{cec}$              | iCMA-ES-dp |          | iCMA-ES-(i)                         |          | iCMA-ES-(ii)                        |          |
|------------------------|------------|----------|-------------------------------------|----------|-------------------------------------|----------|
|                        | Mean       | Median   | Mean                                | Median   | Mean                                | Median   |
| $f_1$                  | 1.00E-08   | 1.00E-08 | 1.00E-08                            | 1.00E-08 | 1.00E-08                            | 1.00E-08 |
| $f_2$                  | 1.00E-08   | 1.00E-08 | 1.00E-08                            | 1.00E-08 | 1.00E-08                            | 1.00E-08 |
| $f_3$                  | 1.00E-08   | 1.00E-08 | 1.00E-08                            | 1.00E-08 | 1.00E-08                            | 1.00E-08 |
| $f_4$                  | 1.43E+04   | 1.27E+04 | 1.00E-08                            | 1.00E-08 | 1.00E-08                            | 1.00E-08 |
| $f_5$                  | 7.41E-02   | 8.61E-08 | 1.00E-08                            | 1.00E-08 | 1.00E-08                            | 1.00E-08 |
| $f_6$                  | 1.00E-08   | 1.00E-08 | 2.79E-03                            | 1.00E-08 | 1.59E-01                            | 1.00E-08 |
| $f_7$                  | 1.00E-08   | 1.00E-08 | 1.00E-08                            | 1.00E-08 | 1.00E-08                            | 1.00E-08 |
| $f_8$                  | 2.09E+01   | 2.11E+01 | 2.10E+01                            | 2.11E+01 | 2.10E+01                            | 2.11E+01 |
| $f_9$                  | 4.36E+00   | 3.98E+00 | 4.10E+00                            | 3.98E+00 | 2.92E+00                            | 2.99E+00 |
| $f_{10}$               | 2.89E+00   | 1.99E+00 | 2.55E+00                            | 2.98E+00 | 2.40E+00                            | 1.99E+00 |
| $f_{11}$               | 9.94E-02   | 1.00E-08 | 4.17E-04                            | 1.00E-08 | 6.00E-02                            | 5.33E-07 |
| $f_{12}$               | 4.25E+04   | 2.78E+04 | 3.34E+03                            | 2.27E+03 | 3.95E+03                            | 3.40E+03 |
| $f_{13}$               | 4.44E+00   | 4.37E+00 | 4.64E+00                            | 4.70E+00 | 4.59E+00                            | 4.60E+00 |
| $f_{14}$               | 2.28E+01   | 2.30E+01 | 2.06E+01                            | 2.09E+01 | 2.09E+01                            | 2.11E+01 |
| $f_{15}$               | 2.00E+02   | 2.00E+02 | 2.00E+02                            | 2.00E+02 | 2.00E+02                            | 2.00E+02 |
| $f_{16}$               | 1.10E+01   | 1.14E+01 | 5.14E+00                            | 5.54E+00 | 8.78E+00                            | 8.66E+00 |
| $f_{17}$               | 1.91E+02   | 1.62E+02 | 5.47E+01                            | 4.81E+01 | 1.34E+02                            | 5.28E+01 |
| $f_{18}$               | 9.13E+02   | 9.16E+02 | 9.13E+02                            | 9.13E+02 | 9.14E+02                            | 9.13E+02 |
| $f_{19}$               | 9.13E+02   | 9.15E+02 | 9.13E+02                            | 9.13E+02 | 9.14E+02                            | 9.13E+02 |
| $f_{20}$               | 9.15E+02   | 9.15E+02 | 9.13E+02                            | 9.13E+02 | 9.14E+02                            | 9.13E+02 |
| $f_{21}$               | 6.64E+02   | 5.00E+02 | 8.07E+02                            | 1.01E+03 | 6.63E+02                            | 5.00E+02 |
| $f_{22}$               | 8.19E+02   | 8.18E+02 | 8.60E+02                            | 8.63E+02 | 8.22E+02                            | 8.21E+02 |
| $f_{23}$               | 6.97E+02   | 5.40E+02 | 6.55E+02                            | 5.40E+02 | 6.55E+02                            | 5.40E+02 |
| $f_{24}$               | 2.00E+02   | 2.00E+02 | 2.00E+02                            | 2.00E+02 | 2.00E+02                            | 2.00E+02 |
| $f_{25}$               | 2.14E+02   | 2.14E+02 | 2.13E+02                            | 2.13E+02 | 2.13E+02                            | 2.13E+02 |
| Mean: iCMA-ES-dp vs.   |            |          | (<, =, >): (5, 8, 12) <sup>†</sup>  |          | (<, =, >): (6, 6, 13) <sup>†</sup>  |          |
| Median: iCMA-ES-dp vs. |            |          | (<, =, >): (4, 11, 10) <sup>†</sup> |          | (<, =, >): (3, 11, 11) <sup>†</sup> |          |

<sup>†</sup> denotes there is a significant difference over the distribution of mean or median errors between iCMA-ES-dp with the corresponding algorithm by a two-sided Wilcoxon matched-pairs signed-ranks test at the 0,05  $\alpha$ -level.