

# Modeling Languages

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SLS 2009

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# The context of SLS 2009

This tutorial has close relationships with the first tutorial *Computer-assisted design of high-performance algorithms* by Holger Hoos

## Similar objectives

- Help the user to design an efficient algorithm
- The user focusses on higher level design issues

## The main differences

- Offering a *modeling language* to the user to design problems
- Dedicated to an application domain
- Focus on *Constraint-Based approaches*
- Algorithm synthesis based on the *structure of the problem*

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# Outline

- 1 Constraint-Based Approaches
  - Modeling and Solving Problems with Constraints
  - The Comet Constraint Programming Language
- 2 Objectives
- 3 Modeling Language for Graph Matching
  - Graph Matching
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# Constraint Programming with Comet

## The CP search

### Branch & Propagate

### Branching

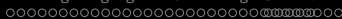
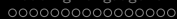
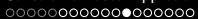
- Decompose into subproblems (e.g., giving a value for a variable)
- Automatic support for backtracking

### Propagation

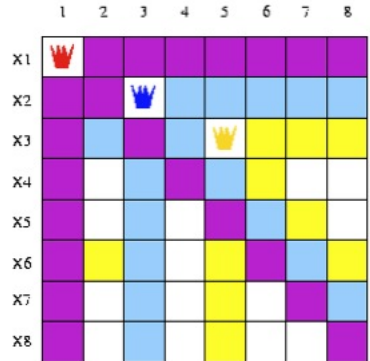
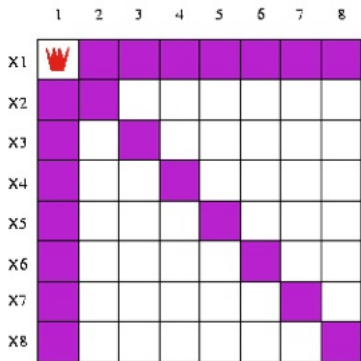
- Reduction of the search space
- Find an equivalent CSP with *smaller domains*
- Based on consistency techniques







# N-Queens : Comet / CP







# Bin-packing data in COMET

```

1 int n = ...; // number of bins
2 int m = ...; // number of objects
3 int W = ...; // capacity
4 range Rbin = 1..n; // range of the bins
5 range Robj = 1..m; // range of the objects
6 range RW = 0..W; // range of the capacity
7 int w[Robj] = ...; // size of each object

```



# CP model in COMET

```

1  import cotfd;
2
3  Solver<CP> cp();
4  var<CP>{int} b[Robj](cp,Rbin); // bin assigned to each object
5  var<CP>{int} load[Rbin](cp,RW); // load of the bins
6  cp.close();
7
8  minimize<cp>
9      max(obj in Robj) b[obj]
10 subject to {
11     forall(j in Rbin)
12         cp.post( load[j] == sum(i in Robj) (b[i] == j)*w[i] );
13         cp.post( sum(j in Rbin) load[j] == sum(i in Robj) w[i] );
14 } using {
15     label(b);
16 }

```



# LS model in COMET

```

1  import cotls;
2
3  Solver<LS> ls();
4  var{int} b[Robj](ls,Rbin);
5  var{int} load[bin in Rbin](ls,RW)
6      <- sum(obj in Robj) (b[obj] == bin) * w[obj];
7  Function<LS> objective = MinBinObjective(ls, b, load, w, W);
8  ls.close();
9
10 // Initialization
11 int curBin = Rbin.getLow();
12 forall (obj in Robj) {
13     if (curBin < Rbin.getUp() && load[curBin] + w[obj] > W)
14         curBin++;
15     b[obj] := curBin;
16 }

```

# LS model in COMET

```

1  // Search
2  int iter = 0;
3  while (iter < 1000) {
4      iter++;
5      selectMin(obj in Robj, bin in Rbin :
6          load[bin] + w[obj] <= W)
7          (objective.getAssignDelta(b[obj], bin)) {
8          b[obj] := bin;
9      }
10 }
```

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# Strength and weakness of constraint-based approaches

The holy grail of constraint programming...

The user states the problem by means of constraints

The computer solves it thanks to embeded solvers

...faced to the reality of NP-hardness

The user often has to help the computer:

- Choose the most appropriate search paradigm
  - CP when constraints are tight enough to prune efficiently
  - CBLS for looser constraints and/or optimization
- Design the "right" model that leads to an efficient search
  - CP: add redundant constraints
  - CBLS: choose appropriate invariants
- "Program" the search
  - CP: design ordering heuristics
  - CBLS: neighborhoods and strategies for escaping local optima





# Generic *versus* dedicated approaches

## Dedicated approaches (operation research)

Design a customized algorithm to solve a problem: very efficient... but cannot be used to solve a slightly different problem

## Modeling languages

Focus on an application domain and design

- a high level modeling language for this domain
- a synthesizer that generates an appropriate solver from the model

## Constraint-based approaches

Design a solver to solve all CSPs: very generic... but not always efficient (unless the user helps the solver !)

# Our goal

Bridge the gap between **high-level modeling** and **efficient solving**:

## High-level modeling

- High level objects and constraints
- Related to an application domain  
(graph matching, scheduling, routing, line balancing, ...)

↪ declarative modeling of problems within this domain

## Efficient solving

Synthesize the appropriate search strategy:

- analyze the structure of the model
- automatically generate a customized solver  
↪ reuse state-of-the-art approaches, combine them, ...







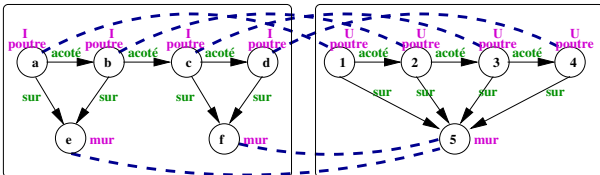
# Graph matching problems

## Why matching graphs ?

- Many applications require to measure object similarity  
 ~> Classification, Search by example, Case-based Reasoning, ...
- Graphs are often used to model objects  
 ~> Images, Molecules, Documents, Design objects, ...
- Graph similarity is measured by matching their vertices

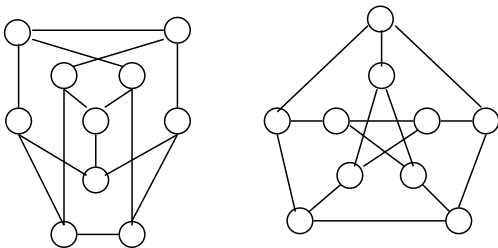
## What is a matching ?

A matching of  $G_1 = (V_1, E_1)$  and  $G_2 = (V_2, E_2)$  is a relation  $m \subseteq V_1 \times V_2$   
 ~>  $(u_1, u_2) \in m \Rightarrow$  vertex  $u_1$  is matched to vertex  $u_2$



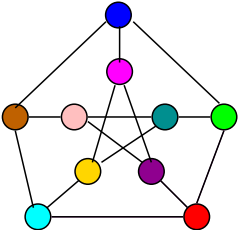
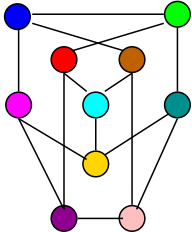
# Well known examples of graph matching problems

- Graph Isomorphism  $\rightsquigarrow$  decide equivalence
- Subgraph Isomorphism  $\rightsquigarrow$  decide inclusion
- Maximum common subgraph  $\rightsquigarrow$  Intersection
- Graph Edit Distance  $\rightsquigarrow$  Best univalent matching
- Extended Graph Edit Distance  $\rightsquigarrow$  Best multivalent matching



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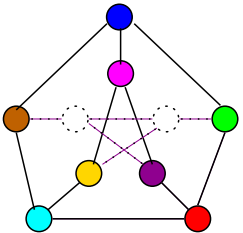
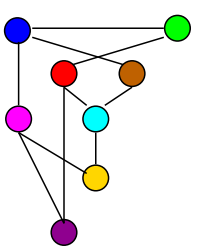
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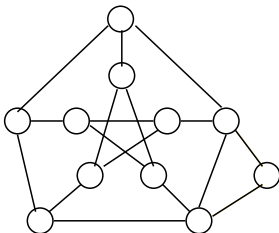
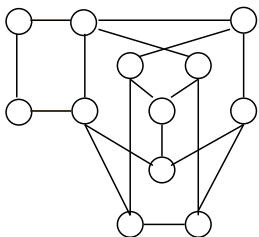
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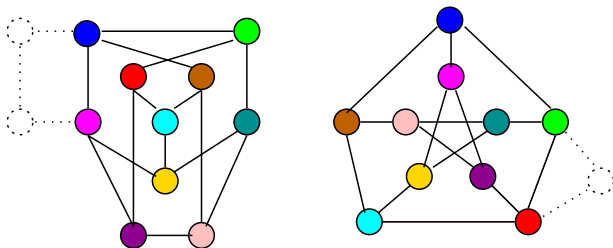
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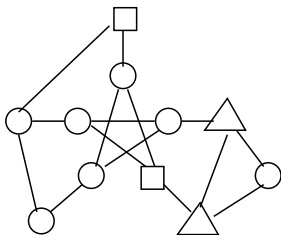
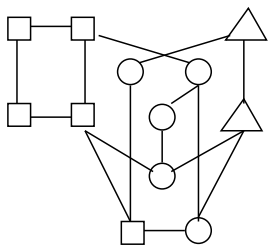
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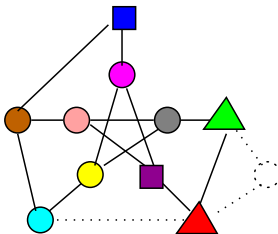
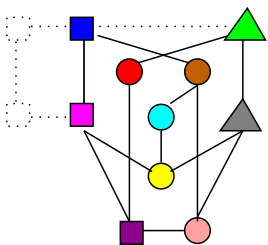
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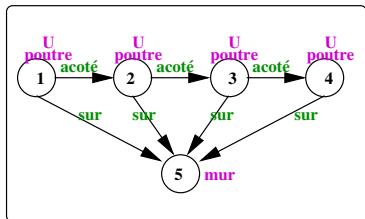
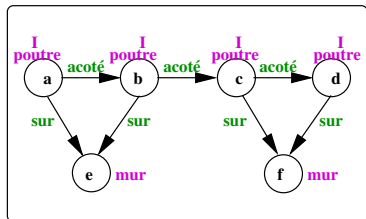
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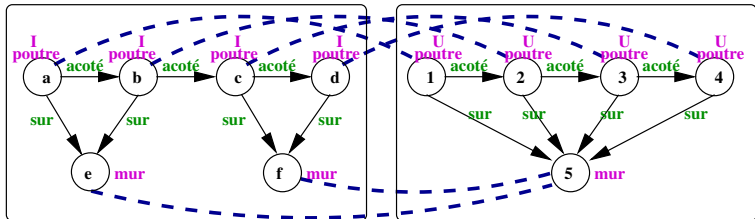
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# Modeling graph matching by means of constraints

## Constraints on the cardinality of the matching

bijjective (1,1), injective (1,0..1), univalent (0..1,0..1), or multivalent (0..n,0..n)

- hard constraints: exact matchings
- soft constraints: error-tolerant matchings

## Constraints on edges

- hard constraints: edges must be matched
- soft constraints: maximize the number of matched edges

## Constraints on labels (in case of labeled graphs)

- hard constraints: matched components must have identical labels
- soft constraints: maximize the similarity of matched component labels

# Example 1: Graph isomorphism

- Declare 2 graph objects g1 and g2 and a matching m

```
bool[,] adj1 = ...
```

```
bool[,] adj2 = ...
```

```
SimpleGraph<Mod> g1(adj1);
```

```
SimpleGraph<Mod> g2(adj2);
```

```
Matching<Mod> m(g1,g2);
```

- Post cardinality constraints on m  $\rightsquigarrow$  bijective matching (1, 1)

```
m.post(cardMatch(g1.getAllNodes(), 1, 1));
```

```
m.post(cardMatch(g2.getAllNodes(), 1, 1));
```

- Post constraints to ensure edge matching

```
m.post(matchedToSomeEdges(g1.getAllEdges()));
```

```
m.post(matchedToSomeEdges(g2.getAllEdges()));
```

- Ask the synthesizer to create the solver... and search a solution

```
m.close();
```

```
DefaultGMSynthesizer synth();
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GMSolution<Mod> sol = synth.solveMatching(m);
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## Example 2: Induced Subgraph Isomorphism

- Declare 2 graph objects g1 and g2 and a matching m

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bool[,] adj1 = ...
```

```
bool[,] adj2 = ...
```

```
SimpleGraph<Mod> g1(adj1);
```

```
SimpleGraph<Mod> g2(adj2);
```

```
Matching<Mod> m(g1,g2);
```

- Post cardinality constraints on m  $\rightsquigarrow$  injective matching (1, 0..1)

```
m.post(cardMatch(g1.getAllNodes(), 1, 1));
```

```
m.post(cardMatch(g2.getAllNodes(), 0, 1));
```

- Post constraints to ensure edges of  $G_1$  to be matched

```
m.post(matchedToSomeEdges(g1.getAllEdges()));
```

- Ask the synthesizer to create the solver... and search a solution

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- Ask the synthesizer to create the solver... and search a solution
 

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```

## Example 3: Largest Common Induced Subgraph

- Declare 2 graph objects `g1` and `g2` and a matching `m`

```
bool[,] adj1 = ...
```

```
bool[,] adj2 = ...
```

```
SimpleGraph<Mod> g1(adj1);
```

```
SimpleGraph<Mod> g2(adj2);
```

```
Matching<Mod> m(g1,g2);
```

- Post cardinality constraints on `m`  $\rightsquigarrow$  univalent matching (0..1, 0..1)

```
m.post(cardMatch(g1.getAllNodes(), 0, 1));
```

```
m.post(cardMatch(g2.getAllNodes(), 0, 1));
```

- Post a soft constraint to maximize the number of matched vertices

```
m.softpost(minMatch(g1.getAllNodes(), 1), 1)
```

- Post constraints to ensure edge matching

```
m.post(matchedToAllEdges(g1.getAllEdges()));
```

```
m.post(matchedToAllEdges(g2.getAllEdges()));
```

- Ask the synthesizer to create the solver... and search a solution

```
m.close(); DefaultGMSynthesizer synth();
```

```
GMSolution Mod sol = synth.solveMatching(m);
```

## Example 3: Largest Common Induced Subgraph

- Declare 2 graph objects `g1` and `g2` and a matching `m`

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```

```
bool[,] adj2 = ...
```

```
SimpleGraph<Mod> g1(adj1);
```

```
SimpleGraph<Mod> g2(adj2);
```

```
Matching<Mod> m(g1,g2);
```

- Post cardinality constraints on `m`  $\rightsquigarrow$  univalent matching  $(0..1, 0..1)$

```
m.post(cardMatch(g1.getAllNodes(), 0, 1));
```

```
m.post(cardMatch(g2.getAllNodes(), 0, 1));
```

- Post a soft constraint to maximize the number of matched vertices

```
m.softpost(minMatch(g1.getAllNodes(), 1), 1)
```

- Post constraints to ensure edge matching

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- Ask the synthesizer to create the solver... and search a solution

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SimpleGraph<Mod> g1(adj1);
SimpleGraph<Mod> g2(adj2);
Matching<Mod> m(g1,g2);
```
- Post cardinality constraints on `m`  $\rightsquigarrow$  univalent matching (0..1,0..1)

```
m.post(cardMatch(g1.getAllNodes(), 0, 1));
m.post(cardMatch(g2.getAllNodes(), 0, 1));
```
- Post a soft constraint to maximize the number of matched vertices

```
m.softpost(minMatch(g1.getAllNodes(), 1), 1)
```

- Post constraints to ensure edge matching

```
m.post(matchedToAllEdges(g1.getAllEdges()));
m.post(matchedToAllEdges(g2.getAllEdges()));
```

- Ask the synthesizer to create the solver... and search a solution

```
m.close(); DefaultGMSynthesizer synth();
GMSolution Mod sol = synth.solveMatching(m);
```

## Example 3: Largest Common Induced Subgraph

- Declare 2 graph objects `g1` and `g2` and a matching `m`

```
bool[,] adj1 = ...
```

```
bool[,] adj2 = ...
```

```
SimpleGraph<Mod> g1(adj1);
```

```
SimpleGraph<Mod> g2(adj2);
```

```
Matching<Mod> m(g1,g2);
```

- Post cardinality constraints on `m`  $\rightsquigarrow$  univalent matching  $(0..1, 0..1)$

```
m.post(cardMatch(g1.getAllNodes(), 0, 1));
```

```
m.post(cardMatch(g2.getAllNodes(), 0, 1));
```

- Post a soft constraint to maximize the number of matched vertices

```
m.softpost(minMatch(g1.getAllNodes(), 1), 1)
```

- Post constraints to ensure edge matching

```
m.post(matchedToAllEdges(g1.getAllEdges()));
```

```
m.post(matchedToAllEdges(g2.getAllEdges()));
```

- Ask the synthesizer to create the solver... and search a solution

```
m.close(); DefaultGMSynthesizer synth();
```

```
GMCSolution Mod sol = synth.solveMatching(m);
```

## Example 3: Largest Common Induced Subgraph

- Declare 2 graph objects g1 and g2 and a matching m

```
bool[,] adj1 = ...
```

```
bool[,] adj2 = ...
```

```
SimpleGraph<Mod> g1(adj1);
```

```
SimpleGraph<Mod> g2(adj2);
```

```
Matching<Mod> m(g1,g2);
```

- Post cardinality constraints on m  $\rightsquigarrow$  univalent matching  $(0..1, 0..1)$

```
m.post(cardMatch(g1.getAllNodes(), 0, 1));
```

```
m.post(cardMatch(g2.getAllNodes(), 0, 1));
```

- Post a soft constraint to maximize the number of matched vertices

```
m.softpost(minMatch(g1.getAllNodes(), 1), 1)
```

- Post constraints to ensure edge matching

```
m.post(matchedToAllEdges(g1.getAllEdges()));
```

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```

- Ask the synthesizer to create the solver... and search a solution

```
m.close(); DefaultGMSynthesizer synth();
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```
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```

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# Synthesizing a solver for graph matching problems (1/3)

Warning: Ongoing research with a very first prototype

↪ many improvements are still to be done !

## Canonical form of modeling constraints

Aggregate all modeling constraints of a same type

- Cardinality (MinMatch, MaxMatch, CardMatch, ...)
- Edge matching (MatchedToSomeEdges, MatchedToAllEdges, ...)
- Label matching (MatchAllNodeLabels, MatchAllEdgeLabels, ...)

↪ Derive characteristics from the canonical model

## Choose a search approach

- CP if no soft constraints and  $\text{MaxCard} \leq 1$  for all vertices of a graph
  - ↪ Maintaining Arc Consistency
- CBLS otherwise
  - ↪ Tabu search

# Synthesizing a solver for graph matching problems (2/3)

## Creation of low level variables

Associate a variable with every vertex of both graphs

- Domains are defined wrt cardinality constraints

MINMATCH	MAXMATCH	Type	Domain
1	1	int	$N$
0	1	int	$N \cup \{\perp\}$
Otherwise		set	$2^N$

- Ensure symmetry ( $X_u$  matched to  $v \Rightarrow X_v$  matched to  $u$ ):
  - CP  $\rightsquigarrow$  Channeling constraints
  - CBLS  $\rightsquigarrow$  invariants

# Synthesizing a solver for graph matching problems (3/3)

## Post the canonical constraints

- CP (hard constraints only)
  - Cardinality constraints
    - ↔ Partly handled by variable domains
    - ↔ Global allDiff for injective and bijective matchings
  - Edge constraints ↔ binary constraints
  - Label constraints on nodes ↔ variable domains
  - Label constraints on edges ↔ binary constraints
- CBLS (hard and soft constraints)
  - Cardinality ↔ neighborhood if hard; invariants if soft
  - Edge ↔ invariants
  - Node labels ↔ neighborhood if hard; invariants if soft
  - Edge labels ↔ invariants

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# (Preliminary) Experimental Results (1/2)

$SI \rightsquigarrow$  Subgraph Isomorphism

$SI+ \rightsquigarrow$  Subgraph Isomorphism + additional distance constraint

	#Nodes	Synthesizer/CP					vf2 [Cordella et al. 99]				
		5%	10%	20%	33%	50%	5%	10%	20%	33%	50%
$SI$	100	0.8	0.5	0.7	0.1	0.2	0.0	0.0	0.0	2.0	0.0
	500	19.3	4.7	10.5	15.8	30.7	0.1	0.1	246.7	192.3	-
	1000	30.6	595.8	119.0	152.3	-	86.7	-	-	-	-
$SI+$	100	0.3	0.1	0.1	0.1	0.2					
	500	3.0	4.4	9.5	16.9	28.9					
	1000	16.1	47.8	82.5	148.0	-					

- Vf2 better for small instances
- Synthesizer outperforms vf2 for larger instances
- Additional constraint improves the search process

# (Preliminary) Experimental Results (2/2)

Maximum common subgraph  $\rightsquigarrow$  CBL5

#nodes	time		iterations		edges%	
25	8.5	(2.5)	7768.1	(2301.3)	48.3	(1.1)
50	33.9	(10.7)	8023.8	(2543.3)	40.2	(0.5)
100	141.5	(46.4)	8398.4	(2755.0)	34.5	(0.2)

- First results to assess feasibility
- Complete approaches cannot handle these instances
- We haven't (yet) compared these results with other approaches

## Further works on modeling for graph matching

- Improve the analysis of the matching characteristics
  - ↪ identify sub-problems that are “easy” to solve
- Integrate dedicated filtering algorithms ↪ CP
  - Iterative partitioning for graph isomorphism (Nauty)
  - Iterative labeling for subgraph iso. (Zampelli et al 2009)
- Integrate reactive search and other meta-heuristics for CBLS
  - ↪ Parameter tuning... !
- Combine CP and CBLS

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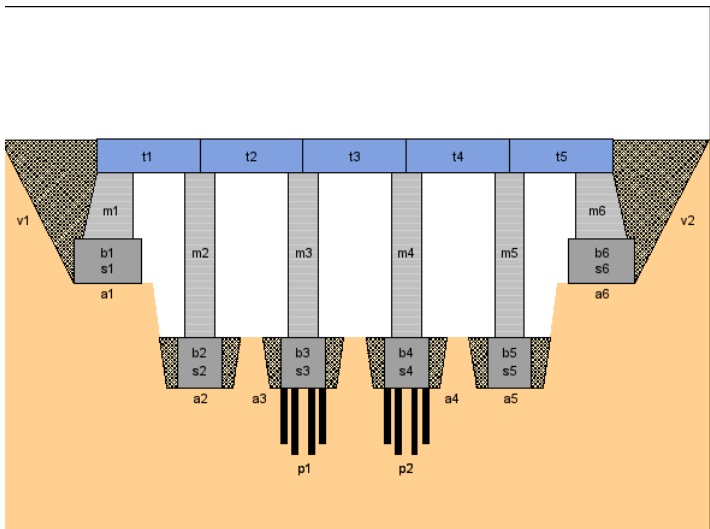
# Scheduling

The goal is to allocate scarce resources to a set of activities over time.

## Scheduling is everywhere

- Products Manufacturing
- Construction Planning
- Code Optimization in Compilers
- Project Management (Pharmaceutic Industry, for instance)
- Trains and Airplanes Scheduling
- Closely Related to Timetabling, Vehicle Routing, Planning...

# Construction Scheduling



# Airport Scheduling





# Scheduling

## There exists a lot of variations

- Models for activities
  - Preemption, Jobs, ...
- Models for resources
  - Cumulative, Machines, Reservoirs, ...
- Constraints
  - Precedences, Max-Slack, ...
- Objective functions
  - Makespan, Sum of Tardiness, ...



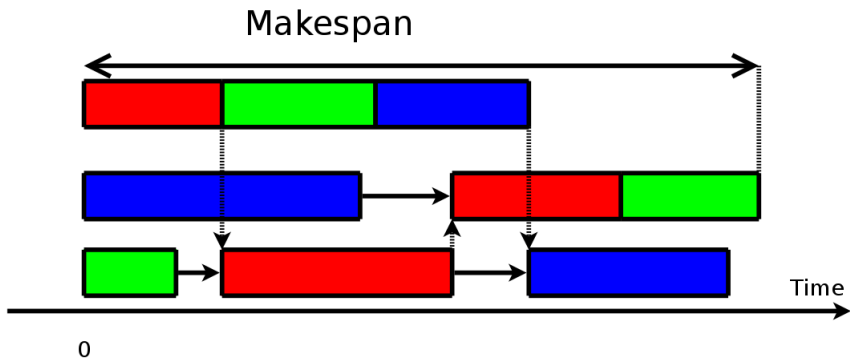
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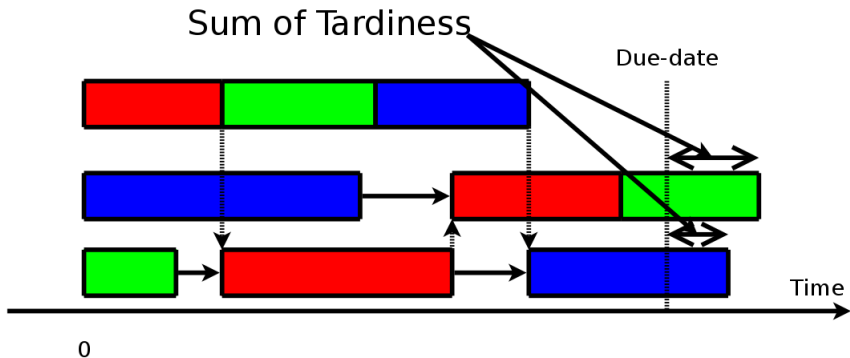




# Job-Shop : Objectives

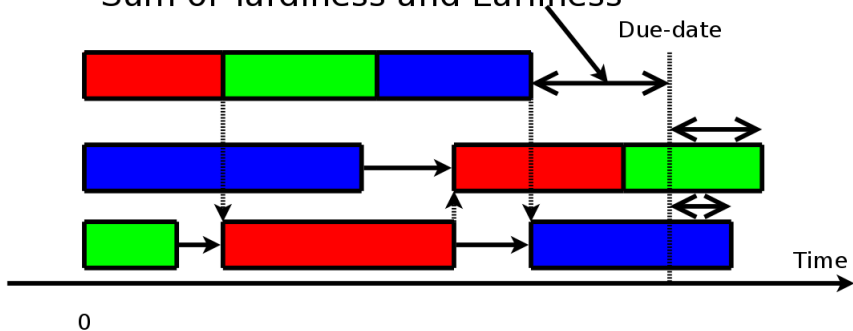


# Job-Shop : Objectives



# Job-Shop : Objectives

## Sum of Tardiness and Earliness



# Modeling : Job-Shop

```
1  range jobs = 1..nbjobs;
2  range machines = 0..nbmachines-1;
3  range tasks = 1..nbjobs*nbmachines;
4  int proc[tasks];
5  int mach[tasks];
6  int job[jobs,machines];
7
8  Schedule<Mod> s();
9  Job<Mod> J[i in jobs](s,"J"+i);
10 Machine<Mod> M[i in machines](s,"M"+i);
11 Activity<Mod> A[i in tasks](s,proc[i],"A"+i);
12 forall(i in tasks)
13     A[i].requires(M[mach[i]]);
14 forall(i in jobs)
15     J[i].containsInSequence(
16         all(j in machines)A[job[i,j]]);
17 s.minimizeObj(makespanOf(s));
```

# Solving

```
1 GreedyTabuSynthesizer synth();
2 // CPSynthesizer synth();
3 Solution<Mod> sol = synth.solve(s);
4 sol.printSolution();
```

```
1 Classifier Set-Up
```

```
2 Models[JobShopWithMakespan,CumulativeJobShopWithMakespan,CumulativeJobShop
```

```
3
```

```
4 ...
```

```
5
```

```
6 930.000000
```

```
7 Time = 12568
```

# Modeling : RCPSP

```

1  range tasks;
2  range resources;
3  int proc[tasks]; //Processing Times
4  int cap[resources]; //Capacities
5  int succ[][tasks]; //Successors
6  int req[tasks,resources]; //Requirements
7  Schedule<Mod> s();
8  Activity<Mod> A[i in tasks](s, proc[i], "J");
9  Resource<Mod> R[i in resources](s, cap[i], "R");
10 forall(i in tasks){
11     forall(j in succ[i].getRange())
12         A[i].precedes(A[succ[i][j]]);
13     forall(j in resources)
14         if(req[i,j]!=0)A[i].requires(R[j],req[i,j]);
15 }
16 s.minimizeObj(Tardiness<Mod>(s,A[sink],duedate)*tardCost);

```

# Available Abstractions

Description	Classes
Schedule	Schedule
Activities	Activity MultiModeActivity
Jobs	Job
Resources	Resource Machine Reservoir StateResource
Objectives	ScheduleObjective CompletionTime, PiecewiseLinearFunction Tardiness, Earliness, Lateness, UnitCost AbsenceCost, AlternativeCost MultObjective, ShiftObjective SumObjective, MaxObjective

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# Context

- **Scheduling** is a large domain of research and application for optimization techniques.
- Among the techniques: Constraint Programming, Local Search, Integer Programming, Genetic Algorithms, Greedy Algorithms
- Most algorithms are specific to a restricted class of problems. A lot of parameters must be tuned.
- It may be hard to recognize problems, find the most appropriate algorithm and code it.

# Available Synthesizers

Description	Classes
Synthesizers	ScheduleSynthesizer CPSynthesizer TSSynthesizer SASynthesizer GreedySynthesizer SequenceSynthesizer ScheduleAnimator
Solutions	Solution

# Represented problems

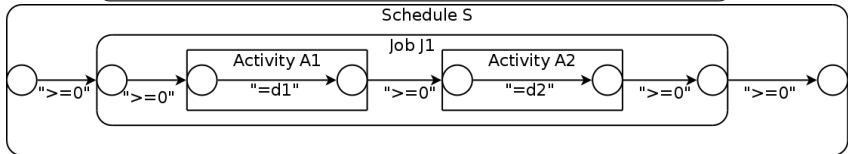
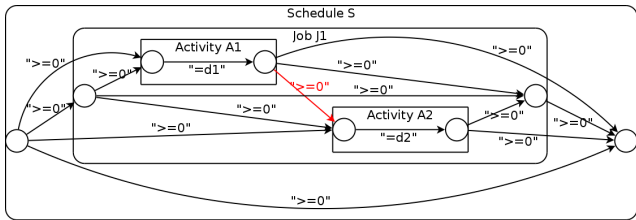
- Job-Shop (Makespan, Tardiness, Earliness-Tardiness)
- Open-Shop
- Cumulative Job-Shop
- RCPSP, RCPSP/max
- MMRCPSPP, MMRCPSPP/max
- Trolley Problem
- MascLib (NCOS and NCGS classes)

# Internal Representation

- **Canonical** : Allow different models of the same problem to be classified in the same way
- **Homogeneous** : Ease the analysis and the information retrieval
- **Structured** : To keep the structure of the problem also helps in the analysis

# Representation : Precedences

- Simplification of the precedences



# Classification of problems

- Goal : Classify the model in one of the classes of problems.
- Based on characteristics.
- A simple “constraint” imposes a value for the characteristic. Its value for a model can be true or false.
- More complex constraints are build as boolean formulas of constraints (using negation, disjunction and conjunction).
- A class of problem is represented by a boolean formula.

# Model Example : Job-Shop & RCPSP

Characteristic	Type	JSP	RCPSP
Unit Processing Time	boolean	–	–
Fixed Processing Time	boolean	true	true
Preemption Allowed	enum	never	never
Common Release Dates	boolean	true	true
Common Deadlines	boolean	–	–
Deadlines Exist	boolean	false	false
Form of the Precedence Graph	enum	chains	DAG
Delay between Activities	boolean	false	false
No wait between Activities	boolean	false	false
Jobs inside Jobs	boolean	false	–
Number Of State Resources	integer	0	0
Maximum Capacity	integer	1	–
All Capacities are Equal	boolean	true	–

# Model Example : Job-Shop & RCPSP

Characteristic	Type	JSP	RCPSP
Reservoir Consumption	boolean	false	false
Reservoir Production	boolean	false	false
Setup Times	boolean	false	false
Disjunctive Requirements	boolean	false	false
All Activities in Jobs	boolean	true	false
Nb of Multi-Mode Activities	integer	0	0
Sum Of Requirements	integer	1	–
Objective Type	enum	minimize	minimize
Objective Form	enum	maximum	total
Objective Components	enum	completion time	lateness
Objective Scope	enum	all activities	one activity
All Due-Dates are equal	enum	–	–





# AEON is an open system

## Extension Mechanisms

- Modelling Abstractions : Requires a lot of work
- Problem Characteristics : Requires to modify several classes
- Problem Classes : Write a XML file
- Synthesizers : Write a subclass of ScheduleSynthesizer
- Strategy : Write a subclass of Strategy

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# Experiments

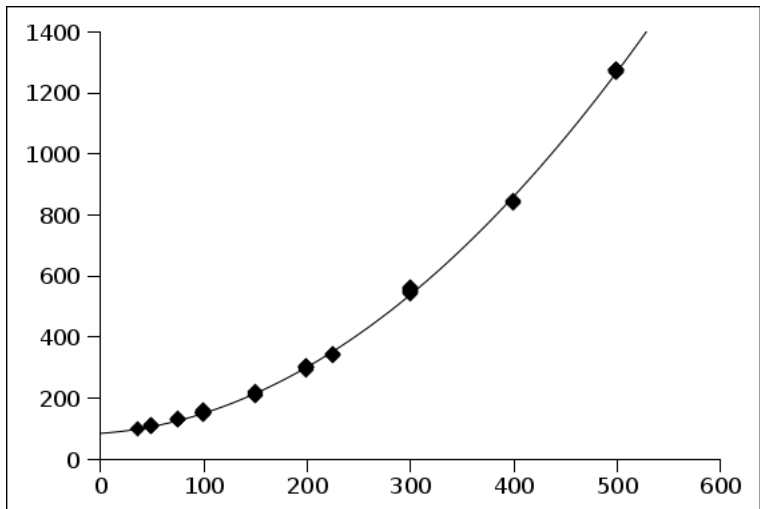
- Goal : assess the practicability of the approach
- Settings : 3 benchmarks:
  - Job-Shop Problem with makespan minimization (JSP)
  - Open-Shop Problem with makespan minimization (OSP)
  - Job-Shop with weighted tardiness minimization (JSPWT)
- Compare three synthesizers together with a specific algorithm:
  - LS (Tabu Search or Simulated Annealing)
  - CP
  - Sequence of LS and CP
  - Reference, a specific algorithm : state of the art algorithm coded in Comet
- Evaluation :
  - MRE (Mean Relative Error) =  $100 * (UB - Opt) / Opt$
  - Time to best found solution

## Experiments: Results

Problem	#Inst.	Average MRE			
		Ref.	LS	CP/LNS	LS+CP
JSP	78	2.08	2.09	54.40	2.03
OSP	80	1.68	1.70	1.58/0.01	0.85
JSPTW	22	4.28	3.87	97.88	4.14

Problem	#Inst.	Average running time to best solution			
		Ref.	LS	CP/LNS	LS+CP
JSP	78	2.6	3.1	4.4(30)	3.4(52)
OSP	80	24.1	25.0	8.0(49)/ <120	30.2(50)
JSPTW	22	24.4	24.3	-(0)	-(0)

# Experiments: Overhead of AEON











## Further Work (2/2)

### New application domains

- Routing problems
- Graph partitionning
- Line balancing
- ...

### Integration of other search paradigms

- Ant Colony Optimization
- Genetic algorithm
- ...

# Modeling Languages

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SLS 2009

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