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# Analysis of long-term swarm performance based on short-term experiments

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**Abstract** Swarm robotics is a branch of collective robotics systems that offers a set of remarkable advantages over other systems. The global behavior of swarm systems emerges from the local rules implemented at the individual level. Therefore, characterizing a global performance obtained at the swarm level is one of the main challenges, especially under complex dynamics such as spatial interferences. In this paper, we exploit the central limit theorem to analyze and characterize the swarm performance over long-term deadlines. The developed model is verified on two tasks: a foraging task and an object filtering task.

**Keywords** Swarm robotics · Time-constrained tasks · Central limit theorem

## 1 Introduction

Swarm robotics is a novel approach of coordinating a large number of robots, in which the global behavior emerges from the local rules implemented on the individual level. Swarm robotics offers a set of advantages over other robotics systems including: fault-tolerance, scalability and flexibility which allow them to represent a promising solution for a wide spectrum of applications. Spatial interferences between robots affect significantly the swarm performance which is defined as the amount of work accomplished by the individuals during a particular time unit. The influence of the spatial interferences is observed on both the performance of the single robot and the global performance of the swarm. A well-known example is the foraging task (Labella et al. 2006; Campo and Dorigo 2007), in which robots are used to retrieve objects scattered over the arena back to a particular area referred to as the *nest*. As noted in Lerman and Galstyan (2002) and Ostergaard et al. (2001), increase in the number of foraging robots decreases the performance of a single robot, i.e., the number of objects retrieved by the robot per time unit. Swarm performance is affected differently from the single robot performance, as increase in the number of robots increases the swarm performance until an optimal performance is reached. Afterwards, the swarm performance starts to decrease influenced by the interferences between robots. Although the influence of spatial interferences has been studied on a limited number of swarm scenarios, mainly foraging, the observations were not surprising and could be interpreted as in the following: increase in the number of robots increases the concurrence between robots and, consequently, the time required by a single robot to accomplish the individual parts. This decreases in turn, the number of parts accomplished within a specific period of time. At the swarm level, the obtained performance keeps increasing as long as the benefit of parallelizing the work is larger than the time penalty paid in escaping the interferences between robots. When robots work in large numbers such as robot swarms they become prone to intensive spatial interferences that are caused by the high density of the system. Those interferences represent complex dynamics that can lead to have an unknown distribution of the swarm performance making its analysis a non-trivial process. Moreover, as in other robotic systems, tasks exe-

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cuted by swarm robotics have mostly long-term durations. Therefore, developing a technique that analyzes the swarm performance for long-term tasks and under the influence of spatial interferences is of a significant importance. However, performing this analysis by running real experiments or computer simulations is a time and resource consuming solution. Therefore, alternative tools are required.

This paper is based on the work presented in Khaluf et al. (2013), in which we have investigated the use of the central limit theorem (CLT)[1] (CLT) for analyzing the long-term performance of a robot swarm. We are focusing on a particular set of robotic tasks, in which the swarm performance can be defined as the sum of a large number of individual or group contributions. This allows us to make use of the CLT in developing our model. This paper extends the previous work and verifies the probabilistic model on a foraging task and an object filtering task (Vardy 2012). The rest of the paper is organized as follows: Sect. 2 lists a set of related works. Section 3 formulates the problem of interest. In Sect. 4, the central limited theorem is introduced in addition to the model proposed for characterizing the swarm performance. Sections 5 and 6 verify the developed model on a foraging task and an object filtering task, respectively. The paper is concluded in Sect. 7.

## 2 Related work

The performance of a swarm robotics system is influenced by the interferences among the working robots (Goldberg and Matarić 2000). Most of the studies performed in this field were focusing on the change in the amount of the work accomplished within a particular time unit, when different swarm sizes are used. The studies were mostly performed on the well-known foraging task and the conclusions were similar and state that increasing the number of robots decreases the performance of individual robots. Whereas for the swarm performance, increasing the number of robots increases the swarm performance up to some point after which the performance starts to decrease influenced by negative effect of the interferences between robots. Several studies were pursued to improve the swarm performance by reducing the density of spatial interferences between robots. In Goldberg (2001), different types of interferences for multi-robot systems have been defined and the work has presented the interactions among robots working together in a common area as the main type of robot interactions. The authors have proposed two techniques to arbitrate the impact of interactions. First, by making sure that robots are working in different areas and second, by scheduling the occupation of shared areas. The first proposal was further investigated under the term bucket-brigade as in Shell and Mataric (2006); Vaughan (2008); Ostergaard et al. (2001), in addition to Lein and Vaughan (2008), in which the approach was extended to consider adaptive working areas. Task partitioning represents another technique, which is used to improve the swarm performance under spatial interferences. In Pini et al. (2009), a task partitioning technique was proposed, in which the shared area was divided into two areas and the robots select their area using a threshold mechanism. The authors of Pini et al. (2011) have studied the role of task partitioning in reducing the concurrent access to the nest area in a harvesting task. Differently from the works listed above, the focus in this paper is not on decreasing the impact of spatial interferences on the swarm performance; however, it is on characterizing this performance under the influence of the complex dynamics associated with the spatial interferences.

Analyzing swarm performance by means of real experiments is not always possible and it is an expensive solution in terms of both time and hardware. Computer simulations, on the other hand, are very time consuming, especially when tasks are associated with long-term deadlines. In such cases, mathematical modeling represents one of the best approaches. In Lerman and Galstyan (2002), a mathematical model has been introduced, in which the authors tried to quantify the effect of spatial interferences on both the single robot and the swarm performances. A list of various mathematical models which can be applied in swarm systems is reported in Muniganti and Pujol (2010). Most of these mathematical studies focus on specific swarm scenarios such as foraging in Liu et al. (2007) or collaborative distributed manipulation in Martinoli et al. (2004). To our best knowledge, no study was considering the mathematical analysis of the collective swarm performance within specific deadlines. In addition, the probability analysis of swarm robotics performance was not considered intensively and only few studies were performed in that field such as in Lerman et al. (2005). The CLT (Rice 2001) is a theorem that is applied widely in several fields related to measurement approximations, hypothesis testing, canceling of communication noise, and statistics (Jacod et al. 2010; Dunsmuir 1979). Nevertheless, it is not investigated yet within the context of swarm robotics. This theorem is exploited in this paper to characterize probabilistically the performance of swarm robotics systems over long-term deadlines and under the influence of complex dynamics.

## 3 Problem formulation

We focus on constructive robotics tasks. Constructive tasks are tasks in which the total swarm contribution is the sum

[1] The central limit theorem, in its classic version, states that the mean of a sufficiently large set of independent and identically distributed random variables each with a finite mean and variance tends to be distributed normally (Rice 2001).

of the robots' individual contributions. We assume that the tasks can be executed additively and that each robot is able to execute an individual part of the task. Consequently, the swarm performance is the accumulative performance of the contributions of the individual robots. Each of the considered tasks can be characterized by its long-term deadline, which refers to the time point after which the robots should stop work on the task.

In swarm robotics systems, the contribution of a single robot is a random variable which can be discrete, as well as continuous based on the type of the task. For example, pushing a box is a task in which the performance is a continuous random variable that represents the distance the box travels within a particular time unit. However, in a foraging task the performance represents the number of objects retrieved back to the nest during the time unit and therefore it belongs to the discrete space. In this paper, we focus on tasks in which the performance is associated with a discrete random variable and the task consists of individual parts that need to be accomplished. The parts of the task are assumed to be generated periodically over time.

Let us have a homogeneous swarm of $N$ simple robots, which is used to execute task $i$, due by its deadline $D_i$. Each of these robots is able to accomplish one part of $i$ at a time. Since $i$ is a constructive task whose successful execution depends on accomplishing a particular number of its parts, robots need to collaborate to achieve this number. We use $\beta_{ij}(D_i)$ to denote the discrete random variable associated with the number of parts that can be accomplished by robot $j$ up to the deadline $D_i$. The performance of the individual robots represents independent variables as the parts of the task are re-generated, thus the parts performed by a robot do not affect the probability of another robot to execute them. The swarm performance on task $i$ up to the same deadline is denoted by $\omega_i(D_i)$, i.e., the total number of parts accomplished by the swarm during the time interval $[0, D_i]$. The swarm performance $\omega_i(D_i)$ is calculated as the sum of the $N$-robot contributions:

$$\omega_i(D_i) = \beta_{i1}(D_i) + \beta_{i2}(D_i) + \cdots + \beta_{iN}(D_i) = \sum_{j=1}^{N} \beta_{ij}(D_i) \tag{1}$$

We divide the time period between the start of the execution $t = 0$ and the task deadline $D_i$ into equal and non-overlapping time windows each with the length $\tau$. The length $\tau$ of the time window is selected under the following constraints: it should be equal to or greater than the average time required by a single robot to accomplish one part on task $i$, the task deadline $D_i$ should be a multiplier of $\tau$ and $\tau$ should be significantly smaller than the task deadline, $\tau \ll D_i$.

The swarm performance at deadline $D_i$ is the sum of the swarm contributions over all the time windows included up to the deadline $D_i$. Hence, we can compute the swarm performance at the deadline $D_i$ as in the following:

$$\omega_i(D_i) = \omega_i(\tau_1) + \omega_i(\tau_2) + \cdots + \omega_i(\tau_K) = \sum_{j=1}^{K} \omega_i(\tau_j) \tag{2}$$

where $K$ is the number of time windows included up to the deadline $D_i$.

On the other hand, the swarm performance at the deadline $D_i$ is the sum of the individual contributions of the robots over all the time windows. Based on Eqs. (1) and (2), the swarm performance can be computed in terms of the robot individual contributions as in the following:

$$\omega_i(D_i) = (\beta_{i1}(\tau_1) + \beta_{i2}(\tau_1) + \cdots + \beta_{iN}(\tau_1)) + \cdots + (\beta_{i1}(\tau_K) + \beta_{i2}(\tau_K) + \cdots + \beta_{iN}(\tau_K)) = \sum_{j=1}^{K} \sum_{l=1}^{N} \beta_{il}(\tau_j) \tag{3}$$

Characterizing the performance obtained by a swarm of $N$ robots at the deadline $D_i$ is performed probabilistically. Both the probability density function (PDF) and the cumulative distribution function (CDF) of the random variable associated with the swarm performance are computed. We aim to perform this analysis with the minimum consumption of time and resources by launching short-term real experiments or computer simulations. The performed analysis helps us to answer questions, such as what is the probability of achieving a specific swarm performance $S_i$ at the deadline $D_i$ under the influence of spatial interferences?

$$\Pr(\omega_i(D_i) \geqslant S_i) \tag{4}$$

## 4 Probabilistic analysis of swarm performance

In this section, we perform a probabilistic analysis of the swarm performance over long-term tasks under the influence of spatial interferences. The swarm performance $\omega_i(\tau_j)$ during the time window $\tau_j$ is the number of parts accomplished by the swarm within the time window $\tau_j$. Concurrently, the single robot contribution $\beta_{il}(\tau_j)$ is the number of parts accomplished by the single robot within the time window $\tau_j$. Those two variables are discrete random variables, whose mean and the variance are influenced by the number of robots working on the task. Additionally, they are affected by the work density, which refers to the number of task parts available at the working arena. The model developed in this paper considers a fixed swarm size while executing the task. However, technical defects may happen while the system is operating and lead to a reduction in the number of functioning robots. However, such a loss is generally small enough in comparison with the number of robots available in the swarm, thus it can be ignored. On the other hand, the assumption of having a constant work density is associated with several real-world applications. Examples could include recycling systems in which robots are responsible to retrieve objects excreted continuously at specific locations to some recycling destination. Another example could be a production-transport system, in which objects are assumed to be produced continuously at different locations and require to be transported to specific delivery points. The analysis of the swarm performance is done based on the well-known CLT. CLT states that the sum of a large enough number of random variables, which are identically distributed with the mean $\mu$ and the variance $\sigma^2$, is normally distributed and can be characterized by the following mean and variance:

$$\mu_n = n\mu \tag{5}$$
$$\sigma_n^2 = n\sigma^2 \tag{6}$$

Let us denote the mean and the standard deviation of the swarm performance $\omega_i(\tau)$ obtained within the time window $\tau$ by: $\mu_{\omega_i(\tau)}$ and $\sigma_{\omega_i(\tau)}$, respectively. According to the CLT, the long-term swarm performance will be normally distributed with the following mean and variance $\mu_{\omega_i(\tau)}$ and $\sigma^2_{\omega_i(\tau)}$:

$$\omega_i(D_i) \sim \text{Norm}(K\mu_{\omega_i(\tau)}, K\sigma^2_{\omega_i(\tau)}) \tag{7}$$

The mean and the standard deviation of the random variable associated with the single robot performance $\beta_{ij}(\tau)$ are denoted by $\mu_{\beta_{ij}(\tau)}$ and $\sigma_{\beta_{ij}(\tau)}$, respectively. Based on the CLT, the long-term swarm performance will be normally distributed with the following mean and variance $\mu_{\beta_{ij}(\tau)}$ and $\sigma^2_{\beta_{ij}(\tau)}$:

$$\omega_i(D_i) \sim \text{Norm}(KN\mu_{\beta_{ij}(\tau)}, KN\sigma^2_{\beta_{ij}(\tau)}) \tag{8}$$

Consequently, the swarm performance can be characterized probabilistically using the cumulative distributed function (CDF) of the normal distribution, which for the mean $\mu$ and the variance $\sigma^2$, which is defined as in the following:

$$\Pr(X \leqslant x) = \frac{1}{2} + \frac{1}{2}\text{erf}\left(\frac{x-\mu}{2\sigma^2}\right) \tag{9}$$

where erf is the error function.

We substitute the random variable $X$ with the swarm performance $\omega_i(D_i)$ and the value of the small $x$ with a desired performance $S_i$ that represents the number of parts required to be accomplished up to the task deadline $D_i$. The probability we are interested to calculate in Eq. (4) can be obtained using the CDF of the normal distribution as in the following:

$$\begin{aligned}\Pr(\omega_i(D_i) \geqslant S_i) &= \Pr(\omega_i(D_i) > (S_i - 1)) \\ &= 1 - \Pr(\omega_i(D_i) \leqslant (S_i - 1))\end{aligned} \tag{10}$$

Hence, CLT can be applied to characterize the long-term performance of swarm robotics system using any of the following inputs:

- The swarm performance over short experiments: the swarm performance at the deadline $D_i$ is the sum of the swarm contributions over all time windows included up to the deadline $D_i$. We map each of these swarm contributions to a random variable with the mean $\mu_{\omega_i(\tau)}$ and the variance $\sigma^2_{\omega_i(\tau)}$ which are measured over short experiments of the length $\tau$. Consequently, CLT can be applied to characterize the swarm performance at the $D_i$ as the sum of these random variables, as in Eq. (2).
- The single robot performance over short experiments: the swarm performance at the deadline $D_i$ is the sum of the individual contributions of the robots over all time windows included up to the deadline $D_i$. As the single robot performance over one time window experiment can be measured by the robot itself, this estimation represents a "self-organized" one. The robot works on the task for short-term experiments to estimate the mean and the standard deviation of its performance during one time window. After that, CLT is applied to characterize the swarm performance obtained at $D_i$ using Eq. (3).

In the following, we verify the model proposed for estimating and characterizing the swarm performance over long-term experiments. The PDF and CDF of the swarm performance which are computed using the model are compared with their counterparts which are obtained empirically from both the foraging task and the object filtering task. Additionally, statistical tests are performed to check the normality of the performance distribution.

## 5 Foraging task

We consider a foraging task, in which a large number of objects (150 in our experiment) are scattered uniformly over an $(9 \times 12\ \text{m}^2)$ object-area. Those objects need to be retrieved back to a $(3 \times 12\ \text{m}^2)$ nest-area, see Fig. 1. The robots are located initially at the nest, where they are uniformly distributed. The nest-area is marked with an array of lights to attract the robots back while they are retrieving the objects. During the task execution, each robot can be in one of the following states: exploring or retrieving. A robot in the exploring

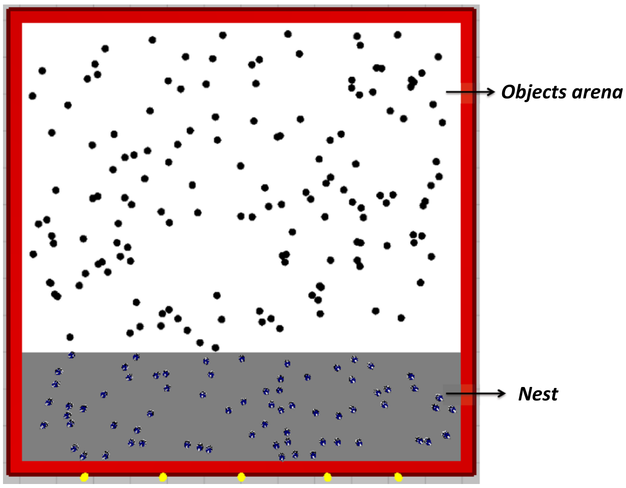


**Fig. 1** A snapshot of the foraging task

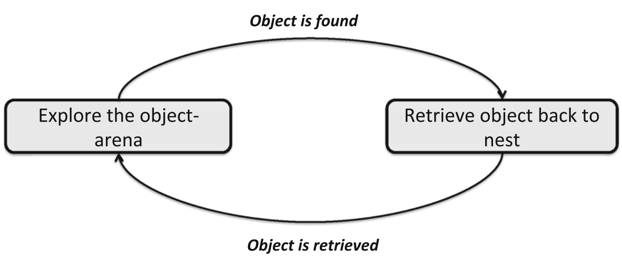


**Fig. 2** The robot controller

state searches the arena for objects to retrieve them back. The search is performed as a random walk that is combined with an obstacle avoidance behavior. As soon as an object is found, the robot changes its state to the retrieving state and starts moving back to the nest-area attracted by the lights, while applying obstacle avoidance. Figure 2 depicts the controller state machine of the foraging robots.

The retrieved objects are assumed to be replaced with new objects, thus, the objects density remains constant throughout the whole experiment. As mentioned earlier, tasks with part replacement could be mapped to several real-world applications such as recycling systems, in which robots are required to collect materials from particular locations and retrieve them to some recycling point. These materials will be generated periodically at the same locations. Another example is the data mules in wireless sensor networks (Shah et al. 2003), in which swarm robotics systems can be deployed to collect data. Data are generated periodically at the sensor nodes and need to be retrieved to a central node for processing. The robotic simulator ARGoS[2] (Pinciroli et al. 2012) is used to compute an average performance function through repeated

[2] ARGoS is a discrete-time physics-based simulation framework developed within the Swarmanoid project. It can simulate various robots at different levels of details, as well as a large set of sensors and actuators.

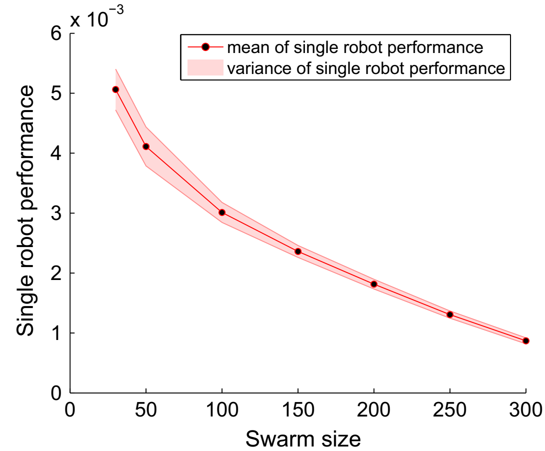


**Fig. 3** Single robot performance during 1 s

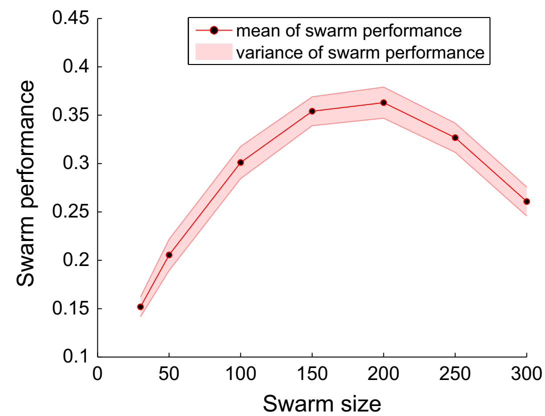


**Fig. 4** Swarm performance during 1 s

high-level simulations to characterize the effect of the spatial interferences on both the single robot performance and the swarm performance. The simulations are repeated for 125 runs. Figure 3 shows how the mean of the single robot performance decreases by increasing the number of robots for different swarm sizes. Whereas, the change in the mean of the swarm performance while applying the same increment in the swarm size is depicted in Fig. 4. We consider a foraging task that is carried out by a swarm of 30 homogeneous robot and is due to the deadline $D_i = 12 \times 10^3$ s. The length of the time window is set to $\tau = 100$ s. The time window $\tau$ is selected long enough to retrieve at least one object back to the nest and enough smaller than the deadline $D_i$. Figure 5 shows the mean $\mu$ in addition to the $3 \times \sigma$ of the random variable associated with the number of objects retrieved over all the time windows included up to $D_i = 12 \times 10^3$ s. Figure 6 shows the time it takes the average of the swarm performance to stabilize. This time is referred to as the *start-up* time. At the beginning, the swarm retrieves a higher number of objects as all robots start synchronized free of objects. They start to search and retrieve objects all together. After-

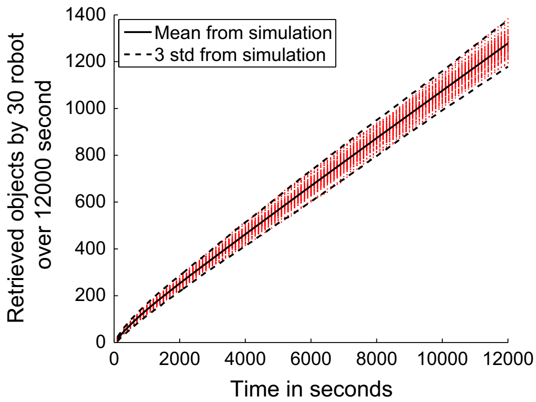


**Fig. 5** $\mu$ and $3\times\sigma$ of the number of objects retrieved during $12\times10^3$ s

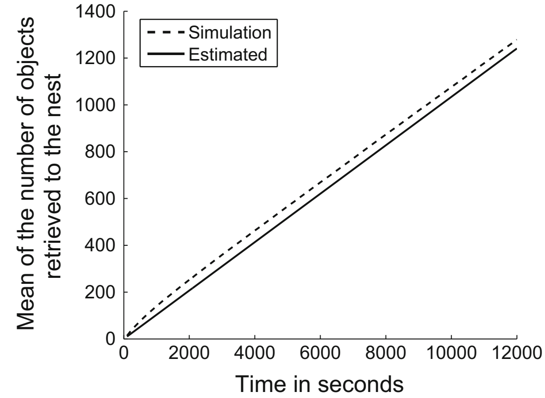


**Fig. 7** Mean of retrieved objects up to $D_i = 12\times10^3$ s vs. the mean predicted by CLT

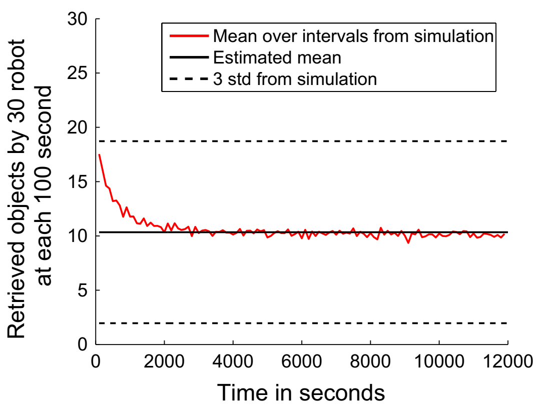


**Fig. 6** $\mu$ and $3\times\sigma$ of the number of objects retrieved over all 100 s intervals

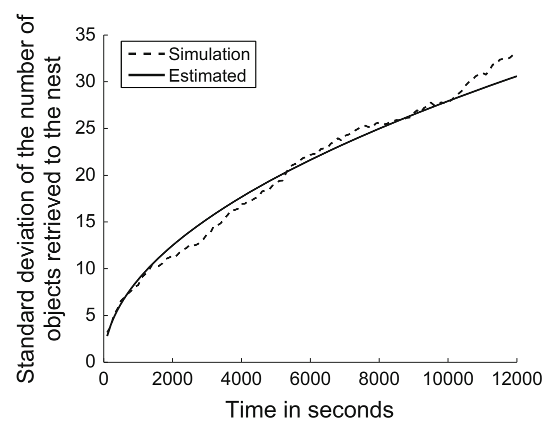


**Fig. 8** Standard deviation of retrieved objects up to $D_i = 12\times10^3$ s vs. the standard deviation predicted by CLT

wards, robots lose their synchronization and become divided into a group that is still searching to retrieve objects and a group that is currently retrieving objects. This is the reason behind the presence of such a start-up time after which the system performance stabilizes around its estimated mean. The accuracy of the CLT estimation for the swarm performance is influenced by including the system performance obtained during the start-up time or excluding it. This influence varies based on the relative relation between the length of both: the deadline $D_i$ and the start-up time.

In the following, the swarm performance will be characterized using each of the two inputs mentioned earlier, the swarm contributions and the single robot contributions both measured over short-term experiments.

### 5.1 Swarm performance over short-term experiments

We substitute the swarm contribution achieved by the swarm within one time window $\tau = 100$ s in Eq. (2). The deadline $D_i = 12\times10^3$ s includes 120 time windows of length 100 s:

$$\omega_i(D_i) = \sum_{j=1}^{K} \omega_i(\tau_j) \Rightarrow \omega_i(12\times10^3) = \sum_{j=1}^{120} \omega_i(100) \quad (11)$$

Based on CLT, the random variable associated with the number of objects retrieved at the deadline $D_i = 12\times10^3$ is normally distributed with the following mean and standard deviation:

$$\mu_{\omega_i(12\times10^3)} = 120\mu_{\omega_i(100)} \quad (12)$$

$$\sigma_{\omega_i(12\times10^3)} = \sqrt{120}\sigma_{\omega_i(100)} \quad (13)$$

Figures 7 and 8 show the mean and the standard deviation of the number of objects retrieved up to the deadline $D_i = 12\times10^3$, as compared to the mean and standard deviation predicted by CLT using Eqs. (12) and (13). The probability of retrieving a number of objects which is equal to or greater than $S_i$ at the deadline $D_i$ can be derived using the CDF of the normal distribution as follows:

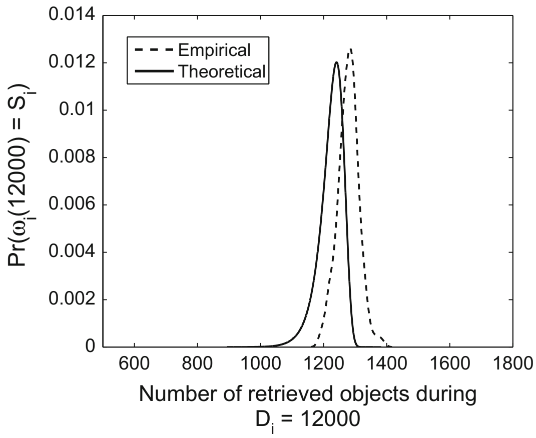


**Fig. 9** PDF of the retrieved number of objects without taking the start-up time into account

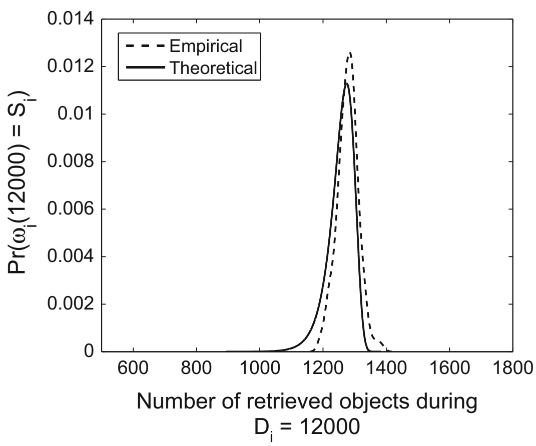


**Fig. 10** PDF of the retrieved number of objects with taking the start-up time into account

$$
\begin{aligned}
&\Pr(\omega_i(12\times10^3)\geqslant S_i)\\
&\quad=1-\left[\frac{1}{2}+\frac{1}{2}\text{erf}\left(\frac{(S_i-1)-\mu_{\omega_i(12\times10^3)}}{\sqrt{2}\sigma_{\omega_i(12\times10^3)}}\right)\right]\\
&\quad=1-\left[\frac{1}{2}+\frac{1}{2}\text{erf}\left(\frac{(S_i-1)-120\,\mu_{\omega_i(100)}}{\sqrt{2}\,\sqrt{120}\,\sigma_{\omega_i(100)}}\right)\right]
\end{aligned}
\tag{14}
$$

Figures 9 and 10 show both the computed and the simulated PDF of the number of objects retrieved by the swarm at the deadline $D_i = 12\times10^3$. In Fig. 9 using the mean $\mu_{\omega_i(100)} = 10.3411$ and standard deviation $\sigma_{\omega_i(100)} = 2.7931$ measured after the system stabilizes and in Fig. 10 using the mean $\mu_{\omega_i(100)} = 10.6239$ and standard deviation is $\sigma_{\omega_i(100)} = 2.9755$ measured with taking the system performance during the start-up time into account. Figures 11 and 12 show the computed as well as the simulated CDF of the number of retrieved objects also in Fig. 11 after the system stabilizes and in Fig. 12 with taking the start-up time into account.

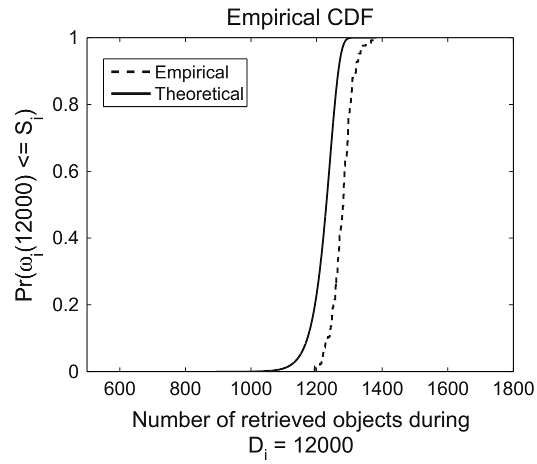


**Fig. 11** CDF of the retrieved number of objects without taking the start-up time into account

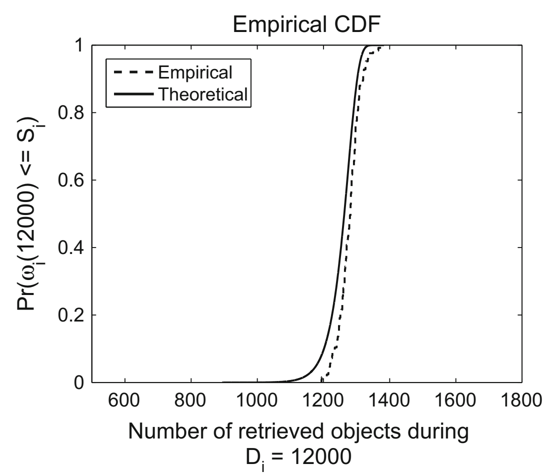


**Fig. 12** CDF of the retrieved number of objects with taking the start-up time into account

### 5.2 Single robot performance over short-term experiments

Here, we characterize the swarm performance at the deadline $D_i = 12\times10^3$ in a self-organized manner using the single robot performance measured during one time window and we substitute it in Eq. (3) as in the following:

$$
\begin{aligned}
\omega_i(D_i) &= \sum_{j=1}^{K}\sum_{l=1}^{N}\beta_{il}(\tau_j) \Rightarrow \omega_i(12\times10^3)\\
&= \sum_{j=1}^{120}\sum_{l=1}^{30}\beta_{il}(100)
\end{aligned}
\tag{15}
$$

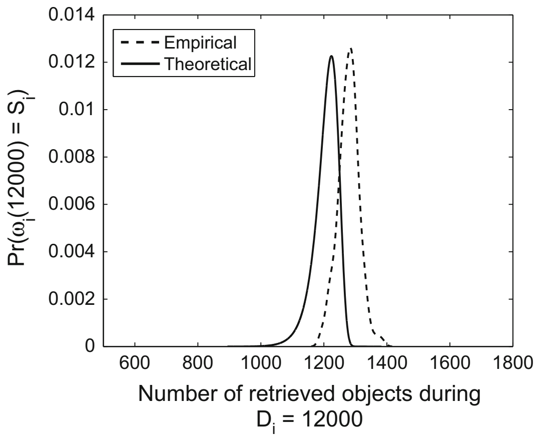


**Fig. 13** PDF of the number of retrieved objects characterized based on single robot performance

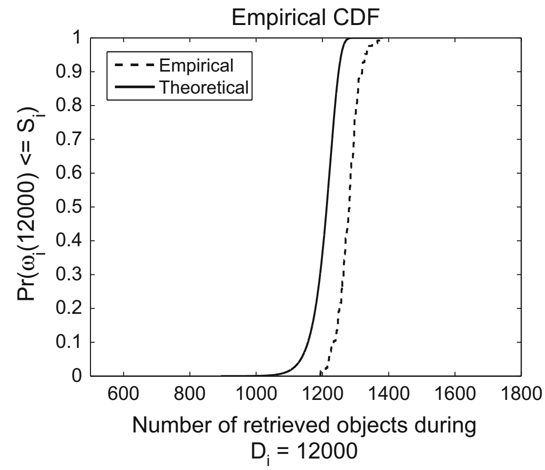


**Fig. 14** CDF of the number of retrieved objects characterized based on single robot performance

According to the CLT, the random variable associated with objects retrieved by the swarm up to the deadline $12 \times 10^3$ is normally distributed with the following mean and standard deviation:

$$\mu_{\omega_i(12\times10^3)} = 120 \times 30\mu_{\beta_{ij}(100)} \quad (16)$$

$$\sigma_{\omega_i(12\times10^3)} = \sqrt{120 \times 30}\sigma_{\beta_{ij}(100)} \quad (17)$$

This estimation can be performed by the robots themselves which may help them in making appropriate allocation decisions. The probability of interest in Eq. (4) can be computed using the CDF of the normal distribution as follows:

$$\Pr(\omega_i(12 \times 10^3) \geqslant S_i) = 1 - \left(\frac{1}{2} + \frac{1}{2}\text{erf}\left(\frac{(S_i - 1) - \mu_{\omega_i(12\times10^3)}}{\sqrt{2}\sigma_{\omega_i(12\times10^3)}}\right)\right) = 1 - \left(\frac{1}{2} + \frac{1}{2}\text{erf}\left(\frac{(S_i - 1) - 120 \times 30\ \mu_{\beta_{ij}(100)}}{\sqrt{2}\ \sqrt{120 \times 30}\ \sigma_{\beta_{ij}(100)}}\right)\right) \quad (18)$$

Measuring the swarm performance after the system stabilizes is straightforward as mentioned above. Therefore, the swarm performance is simulated only in the case of taking the start-up time into account. Figure 13 shows a comparison between the computed and the simulated PDF of the number of objects retrieved by the 30 robot at $D_i = 12 \times 10^3$. Additionally, Fig. 14 shows the same comparison, however, of the CDF.

The analysis performed using the single robot contributions is not the same accurate as the one performed using the swarm contributions. The reason behind is that when swarm contributions are used, the performance is averaged over $N$ robots rather than using a single trail, i.e., the contribution of a single robot.

## 6 Objects filtering task

In this section, we focus on an object filtering task, in which an arena of ($6 \times 6$ m$^2$) is divided into two equal parts, the white part and the black part. The two parts are separated by a wall, which has four passages to connect them. The passages are marked by lights to attract the robots while they are filtering the objects. Two types of objects, red and yellow, are scattered uniformly on both parts of the arena. The goal is to transport the red objects to the white part of the arena and the yellow objects to the black part of it, see Fig. 15. Objects filtering is a more complicated task than a simple foraging. In this task, robots are required to distinguish between the different types of objects and to recognize the arena they are working at. Additionally, the arenas are built in a way that limits the access from the one to the other and restricts

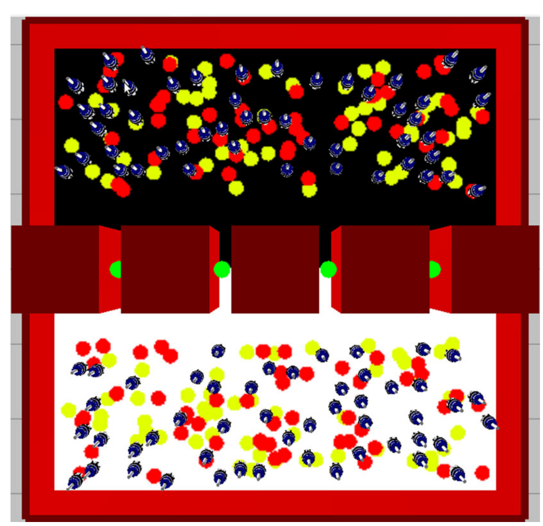

**Fig. 15** A snapshot of the object filtering task

the flow of robots between them. Such factors have a direct influence on the swarm performance and our goal here is to show that the presented approach is still representing an efficient tool to estimate the long-term performance of the swarm even under such conditions.

We use a swarm of homogeneous robots to execute the objects filtering task. Initially, an equal number of robots is assigned to each of the arena parts (20 robots in our experiment). The robots of each part start from uniformly distributed positions. Each robot starts the execution by identifying the arena color it is placed on and based on that, the color of the item it should filter. The robot starts to explore performing a random walk combined with an obstacle avoidance behavior. As soon as it finds one of the objects that needs to be filtered, it picks the object and starts moving towards the light source. Approaching the destination part of the arena requires the robot to move through one of the passages which connect the two arena parts. In case of conflicts with other robots which are trying to use the same passage, ties are broken in terms of the longer distance traveled within the passage. I.e., the robot which has traveled a longer distance within the passage is given the passage free and all other robots should move back. After the robot arrives to the part of the arena to which the transported object belongs, it starts searching for a free loading location to drop the object. This is done using a combined behavior of obstacle avoidance and ground detection, in which the robot uses its ground sensors to detect free patches on the ground where the object can be dropped. As soon as the object is dropped, the robot starts working to filter objects of its current arena part. The controller state machine of the robots is shown in Fig. 16.

Filtered objects are replaced with new objects and, therefore, the density of the objects remains constant throughout the whole experiment. The simulator ARGoS is used to compute an average performance function through repeated

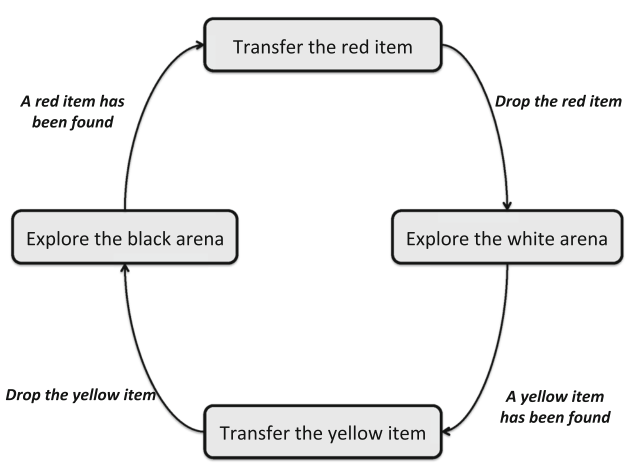


**Fig. 16** The robot controller

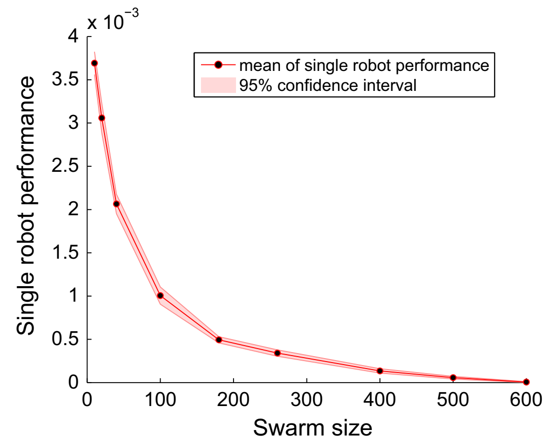


**Fig. 17** Single robot performance during 1 s

simulations. The obtained performance function characterizes the effect of the spatial interferences on both the single robots performance and the swarm performance. The simulations are repeated 20 runs for each swarm size and the obtained dynamics of the single robot and the swarm performances are similar to those of the foraging task reported in Sect. 5. The mean of the single robot performance decreases while increasing the swarm size as we can see in Fig. 17. On the other hand, the mean of the swarm performance increases while increasing the number of robots up to some optimal performance after that it starts to decrease affected by the influence of the spatial interferences. The decrement in the swarm performance is slower in comparison to the one observed in the foraging task.

This is caused by the limited possibility of using the passages between the two parts of the arena. Therefore, increasing the number of robots, increases the number of objects found by the robots, but not the number of filtered objects. Continuing to increase the number of robots leads to decrease the swarm performance since the exploration process becomes difficult because of the high intensity of

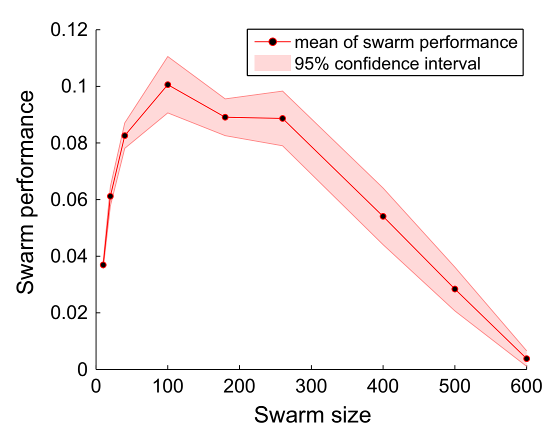


**Fig. 18** Swarm performance during 1 s

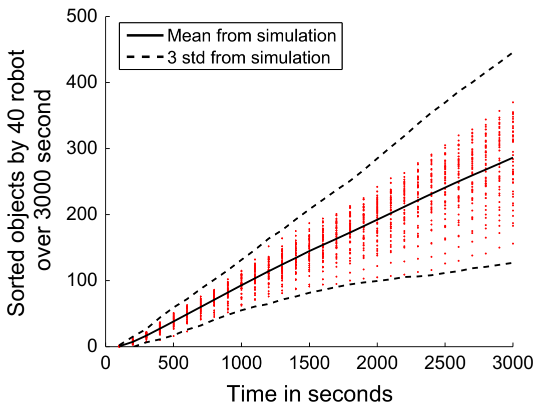


**Fig. 19** $\mu$ and $3 \times \sigma$ of the number of objects filtered during 3000 s

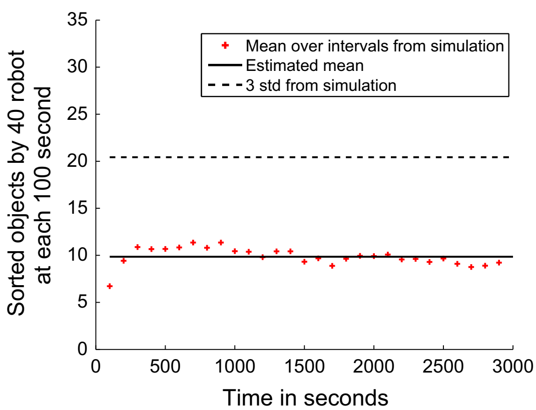


**Fig. 20** $\mu$ and $3 \times \sigma$ of the number of objects filtered over all 100 s intervals

interferences between robots. Figure 18 shows the swarm performance obtained for different swarm sizes.

We consider an object filtering task which is executed by a swarm of 40 robots, 20 robots on each arena part. The task deadline is set to $D_i = 3000$ s and we use the same time window as for the foraging task in Sect. 5 with the length $\tau = 100$ s. Figure 19 shows the mean $\mu$ and the $3 \times \sigma$ of the random variable associated with the number of objects filtered over all the time windows included up to the deadline of 3000 s. In Fig. 20, we can notice how the swarm performance is stabilized around its estimated mean starting from the beginning of the execution up to the deadline. Therefore, we do not have the same problem as in the foraging task, in which the system performance requires a particular start-up time to stabilize.

### 6.1 Swarm performance over short-term experiments

The swarm performance is analyzed using the swarm contributions measured over short-term experiments, i.e., during one time window $\tau = 100$ s. Equation (2) is used after substituting the deadline with $D_i = 3000$ s, which includes 30 time windows of length 100 s:

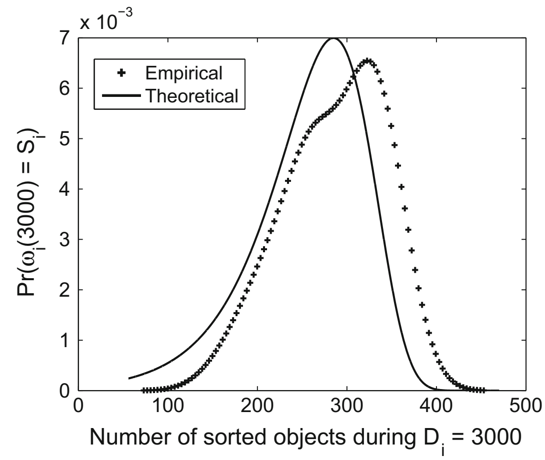


**Fig. 21** PDF of the number of filtered objects characterized based on swarm performance

$$\omega_i(D_i) = \sum_{j=1}^{K} \omega_i(\tau_j) \Rightarrow \omega_i(3000) = \sum_{j=1}^{30} \omega_i(100) \tag{19}$$

The random variable associated with the number of objects filtered up to the deadline $D_i = 3000$ s is normally distributed with the mean and the standard deviation computed using the CLT, as in the following:

$$\mu_{\omega_i(3000)} = 30\mu_{\omega_i(100)} \tag{20}$$

$$\sigma_{\omega_i(3000)} = \sqrt{30}\sigma_{\omega_i(100)} \tag{21}$$

Our goal is to provide a probabilistic characterization of the swarm performance over the deadline of 3000 s, performed using both the PDF and the CDF. The probability of filtering a number of objects which is equal to or more than $S_i$ due to the deadline $D_i$ can be computed using the CDF of the normal distribution as follows:

$$\begin{aligned} &\Pr(\omega_i(3000) \geqslant S_i) \\ &= 1 - \left[\frac{1}{2} + \frac{1}{2}\mathrm{erf}\left(\frac{(S_i - 1) - \mu_{\omega_i(3000)}}{\sqrt{2}\sigma_{\omega_i(3000)}}\right)\right] \\ &= 1 - \left[\frac{1}{2} + \frac{1}{2}\mathrm{erf}\left(\frac{(S_i - 1) - 30\,\mu_{\omega_i(100)}}{\sqrt{2}\,\sqrt{30}\,\sigma_{\omega_i(100)}}\right)\right] \end{aligned} \tag{22}$$

The PDF is measured over 50 runs of ARGoS simulations. After that, it is compared with the PDF computed using the CLT and Fig. 21 illustrates their consistency. Concurrently, Fig. 22 depicts both the computed and the simulated CDF of the number of objects filtered by the swarm up to the deadline.

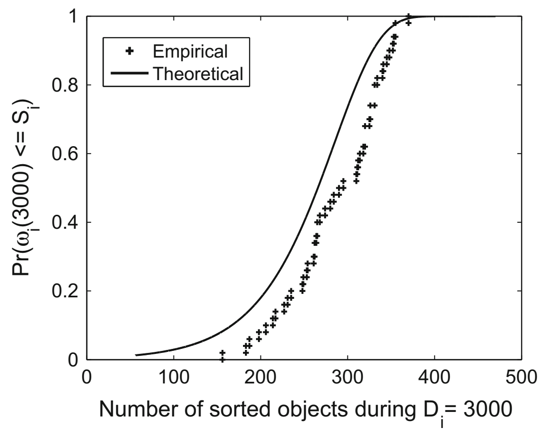


**Fig. 22** CDF of the number of filtered objects characterized based on swarm performance

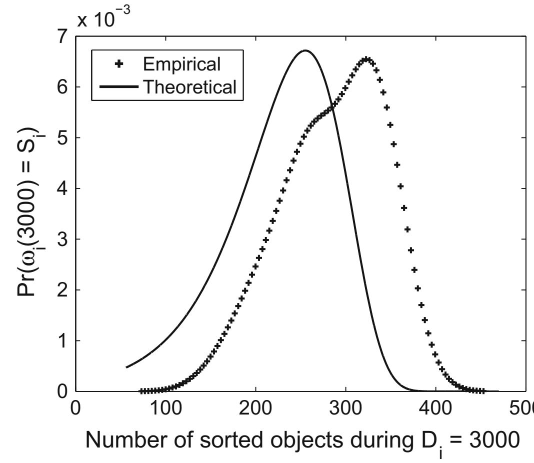


**Fig. 23** PDF of the number of filtered objects characterized based on single robot performance

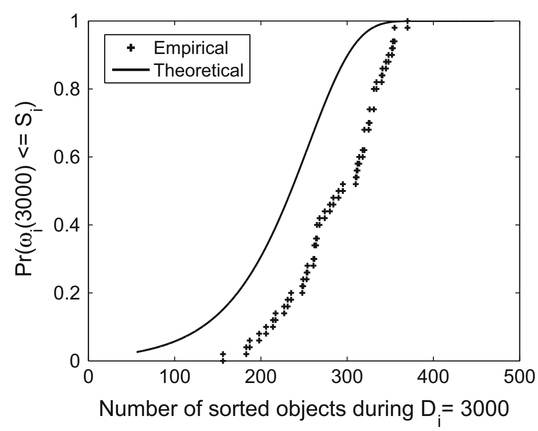


**Fig. 24** CDF of the number of filtered objects characterized based on single robot performance

### 6.2 Single robot performance over short-term experiments

The swarm performance obtained at the deadline $D_i = 30{,}000$ can be estimated as explained in Sect. 5.2 using the performance of the single robot measured during short-term experiments. The number of filtered objects by the swarm during the deadline 3000 is normally distributed with the following mean and standard deviation:

$$\mu_{\omega_i(3000)} = 30 \times 40\mu_{\beta_{ij}(100)} \tag{23}$$

$$\sigma_{\omega_i(3000)} = \sqrt{30 \times 40}\sigma_{\beta_{ij}(100)} \tag{24}$$

Thus, the probability in Eq. (4) can be computed using the CDF of the normal distribution as in the following:

$$\begin{aligned}
&\Pr(\omega_i(3000) \geqslant S_i) \\
&= 1 - \left(\frac{1}{2} + \frac{1}{2}\mathrm{erf}\left(\frac{(S_i - 1) - \mu_{\omega_i(3000)}}{\sqrt{2}\,\sigma_{\omega_i(3000)}}\right)\right) \\
&= 1 - \left(\frac{1}{2} + \frac{1}{2}\mathrm{erf}\left(\frac{(S_i - 1) - 30 \times 40\,\mu_{\beta_{ij}(100)}}{\sqrt{2}\,\sqrt{30 \times 40}\,\sigma_{\beta_{ij}(100)}}\right)\right)
\end{aligned} \tag{25}$$

Comparisons between the computed and the simulated PDF and CDF are depicted in Figs. 23 and 24, respectively.

## 7 Conclusion

In this paper, we have presented a probabilistic analysis of the swarm performance obtained over long-term deadlines and under the influence of complex dynamics (spatial interferences). Estimating the performance of a swarm robotics system is an important process, especially for cases in which this performance needs to be planned under specific constraints, e.g., temporal constraints. In such cases, an early estimation of the long-term performance of a swarm is significantly important. This paper investigates the use of the CLT for analyzing the swarm performance probabilistically under the complex dynamics of spatial interferences. This represents an efficient analysis in terms of preserving time and resources which are required when real-time experiments or computer simulations are performed. The performance analysis can be accomplished using either the swarm contributions or the single robot contributions both over short-term experiments. The normality of the obtained swarm performance was tested for all the conducted experiments using the well-known statistical test, the Jarque–Bera test (Jarque and Bera 1980). The result of the test was for all data sets $h = 0$. I.e., that the test does not reject the null hypothesis at the 5 % significance level. The null hypothesis states that the performance data come from a normal distribution.

Characterizing the swarm performance probabilistically is useful, particularly for tasks in which the swarm performance and/or the single robot performance do not follow a well-known distribution (complex dynamics). In such cases, the performed analysis can be applied efficiently to characterize the swarm performance. In addition, having the possibility to analyze the swarm performance on the global level by performing short-term experiments allows for launching repair mechanisms at an early stage of the execution.

As a future work, we are planning to use the probabilistic model presented in this paper for developing autonomous task allocation mechanisms for time-constrained tasks. Those mechanisms aim to optimize the assignment of robots to a set of tasks based on the amount of work the swarm can accomplish (estimated by the model) in comparison to a required amount of work. In addition, we are planning to study the validity of the proposed model on different kinds of tasks apart from the constructive tasks.

**Compliance with ethical standards**

**Conflict of interest** The authors declare that they have no conflict of interest.

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