# LOCAL LEARNING FOR NONLINEAR CONTROL

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Abstract: This paper presents a local method for modeling and control of nonlinear dynamical systems, when only a limited amount of input-output data is available. The proposed methodology couples a local model identification inspired by the lazy learning technique, with a control strategy based on linear optimal control theory. The local modeling procedure uses a query-based approach to select the best model configuration by assessing and comparing different alternatives.
The control method combines the linearization provided by the local learning techniques with optimal linear control theory, to control nonlinear systems in far from equilibrium configurations. Simulation results of the control of a complex nonlinear system (the bioreactor) are presented.

Keywords: Discrete-time nonlinear control systems, Optimal control of nonlinear systems

## 1. INTRODUCTION

In this paper we present a local method to model and control an unknown dynamical system starting from a limited amount of input-output data. Local techniques are an old idea in time series prediction (Farmer and Sidorowich, 1987), classification (Cover and Hart, 1967) and regression (Cleveland, 1979). The idea of local approximators as alternative to global models originated in non-parametric statistics (Epanechnikov, 1969), (Benedetti, 1977) to be later rediscovered and developed in the machine learning field (Aha, 1989), (Bottou and Vapnik, 1992). Recent work on lazy learning (a.k.a memory-based or instance-based learning) gave a new impetus to the adoption of local techniques for modeling (Atkeson *et al.*, 1997*a*) and control problem (Atkeson *et al.*, 1997*b*).

Here, we propose a model identification methodology based on the use of an iterative optimization procedure to select the best local model among a set of different candidates. Modeling a nonlinear mapping using a limited number of observations requires the data analyst to make several choices involving the set of relevant variables and observations, the model structure, the learning algorithm, and the validation protocol. Our method defers all of these decisions until a prediction or a local description is requested (query-based approach). In classical methods the several options of a local model are designed according to heuristic criteria and a priori assumptions. Here we propose an automatic tuning procedure which searches for the optimal model configuration, by returning for

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each candidate model its parameters and a statistical description of its generalization properties.

The second contribution of the paper is a nonlinear control design technique, which uses extensively analysis and design tools imported from linear control. The idea of employing linear techniques in a nonlinear setting is not new in control literature but had recently a new popularity thanks to methods for combining multiple estimators and controllers in different operating regimes of the system (Murray-Smith and Johansen, 1997). Gain scheduling (Shamma and Athans, 1992), fuzzy inference systems (Takagi and Sugeno, 1985), local model networks (Johansen and Foss, 1993) are well-known examples of control techniques for nonlinear systems inspired to linear control. However, two strong assumptions underlie linearization control methods: an analytical description of the equilibrium points locus is available and the system is supposed to evolve in a sufficiently restricted neighborhood of the desired regime. Here we propose an indirect control method for performing finite-time horizon control which requires only a limited amount of input-output data from the observed system behavior. The controller is designed with optimal control techniques parameterized with the values returned by the linear local estimator. The idea is that a combination of a local estimator with a time varying optimal control can take into account the nonlinearity of a system over a wider range than conventional linearized quadratic regulators (LQR).

The remainder of the paper is organized as follows. In section 2 we will introduce our modeling technique based on an iterative selection procedure. Details on the algorithm for local optimal control can be found in section 3. Finally, in section 4 an example of control of a simulated nonlinear systems is given.

## 2. LOCAL MODELING AS AN OPTIMIZATION PROBLEM

Modeling from data involves integrating human insight with learning techniques. In many real cases, the analyst faces a situation where a limited set of data is available and an accurate prediction is required. Often, information about the order, the structure or the set of relevant variable is missing or not reliable. The process of learning consists of a trial and error procedure during which the model is properly tuned on the available data. In the lazy learning approach, the estimation of the value of the unknown function is solved giving the whole attention to the region surrounding the point where the estimation is required. The classical non-adaptive memory-based procedure essentially consists of these steps:

- for each query point $x_q$, defining a set of neighbors, each weighted according to some relevance criterion (e.g. the distance)
- choosing a regression function $f$ in a restricted family of parametric functions estimating the local weighted regression
- computing the regression value $f(x_q)$.

The data analyst who adopts a local regression approach, has to take a set of decisions related to the model (e.g. the number of neighbors, the weight function, the parametric family, the fitting criterion to estimate the parameters). We extend the classical approach with a method that automatically selects the adequate configuration. To this aim, we simply import tools and techniques from the field of linear statistical analysis. The most important of these tools is the PRESS statistic (Myers, 1990), which is a simple, well-founded and economical way to perform *leave-one-out* cross validation (Efron and Tibshirani, 1993) and therefore to assess the performance in generalization of local linear models. Due to its short computation time which allows its intensive use, it is the key element of our approach to modeling data. In fact, if PRESS can assign a quantitative performance to each linear model, alternative models with different configurations can be tested and compared in order to select the best one. This same selection strategy is indeed exploited to select the training subset among the neighbors, as well as various structural aspects like the features to treat and the degree of the polynomial used as a local approximator. The general ideas of the approach can be summarized in the following way.

(1) The task of learning an input output mapping is decomposed in a series of linear estimation problems
(2) Each single estimation is treated as an optimization problem in the space of alternative model configurations
(3) The estimation ability of each alternative model is assessed by the cross-validation performance computed using the PRESS statistic.

## 3. LAZY LEARNING OPTIMAL CONTROL

Although nonlinearity characterizes most real control problems, methods for analysis and control design are considerably more powerful and theoretically founded for linear systems than for nonlinear ones. Here we propose a hybrid architecture for the indirect control of nonlinear discrete time plants from their observed input-output behavior. This approach combines the local learning identification procedure described in the previous section with control techniques borrowed from conventional linear optimal control

Consider a class of discrete time dynamic systems whose equations of motion can be expressed in the form

$$\begin{aligned} y(k) = f\big(&y(k-1),\ldots,y(k-ny), \\ &u(k-d),\ldots,u(k-d-nu), \\ &e(k-1),\ldots,e(k-ne)\big) + e(k) \ , \quad (1) \end{aligned}$$

where $y(k)$ is the system output, $u(k)$ the input, $e(k)$ is a zero-mean disturbance term, $d$ is the relative degree and $f(\cdot)$ is some nonlinear function. This model is known as the NARMAX model (Leontaritis and Billings, 1985). Let us assume we have no physical description of the function f but a limited amount of pairs $[u(k), y(k)]$ from the observed input-output behavior. Defining the information vector

$$\begin{aligned} \varphi(k-1) = \big[&y(k-1),\ldots,y(k-ny), \\ &u(k-d),\ldots,u(k-d-nu), \\ &e(k-1),\ldots,e(k-ne)\big] \ , \quad (2) \end{aligned}$$

the system (1) can be written in the input-output form $y(k) = f\big(\varphi(k-1)\big) + e(k)$.

Consider the optimal control problem of a nonlinear system over a finite horizon time. Using a quadratic cost function, the solution to an optimal control problem is the control sequence $U$ that minimizes

$$J = \frac{1}{2}y(t_f)^T P(t_f) y(t_f) + \frac{1}{2}\sum_k \left| y(k)^T u(k)^T \right| \begin{vmatrix} Q_k & M_k \\ M_k^T & R_k \end{vmatrix} \left| y(k) u(k) \right| \quad (3)$$

with $Q_k, M_k, R_k, P_f$ weighting terms designed a priori. While analytic results are not available for a generic nonlinear configuration, optimal control theory (Stengel, 1986) provides the solution for the linear case. Hence, we will now present the nonlinear problem in a linear time varying setting.

Consider the trajectory of the dynamical system once forced by an input sequence $U = [u(1), u(2), \ldots, u(t_f)]$. Assume that the system can be linearized about each state of the trajectory. Neglecting the residual errors due to the first order Taylor series approximation, the behavior of the linear system along a generic trajectory is the behavior of a linear time varying system whose state equations can be written in the form

$$\begin{aligned} y(k+1) &= A\big(\varphi(k)\big)y(k) + B\big(\varphi(k)\big)u(k) \\ &\quad + K\big(\varphi(k)\big) \qquad (4) \\ &= A_k y(k) + B_k u(k) + K_k \end{aligned}$$

with $A_k, B_k, K_k$ parameters of the system linearized about the query point $\varphi(k)$. $K_k$ is an offset term that equals zero in equilibrium points. This term requires a slight modification in the linear controller formulation. However, in order to simplify the notation, in the following we will neglect the constant term.

Optimal control theory provides the solution for the linear time varying system (4). At each time step the optimal control action is

$$u(k) = -(R_k + B_k^T P_{k+1} B_k)^{-1} (M_k^T + B_k^T P_{k+1} A_k) y(k) \quad (5)$$

where $Pk$ is the solution to the backward Riccati equation.

$$\begin{aligned} P_k = Q_k &+ A_k^T P_{k+1} A_k \\ &- (M_k + A_k^T P_{k+1} B_k)(R_k + B_k^T P_{k+1} B_k)^{-1} \\ &\qquad (M_k^T + B_k^T P_{k+1} A_k) \quad (6) \end{aligned}$$

having as final condition

$$P(t_f) = P_f \quad (7)$$

The piecewise-constant optimal solution is obtained by solving the Euler-Lagrange equations, the three necessary and sufficient conditions for optimality when the final time is fixed.

$$0 = \frac{\partial H_k}{\partial u_k} = y_k^T M_k + u_k^T R_k + \lambda_{k+1}^T B_k \quad (8)$$

$$\lambda_k^T = \frac{\partial H_k}{\partial y_k} = y_k^T Q_k + u_k^T M_k^T + \lambda_{k+1}^T A_k \quad (9)$$

$$\lambda_f^T = y_f^T P_f \quad (10)$$

with $\lambda_k = P_k y_k$ adjoint term in the augmented cost function (Hamiltonian)

$$H_k = J + \lambda_{k+1}^T \big(A_k y(k) + B_k u(k)\big) \quad (11)$$

The Euler-Lagrange equations do not hold for nonlinear systems. Anyway, if the system can be represented in the form (4), formula (8) can be used to compute the derivative of the cost function (3) with respect to a control sequence $U$. This requires at each time $k$ the matrices $A_k, B_k$ that can be obtained by linearizing the system dynamics along the trajectory forced by the input sequence.

As discussed in section 2, our modeling procedure performs system linearization with minimum effort, no a priori knowledge and only a reduced amount of data. Hence, we propose an algorithm for nonlinear optimal control, formulated as a gradient based optimization problem and based on the local system linearization.

The algorithm searches for the sequence of input actions

$$U^{opt} = arg \min_{U^i} J(U^i) \quad (12)$$

that minimizes the finite-horizon cost function (3) along the future $t_f$ steps. The cost function $J(U^i)$ for a generic sequence $U^i$ is computed simulating forward for $t_f$ steps the model identified by the local learning method. The gradient of $J(U^i)$ with respect to $U^i$ is returned by (8).

These are the basic operations of the optimization procedure (described in detail in Fig. 1) executed each time a control action is required:

- forward simulation of the lazy model forced by a finite control sequence $U^i$ of dimension $t_f$
- linearization of the simulated system about the resulting trajectory
- computation of the resulting finite cost function $J(U^i)$
- computation of the gradient of the cost function with respect to simulated sequence
- updating of the sequence with a gradient based algorithm.

Once the search algorithm returns $U^{opt}$, the first action of the sequence is applied to the real system (receding horizon control strategy (Clarke, 1994)). Let us remark how the lazy learning model has a twofold role in the algorithm in Fig. 1: (i) at step 2 it behaves as an approximator which predicts the behavior of the system once forced with a generic input sequence (ii) at step 3 it returns a linear approximation to the system dynamics.

Atkeson et al. (Atkeson *et al.*, 1997*b*) and Tanaka (Tanaka, 1995) applied infinite-time LQR regulator to nonlinear systems linearized with lazy learning and neuro-fuzzy models. The drawback of these approaches is that an equilibrium point or a reference trajectory is required. Also, they make the strong assumption that the state of the system will remain indefinitely in a neighborhood of the linearization point. As discussed above, the advantage of the proposed approach is that these requirements do not need to be satisfied. First, lazy learning is able to linearize a system in points far from equilibrium. Secondly, the time varying approach makes possible the use of a linear control strategy even though the system operates within different linear regimes.

*Remark*: we make the assumption that the parameters returned by the local models are a real description of the local behavior (certainty equivalence principle). This is a restricting assumption which requires a sufficient degree of accuracy in the approximation. However, we see in the optimal control theory a possible solution to this limitation. In fact, stochastic optimal control theory provides a formal solution to the problem of parameter uncertainty in control systems (dual control (Fel'dbaum, 1965)). Further, our modeling procedure can return at no additional cost a statistical description of the estimated parameters . Hence, future work will focus on the extension of the technique to the stochastic control case.

## 4. THE CONTROL OF THE BIOREACTOR

Consider the bioreactor system, a well-known benchmark in nonlinear control (MillerIII *et al.*, 1990). The bioreactor is a tank containing water, nutrients, and biological cells. Nutrients and cells are introduced into the tank where the cells mix with the nutrients. The state of this process is characterized by the number of cells ($c1$) and the amount of nutrients ($c2$). Bioreactor equations of motion are the following:

$$\begin{cases} \dfrac{dc_1}{dt} & = -c_1 u + c_1(1-c_2)\,e^{\frac{c_2}{\gamma}} \\ \dfrac{dc_2}{dt} & = -c_2 u + c_1(1-c_2)\,e^{\frac{c_2}{\gamma}}\left(\dfrac{1+\beta}{1+\beta-c_2}\right) \end{cases} \tag{13}$$

with $\beta = 0.02$ and $\gamma = 0.48$. In our experiment the goal was to stabilize the multi-variable system about the unstable state $(c_1^*, c_2^*) = (0.2107, 0.726)$ by performing a discrete control action each 0.5 seconds.

We use the control algorithm described in section 3. The system is modeled in the input-output form (1) having the orders $ny = 2, nu = 1, ne = 0, d = 1$. The horizon of the control algorithm is fixed to $t_f = 5$. The initial state conditions are set by to the random initialization procedure defined in (MillerIII *et al.*, 1990). We initialize the lazy learning database with a set of 1000 points collected by preliminarly exciting the system with a random uniform input. The database is then updated on-line each time a new input-output pairs is returned by the simulated system. The plot in Fig. 2a) shows the output of the two controlled state variables, while the plot in Fig. 2b)below the control action.

The bioreactor is considered as a challenging problem for its nonlinearity and because small changes in parameters value can cause the bioreactor to become unstable. These results show how using local techniques it is possible to control complex systems on a wide nonlinear range, with only a limited amount of points and no a priori knowledge about the underlying dynamics.

## 5. CONCLUSIONS AND FUTURE DEVELOPMENTS

In control literature local controllers have generally a restricted range of operating conditions. Here, we proposed a controller, which although making extensive use of local techniques, works on an extended range of operating conditions. These characteristics makes of it a promising tool for intelligent control systems, inspired to traditional engineering methods but able to deal with complex nonlinear systems. An application

(1) Initialization of the algorithm with a random sequence of actions $U_s^0$.
(2) Forward simulation of the system forced by the sequence $U_s^i = [u_s^i(k), u_s^i(k+1), \ldots, u_s^i(k+t_f)]$ where $u_s^i(j)$ denotes the action applied to the simulated system at time $j$. The system behavior is predicted using the model identified by the local learning method.
(3) Formulation of the nonlinear system in the time varying form. The parameters $A_j, B_j, K_j$ with
$j = k, \ldots, k+t_f$ are returned by the local model identification.
(4) Backward resolution of the discrete-time Riccati equation (6) for the resulting time varying system.
(5) Computation of the cost function (3).
(6) Computation of the gradient vector $\frac{\partial J}{\partial U_s^i} = [\frac{\partial J}{\partial u_s^i(k)}, \frac{\partial J}{\partial u_s^i(k+1)}, \ldots, \frac{\partial J}{\partial u_s^i(t_f)}]$ by using formula (8).
(7) Updating of the control sequence $U_s^i \to U_s^{i+1}$. The optimization step is performed by a constrained gradient based algorithm implemented in the Matlab function `constr` (Grace, 1994).
(8) If the minimum has been reached ($U_s^i = U_s^{i+1}$) goto 9 else goto 2.
(9) Control action execution. The first action $u_s^{opt}(k)$ of the sequence $U_s^{opt}$ is applied to the real system.

Fig. 1. The lazy learning optimal control algorithm

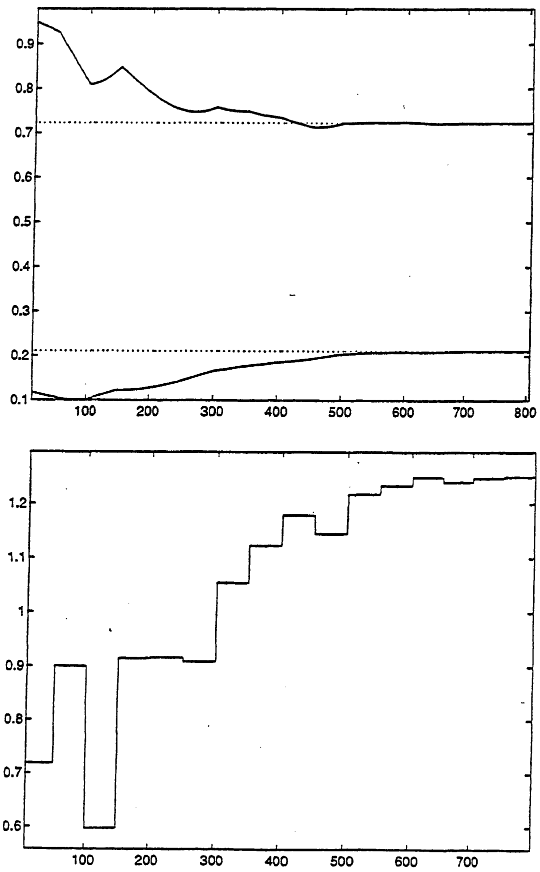


Fig. 2. Control results a) controlled variables (solid) and references (dotted) b) control action. (The x-axis unit corresponds to 0.01 seconds)

of the method to a simulated control problem was presented. Future developments will concern the combination of the local modeling technique with other certainty equivalence controllers (e.g. minimum variance controller, pole placement) and the extension of the method to stochastic dual control.

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