

Université Libre de Bruxelles

*Institut de Recherches Interdisciplinaires
et de Développements en Intelligence Artificielle*

Self-Organized Flocking with a Heterogeneous Mobile Robot Swarm

A. Stranieri, E. Ferrante, A. E. Turgut, V. Trianni, C.
Pincioli, M. Birattari and M. Dorigo
Université Libre de Bruxelles

IRIDIA – Technical Report Series

Technical Report No.
TR/IRIDIA/2011-012

April 2011
Last revision: April 2011

IRIDIA – Technical Report Series
ISSN 1781-3794

Published by:

IRIDIA, *Institut de Recherches Interdisciplinaires
et de Développements en Intelligence Artificielle*
UNIVERSITÉ LIBRE DE BRUXELLES
Av F. D. Roosevelt 50, CP 194/6
1050 Bruxelles, Belgium

Technical report number TR/IRIDIA/2011-012

Revision history:

TR/IRIDIA/2011-012.001 April 2011

The information provided is the sole responsibility of the authors and does not necessarily reflect the opinion of the members of IRIDIA. The authors take full responsibility for any copyright breaches that may result from publication of this paper in the IRIDIA – Technical Report Series. IRIDIA is not responsible for any use that might be made of data appearing in this publication.

Self-Organized Flocking with a Heterogeneous Mobile Robot Swarm

Alessandro Stranieri
Vito Trianni

Eliseo Ferrante
Carlo Pinciroli
Marco Dorigo

Ali Emre Turgut
Mauro Birattari

Université Libre de Bruxelles, 1050, Belgium
{astranie, eferrante, aturgut, vtrianni, cpinciro, mbiro, mdorigo}@ulb.ac.be

April 28, 2011

Abstract

In this paper, we study self-organized flocking in a swarm of behaviorally heterogeneous mobile robots: aligning and non-aligning robots. Aligning robots are capable of agreeing on a common heading direction with other neighboring aligning robots. Conversely, non-aligning robots lack this capability. Studying this type of heterogeneity in self-organized flocking is important as it can support the design of a swarm with minimal hardware requirements. Through systematic simulations, we show that a heterogeneous group of aligning and non-aligning robots can achieve good performance in flocking behavior. We further show that the performance is affected not only by the proportion of aligning robots, but also by the way they integrate information about their neighbors and also by heterogeneity in the way robots move.

1 INTRODUCTION

Flocking is the cohesive and aligned motion of a group of individuals along a common direction. All studies about flocking within computer science and robotics root back to the seminal work of Reynolds (1987). He was the first to simulate flocking of birds based on three behaviors: *separation* — individuals try to keep a minimum distance between their neighbors, *cohesion* — individuals try to stay together with their neighbors, and *alignment* — individuals try to match their velocities to the average speed of their neighbors. The vast majority of the studies about flocking assume that all the robots in the swarm are behaviorally identical and exploit the three behaviors described above.

In this paper, we consider flocking in a behaviorally heterogeneous swarm of robots. All robots in the swarm use the separation and the cohesion behavioral rule. However, only a fraction of the robots, which we call the *aligning* robots, uses the alignment behavior. The rest of the robots, which we call the *non-aligning* robots, do not use the alignment behavior.

We believe that studying heterogeneity in alignment in self-organized flocking is very important from the practical point of view. In fact, the alignment behavior is more demanding in terms of robotics hardware than the separation and cohesion behaviors. In fact, the alignment behavior requires special hardware able to detect the orientation of neighboring robots or, as we do in this paper, a communication device. Therefore, understanding if a swarm can achieve flocking with only a few aligning robots can support the design of swarms with minimal hardware requirements.

We conduct simulation-based experiments and we measure self-organized flocking performance in terms of the degree of group order, group cohesiveness and average group speed. With respect to these criteria, we found that the swarm achieves good flocking performance when the proportion

of aligning robots is high. Conversely, this performance decreases as the proportion gets lower. To tackle this problem, we propose a new model of robot motion. In the new model, non-aligning robots modulate their forward speed, instead of moving at a fixed forward speed as the other robots.

The rest of the paper is organized as follows: we first present related works in flocking, starting from studies in biology and then in robotics. We then introduce our heterogeneous flocking model. After, we introduce the robots and we explain how we implement flocking on the robots. Subsequently, we describe the experimental setup, metrics and the results. Finally, we conclude the paper and propose future directions of research.

2 RELATED WORK

Flocking is a widely observed phenomenon in social animals (Camazine et al., 2001) such as locusts (Buhl et al., 2006), birds (Ballerini et al., 2007) or human beings (Dyer et al., 2008). Animal groups show a great diversity in their population due to the differences in age, morphology (Krause et al., 1998), nutritional state (Krause, 1993), personality (Michelena et al., 2010), and leadership status (Reebs, 2000) of the individuals. This diversity mainly results in behavioral differences among the individuals. Couzin et al. (2002) showed that behavioral differences between the individuals in a group change both the dynamics and the organization of the group. Subsequently, Couzin et al. (2005) conducted a seminal study about leadership in animal groups. They modeled a heterogeneous group of individuals of which only a few are aware of a target direction. They showed that the few informed individuals are able to move the whole group along the target direction.

In robotics, most of the studies about flocking assume a homogeneous set of behaviorally equivalent individuals. One of the earliest studies in robotics was performed by Mataric (1994). She devised a set of “basis behaviors” to implement flocking in a group of robots: safe-wandering, aggregation, dispersion and homing. With the proposed set of behaviors, robots are able to move cohesively towards a homing direction. Kelly and Keating (1996), following a behavior-based approach, designed a leader-following behavior to realize flocking. Hayes and Dormiani-Tabatabaei (2002) proposed a flocking behavior having collision avoidance and alignment behaviors based on local range and bearing measurements. Spears et al. (2004) proposed a framework based on artificial physics. The robots were able to form a regular lattice structure using attraction/repulsion virtual forces and move along a direction indicated by a light source in the environment. Holland et al. (2005) proposed a flocking behavior for unmanned ground vehicles based on separation, cohesion and alignment behaviors. Turgut et al. (2008) proposed a flocking behavior based on separation/cohesion and heading alignment behaviors. They implemented this behavior in robots with limited sensing capabilities and conducted a systematic study on the effect of sensing noise in heading measurement on flocking. In a recent study, Moslinger et al. (2009) proposed a flocking behavior for robots with limited sensing capabilities. It is based on only attraction and repulsion behaviors. By adjusting the sizes of attraction and repulsion zones, they achieved flocking for a small group in a constrained environment.

Other works in robotics considered a group of behaviorally heterogeneous robots. Momen et al. (2007) studied flocking with a heterogeneous robotic swarm inspired by mixed-species foraging flocks of birds (Graves and Gotelli, 1993). Using simulations, they showed some aspects of mixed-species flocking, such as behavioral differences in their attraction and repulsion rules. Çelikkanat and Şahin (2010), inspired by Couzin et al. (2005) extended the flocking behavior proposed by Turgut et al. (2008) and created a heterogeneous robot swarm by informing some of the robots about a target direction. Recently, in another follow-up study, Ferrante et al. (2010) introduced a new communication strategy to improve flocking performance in case of both static and changing target directions.

To the best of our knowledge, most of the studies in swarm robotics about self-organized flocking have not considered diversity in alignment.

3 METHOD

We follow a design method based on artificial physics framework introduced by [Spears et al. \(2004\)](#). According to this method, robots exert virtual forces on each other. The swarm consists of aligning and non-aligning robots. Aligning robots are subject to the following virtual forces

$$\mathbf{f} = \alpha_1 \mathbf{p} + \beta_1 \mathbf{h},$$

whereas for the non-aligning robots the virtual force is computed as

$$\mathbf{f} = \alpha_2 \mathbf{p}.$$

We define \mathbf{p} as the *proximal control vector* and \mathbf{h} as the *alignment control vector*. The proximal control vector \mathbf{p} accounts for attraction and repulsion rules for keeping the robot together with its neighbors and to avoid collisions.

The alignment control vector \mathbf{h} is used to make the aligning robots match the average heading direction of its neighboring aligning robots. The parameters α_1 , β_1 and α_2 are used to adjust the contribution of the corresponding vectors.

3.1 Proximal control

Let m_p denote the number of neighbors of a robot within a range D_p . Let also d_i and ϕ_i denote the relative range and bearing of the i^{th} neighbor, respectively. The proximal control vector \mathbf{p} is given by:

$$\mathbf{p} = \sum_{i=1}^{m_p} p_i(d_i) e^{j\phi_i}.$$

p_i is calculated as a function of d_i using a force function derived from the Lennard-Jones potential function, which results in the formation of regular structures as shown in [Hettiarachchi and Spears \(2009\)](#):

$$p_i(d_i) = 12\epsilon \left[\frac{d_{des}^{12}}{d_i^{13}} - \frac{d_{des}^6}{d_i^7} \right].$$

The parameter ϵ determines the strength of the attractive and repulsive force, and d_{des} is the desired distance between the robots.

3.2 Alignment control

Let θ_0 denote the orientation of a given robot. Furthermore, let m_a denote the number of aligning robots within the range D_a of this robot, and $\theta_i, i \in \{1, \dots, m_a\}$ their orientation. All orientations are expressed in the body-fixed reference frame of the robot under consideration¹. The robot calculates the alignment control vector, that is, the average orientation of the m_a robots, including its own:

$$\mathbf{h} = \frac{\sum_{i=0}^{m_a} e^{j\theta_i}}{\left\| \sum_{i=0}^{m_a} e^{j\theta_i} \right\|},$$

where $\| \cdot \|$ denotes the norm of a vector.

¹In our study, we define two reference frames, both of which use the right-hand convention. One is the reference frame common to all of the robots, which is available due to the light source. The other is the body-fixed reference frame specific to each robot. The body-fixed reference frame is fixed to the center of a robot: its x -axis points to the front of the robot and its y -axis is coincident with the rotation axis of the wheels.

3.3 Motion control

We present two motion control rules. The two rules differ in the way the forward speed u and the angular speed ω are determined. The first rule is denoted as *constant forward speed motion control* (henceforth CMC). In CMC, robots are always moving at a constant forward speed, but can change their angular speed. According to the second rule, denoted as *variable forward speed motion control* (henceforth VMC), robots move not only at a variable angular speed but also at a variable forward speed.

CMC: The forward speed is kept constant at

$$u = U.$$

The angular speed is proportional to the angular component of the total force \mathbf{f} . Hence, it ignores the magnitude $\|\mathbf{f}\|$ of the force:

$$\omega = K \angle \mathbf{f}.$$

VMC: First, let $\mathbf{f}_x = \|\mathbf{f}\| \cos(\angle \mathbf{f})$ and $\mathbf{f}_y = \|\mathbf{f}\| \sin(\angle \mathbf{f})$ denote the projection of the total force \mathbf{f} on the x -axis and y -axis of the robot body-fixed reference frame respectively. Accordingly, the forward speed u is directly proportional to the x component of the total force and the angular speed ω is directly proportional to the y component of the force. Hence:

$$\begin{aligned} u &= K_1 \mathbf{f}_x \\ \omega &= K_2 \mathbf{f}_y. \end{aligned}$$

In our experiments, we study both the *homogeneous motion* and the *heterogeneous motion* case. In the homogeneous motion case, all robots share the same motion control rule, that is, CMC. In the heterogeneous motion case, aligning robots use CMC, whereas non-aligning robots use VMC.

4 FLOCKING WITH ROBOTS

In this study, the swarm is composed of simulated versions of the foot-bot robot developed by [Bonani et al. \(2010\)](#). The foot-bot is a differentially-driven mobile robot with the following sensors and actuators: i) A light sensor used to measure the orientation of robot (θ_0) with respect to a light source present in the environment perceived by all robots. ii) A range and bearing sensing and communication device (henceforth called RAB), with which a robot can communicate with its neighbors and perceive their range and bearing measurements ([Roberts et al., 2009](#)). iii) Two wheels actuators, that are used to control independently the left and right wheels speed of the robot.

To achieve proximal control with the foot-bot the RAB is used for measuring the relative range and bearing d_i and ϕ_i of the i_{th} neighbor. For achieving alignment control, we use communication to simulate orientation sensing as in [Turgut et al. \(2008\)](#). In particular, each aligning robot sends its orientation, expressed in the global reference frame, using the communication unit present in the RAB. At the same time, it receives the orientation θ_i of its i^{th} neighboring aligning robot. It transforms this angle into its body-fixed reference frame. In this way, we are able to simulate a robot sensing the orientation of its neighboring aligning robots.

To achieve motion control, we first limit the forward speed within $[0, U_{max}]$, and the angular speed within $[-\Omega_{max}, \Omega_{max}]$. We then use the differential drive model used in [Turgut et al. \(2008\)](#) to convert the forward speed u and the angular speed ω into the linear speeds of the left (N_L) and right (N_R) wheel:

$$N_L = \left(u + \frac{\omega}{2} l \right),$$

$$N_R = \left(u - \frac{\omega}{2} l \right),$$

where l is the distance between the wheels.

The values of the constants, which we used in our experiments are given in Table 1.

5 EXPERIMENTS

We execute simulation-based experiments with a swarm of foot-bots using the ARGoS simulator (Pinciroli et al., 2011), which is an open-source², plug-in based, multi-physics engine simulator.

5.1 Experimental setup

At the beginning of each experiment, N mobile robots are randomly placed (position and orientation-wise) with a proportion $\rho \in [0, 1]$ of aligning robots. The density of robots is kept fixed and equal to 6 robots per square meter on a square shaped area. A light source is placed at a fixed position in the arena, far away from the swarm, to provide the common reference frame.

In the experiments, noise is added to the orientation measurement and the angle of the proximal control vector. Noise is modeled as a uniformly distributed random variable within the range $[-\sigma\pi, \sigma\pi]$.

We conduct experiments considering the two different models of motion control.

Homogeneous motion: In this case, all robots use CMC. Here, we study the effect of the ratio $\frac{\beta_1}{\alpha_1}$, and we do not change α_1 and β_1 independently, since CMC does not utilize the magnitude of \mathbf{f} , but only its angular component. As such, multiplying both α_1 and β_1 with the same constant value will produce no difference in the robot motion. For the same reason, α_2 does not effect the robot motion.

Heterogeneous motion: In this case, aligning robots use CMC whereas non-aligning robots are using VMC. For the non-aligning robots, the magnitude of \mathbf{f} plays a role in their motion. Thus, additionally to the effect of changing $\frac{\beta_1}{\alpha_1}$ of the aligning robots, we study the effect of changing α_2 of the non-aligning robots.

We show the results in heterogeneous self-organized flocking with medium ($N = 25$) and large ($N = 100$) swarm sizes and with low ($\rho = 0.4$) and high ($\rho = 0.8$) proportion of aligning robots. We study the effect of changing the ratio $\frac{\beta_1}{\alpha_1} \in \{1, 2, 4, 6, 8, 10\}$ and, for the heterogeneous case, we also study the effect of changing $\alpha_2 \in \{1, 2, 4, 6, 8, 10\}$, but we report here only the results obtained with the best case, that is, $\alpha_2 = 10$ (refer to Stranieri et al. (2011) for the complete set of results). For each experimental setting, we execute R runs and report average and standard deviation of the results. The duration of one run is T simulated seconds.

We study how the heterogeneous flocking performance is influenced by: i) the way robots implement their motion (homogeneous motion versus heterogeneous motion), ii) the parameters that affect the strength of the proximal control vector and of the alignment control vector, that is, $\frac{\beta_1}{\alpha_1}$ and α_2 and iii) the ratio of aligning robots ρ .

5.2 Metrics

In this study, we are interested in having a swarm of robots that move cohesively as single group. Furthermore, the swarm should be aligned towards the same direction and move towards it as fast as possible. We use three metrics to measure the degree of attainment of these objectives: order, group cohesion and rescaled group speed.

²<http://iridia.ulb.ac.be/argos/>

Variable	Description	Value(s) / Range
N	Number of robots	{25, 100}
ρ	Prop. of aligning robots	{0.4, 0.8}
β_1/α_1	Alig. robots parameters	{1, 2, 4, 6, 8, 10}
α_2	Non alig. robots parameter	{1, 2, 4, 6, 8, 10}
U	Maximum forward speed	1.5 cm/s
K	CMC angular gain	0.5 1/s
K_1	VMC linear gain	0.25 s/kg
K_2	VMC angular gain	0.1 s/(kg · m)
l	Inter-wheel distance	0.1 m
U_{max}	VMC max forward speed	20 cm/s
Ω_{max}	VMC max angular speed	$\pi/2$ rad/s
ϵ	Strength of pot. function	0.5
d_{des}	Inter-robot distance	0.6 m
σ	Amount of noise	0.1
T	Experiment duration	600 secs

Table 1: Experimental values or range of values for all constants and variables

Order: The order metric ψ measures the angular order of the robots (Vicsek et al., 1995), $\psi \approx 1$ when the group shares a common heading and $\psi \ll 1$ when each robot is pointing in a different direction. The order is defined as:

$$\psi = \frac{1}{N} \left\| \sum_{i=1}^N e^{j\theta_i} \right\|.$$

Group cohesion: To measure group cohesion ξ , we determine the number of groups g present at the end of each experiment (Couzin et al., 2005). Group cohesion is computed as:

$$\xi = 2 - \min(2, g).$$

Rescaled Group speed: We calculate the average group speed as:

$$s = \left\| \frac{\mathbf{c}_T - \mathbf{c}_0}{T} \right\|,$$

where \mathbf{c}_T and \mathbf{c}_0 are the position of the center of mass of the swarm at the end and at the beginning of the experiment, respectively. We then rescale the average group speed:

$$s_r = \frac{s}{U},$$

where U is the maximum forward speed of CMC.

5.3 Results with homogeneous motion

The experimental results for homogeneous motion are depicted in Figure 1. We first focus on the $\rho = 0.8$ case, for both $N = 25$ (Figure 1a) and $N = 100$ (Figure 1b). Results show that the swarm is cohesive in most runs. However, order and speed are high only when $\frac{\beta_1}{\alpha_1} \geq 2$. Furthermore, while order is high at different values of the ratio $\frac{\beta_1}{\alpha_1}$, speed increases with increasing values of $\frac{\beta_1}{\alpha_1}$, until it saturates at around $\frac{\beta_1}{\alpha_1} = 6$. This shows that, when the alignment control vector is higher, robots tend to move faster. This is explained by the fact that the alignment control vector is more stable, over time, than with respect to the proximal control vector. Thus, the higher the weight

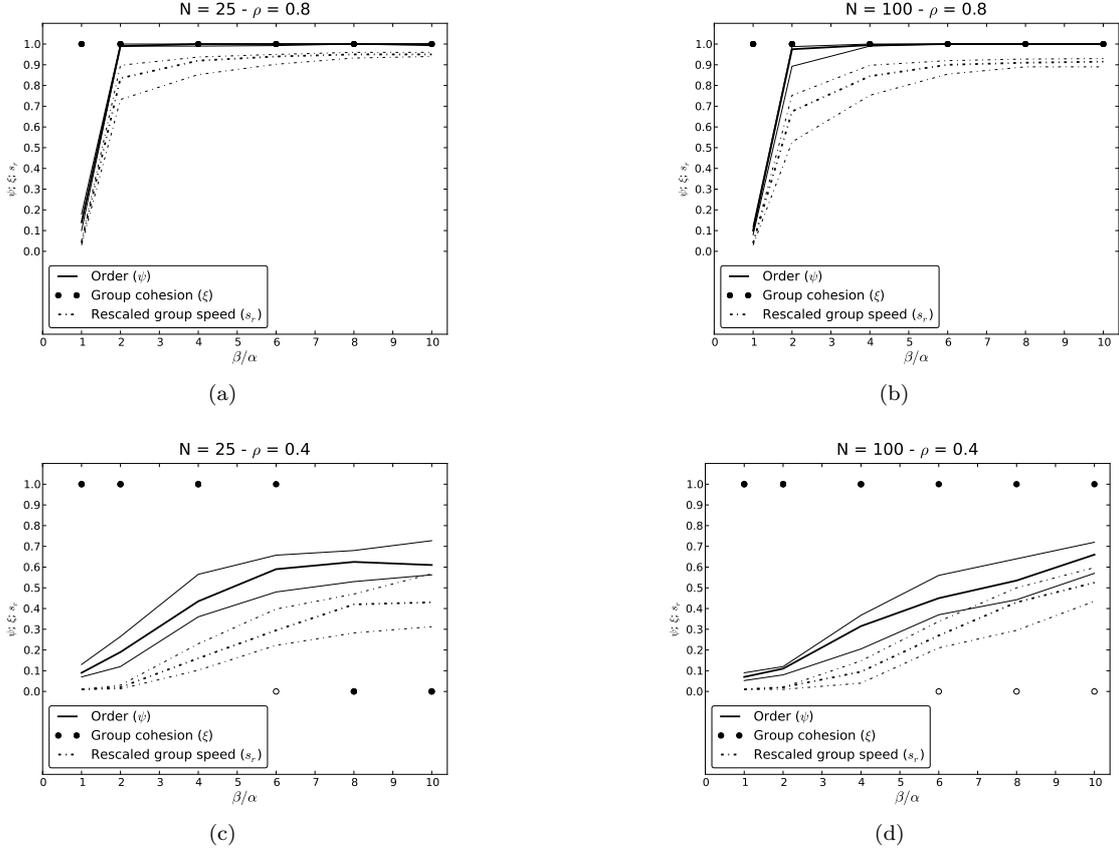


Figure 1: Homogeneous motion experiments for varying swarm size ($N \in \{25, 100\}$) and ratio of aligning robots ($\rho \in \{0.4, 0.8\}$). Thick lines show the median values, whereas thin lines show the 25% and the 75% percentile score of the data. For group cohesion, filled circles correspond to median values and empty circles to the 25% percentile score of the data.

of the alignment control vector, the more the robots tends to move forward rather than to turn. This allows the swarm to move faster, until speed saturates at the maximum forward speed U_{max} .

When the proportion of aligning robots is $\rho = 0.4$, performance gets sensibly worse (Figures 1c and 1d). In both cases ($N = 25$ and $N = 100$), we observe two possible outcomes: for small values of the ratio $\frac{\beta_1}{\alpha_1}$, the swarm remains cohesive, but does not move. This happens because the relative contribution of the alignment control vector is not enough for the aligning robots to pull the entire swarm towards the agreed goal direction. For larger values of the ratio $\frac{\beta_1}{\alpha_1}$, group speed and order get higher. However, in at least 25% of the runs, the swarm has split. This happens because, in those runs, clusters of non-aligning robots are present. Since the motion of these robots is governed only by the proximal control vector, they are not able to match the higher speed of the aligning robots since they tend to turn more rather than to move forward, thus they remain disconnected from the group.

5.4 Results with heterogeneous motion

In the heterogeneous motion case, results with $\rho = 0.8$ (Figures 2a and 2b), are similar to the results obtained, with the same ratio, in the homogeneous motion case. The advantage of employing heterogeneous motion is more evident with $\rho = 0.4$ (Figures 2c and 2d). With both swarm sizes we have that, when $\frac{\beta_1}{\alpha_1} > 2$, the swarm is able to effectively flock together at the cost of a reduced

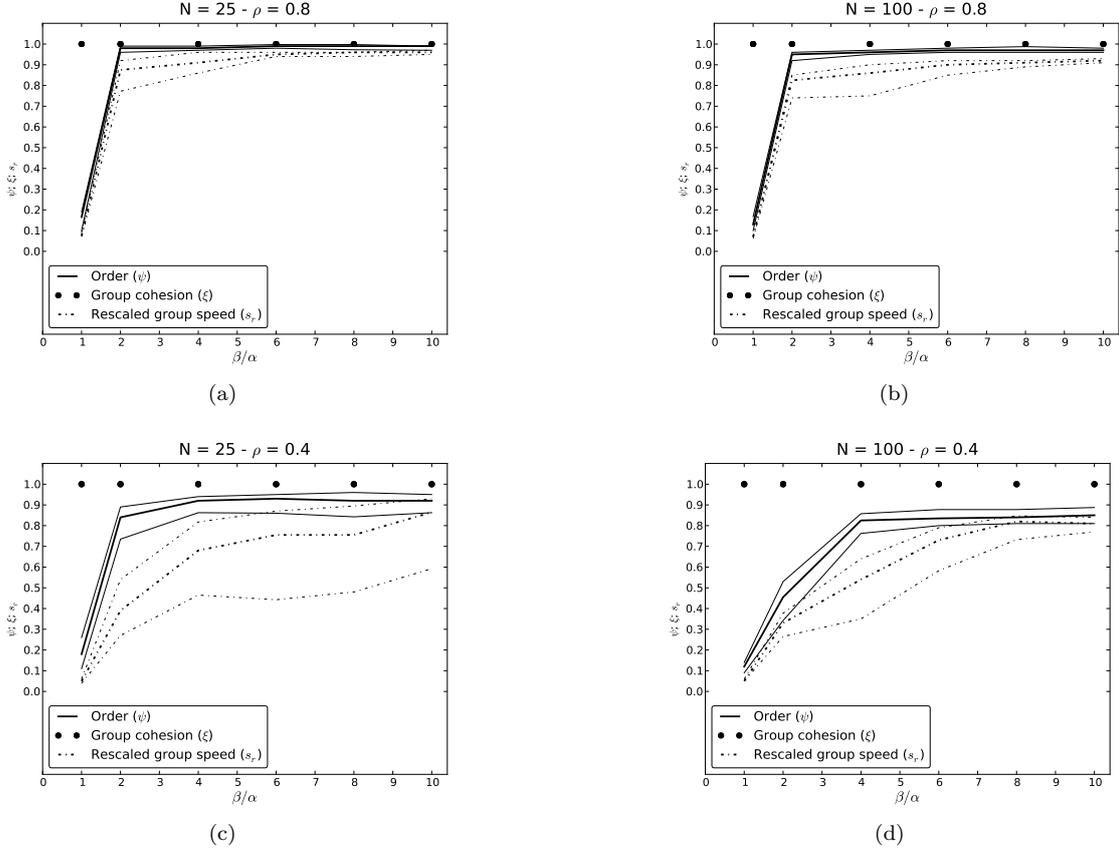


Figure 2: Heterogeneous motion experiments for varying swarm size ($N \in \{25, 100\}$) and ratio of aligning robots ($\rho \in \{0.4, 0.8\}$). Thick lines show the median values, whereas thin lines show the 25% and the 75% percentile score of the data. For group cohesion, filled circles correspond to median values and empty circles to the 25% percentile score of the data.

speed.

The improved capability of the swarm to stay together is due to the advantage of using VMC in the non-aligning robots. In fact, non-aligning robots are able to respond to the high variations in the proximal control vector much more when they can also change their forward speed. As such, they are also able to stay together with the aligning robots, both when they are alone and when they are in small or big clusters. Finally, the reduced speed and the high variation of speed among runs is due to the following fact. In presence of a low proportion of aligning robots, we observed that the group heading direction is stable over short periods of time but changes over long periods of time due to the disturbances caused by the non-aligning robots. This results in a non-linear trajectory executed by the entire swarm, which is different for each run. Since the rescaled group speed is computed assuming a linear trajectory, this measurement has large variation in the total displacement changes from run to run.

6 CONCLUSIONS AND FUTURE WORKS

In this paper, we studied self-organized flocking in a swarm composed of behaviorally heterogeneous mobile robots. The swarm is composed of aligning robots, which are able to agree on a common heading direction, and non-aligning robots which lack this capability. We furthermore propose

a new model for achieving motion in self-organized flocking. According to this model, aligning robots only change their angular speed, whereas non-aligning robots change both their forward and their angular speed.

We study the performance in terms of group alignment order, cohesiveness and speed. Results show that self-organized flocking is also possible when some individuals in the swarm lack the capability to agree on a common direction. More in particular, we showed that: i) higher proportion of aligning robots always correspond to better performance; ii) performance are affected by the relative contribution of alignment and proximal control and iii) when the proportion of aligning robots is small, flocking is possible only when the non-aligning robots also change their forward speed .

Possible directions for future work are the following: First, we plan to study energy efficiency within the same framework of study. In particular, the use of a heterogeneous group of aligning and non-aligning robots poses a trade-off between efficiency of the motion and energy utilized for communication. Second, we would like to study the correlation between spatial aspects of the swarm composition. In particular, we would like to study whether particular configurations (i.e. topology, connectivity, ...) have different effects on the flocking performance. Third, we plan to propose an algorithm for letting the robot decide when to employ alignment control, according to their local environment. Also for this study, we will need to take into account energy consumption.

7 ACKNOWLEDGMENTS

This work was partially supported by the European Union through the ERC Advanced Grant "E-SWARM: Engineering Swarm Intelligence Systems" (contract 246939), and by the Meta-X project, funded by the Scientific Research Directorate of the French Community of Belgium. Carlo Pinciroli acknowledges support from ASCENS, a project funded by the Future and Emerging Technologies programme of the European Commission. Mauro Birattari, and Marco Dorigo acknowledge support from the F.R.S.-FNRS of Belgium's French Community, of which they are a Research Associate and a Research Director, respectively. The information provided is the sole responsibility of the authors and does not reflect the European Commission's opinion. The European Commission is not responsible for any use that might be made of data appearing in this publication.

References

- Ballerini, M., Cabibbo, N., Candelier, R., Cavagna, A., Cisbani, E., Giardina, I., V.Lecomte, Orlandi, A., Parisi, G., Procaccini, A., Viale, M., and Zdravkovic, V. (2007). Interaction ruling animal collective behavior depends on topological rather than metric distance: Evidence from a field study. *Proceedings of the National Academy of Sciences of the United States of America*, 105(4):1232–1237.
- Bonani, M., Longchamp, V., Magnenat, S., Rétornaz, P., Burnier, D., Roulet, G., Vaussard, F., Bleuler, H., and Mondada, F. (2010). The MarXbot, a Miniature Mobile Robot Opening new Perspectives for the Collective-robotic Research. In *2010 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS 2010)*, pages 4187–4193, Piscataway, NJ. IEEE Press.
- Buhl, J., Sumpter, D. J. T., Couzin, I., Hale, J., Despland, E., Miller, E., and Simpson, S. J. (2006). From disorder to order in marching locusts. *Science*, (312):1402–1406.
- Camazine, S., Franks, N., Sneyd, J., Bonabeau, E., Deneubourg, J.-L., and Theraulaz, G. (2001). *Self-Organization in Biological Systems*. Princeton University Press, Princeton, NJ.
- Çelikkanat, H. and Şahin, E. (2010). Steering self-organized robot flocks through externally guided individuals. *Neural Computing and Applications*, 19(6):849–865.

- Couzin, I. D., Krause, J., Franks, N. R., and Levin, S. A. (2005). Effective leadership and decision-making in animal groups on the move. *Nature*, 433:513–516.
- Couzin, I. D., Krause, J., James, R., Ruxton, G. D., and Franks, N. R. (2002). Collective memory and spatial sorting in animal groups. *Journal of theoretical biology*, 218:1–11.
- Dyer, J. R. G., Ioannou, C. C., Morrell, L. J., Croft, D. P., Couzin, I. D., A., W. D., and Krause, J. (2008). Consensus decision making in human crowds. *Animal Behavior*, 75(2):461–470.
- Ferrante, E., Turgut, A. E., Mathews, N., Birattari, M., and Dorigo, M. (2010). Flocking in stationary and non-stationary environments: A novel communication strategy for heading alignment. In Schaefer, R., Cotta, C., Kołodziej, J., and Rudolph, G., editors, *Parallel Problem Solving from Nature – PPSN XI*, volume 6239 of *Lecture Notes in Computer Science*, pages 331–340. Springer.
- Graves, G. R. and Gotelli, N. J. (1993). Assembly of avian mixed-species flocks in amazonia. *Proceedings of the National Academy of Sciences of the United States of America*, 90(4):1388–1391.
- Hayes, A. and Dormiani-Tabatabaei, P. (2002). Self-organized flocking with agent failure: Off-line optimization and demonstration with real robots. In *Proc. of the IEEE Int. Conf. on Robotics and Automation*, pages 3900–3905, Piscataway, NJ. IEEE Press.
- Hettiarachchi, S. and Spears, W. (2009). Distributed adaptive swarm for obstacle avoidance. *International Journal of Intelligent Computing and Cybernetics*, 2(4):644–671.
- Holland, O., Woods, J., Nardi, R., and Clark, A. (2005). Beyond swarm intelligence: the ultra-swarm. In *Proc. of the IEEE Swarm Intelligence Symposium*, pages 217–224, Piscataway, NJ. IEEE Press.
- Kelly, I. and Keating, D. (1996). Flocking by the fusion of sonar and active infrared sensors on physical autonomous robots. In *Proc. of the Third Int. Conf. on Mechatronics and Machine Vision in Practice*, pages 14–17.
- Krause, J. (1993). The relationship between foraging and shoal position in a mixed shoal of roach (*rutilus rutilus*) and chub (*leuciscus cephalus*): a field study. *Oecologia*, 93:356–359.
- Krause, J., Reeves, P., and Hoare, D. (1998). Positioning behaviour in roach shoals : The role of body length and nutritional state. *Behavior*, 135(8-9):1031–1039.
- Matarić, M. J. (1994). *Interaction and Intelligent Behavior*. PhD thesis, MIT, MA.
- Michelena, P., Jeanson, R., Deneubourg, J.-L., and Sibbald, A. M. (2010). Personality and collective decision-making in foraging herbivores. *Proceedings of the Royal Society B*, 277:1093–1099.
- Momen, S., Amavasai, B. P., and Siddique, N. H. (2007). Mixed species flocking for heterogeneous robotic swarms. In *The International Conference on Computer as a Tool (EUROCON 2007)*, pages 2329 – 2336, Piscataway, NJ. IEEE Press.
- Moslinger, C., Schmickl, T., and Crailsheim, K. (2009). A minimalistic flocking algorithm for swarm robots. In *European Conference of Artificial Life (ECAL 2009)*.
- Pinciroli, C., Trianni, V., O’Grady, R., Pini, G., Brutschy, A., Brambilla, M., Mathews, N., Ferrante, E., Caro, G. D., Ducatelle, F., Stirling, T., Gutiérrez, A., Gambardella, L. M., and Dorigo, M. (2011). ARGoS: a modular, multi-engine simulator for heterogeneous swarm robotics. Technical Report TR/IRIDIA/2011-009, IRIDIA, Université Libre de Bruxelles, Brussels, Belgium.
- Reebs, S. G. (2000). Can a minority of informed leaders determine the foraging movements of a fish shoal. *Animal Behavior*, 59:403–409.

- Reynolds, C. (1987). Flocks, herds and schools: A distributed behavioral model. In Stone, M. C., editor, *SIGGRAPH '87: Proc. of the 14th annual conference on computer graphics and interactive techniques*, pages 25–34, New York. ACM Press.
- Roberts, J., Stirling, T., Zufferey, J., and Floreano, D. (2009). 2.5d infrared range and bearing system for collective robotics. In Papanikolopoulos, N., Sugano, S., Chiaverini, S., and Meng, M., editors, *IEEE/RSJ International Conference on Intelligent Robots and Systems*, Piscataway, NJ. IEEE Press.
- Spears, W. M., Spears, D. F., Hamann, J. C., and Heil, R. (2004). Distributed, physics-based control of swarms of vehicles. *Autonomous Robots*, 17:137–162.
- Stranieri, A., Ferrante, E., Turgut, A. E., Trianni, V., Pinciroli, C., Birattari, M., and Dorigo, M. (2011). Oself-organized flocking with a heterogeneous mobile robot swarm. Supplementary information page at <http://iridia.ulb.ac.be/supp/IridiaSupp2011-011/>.
- Turgut, A. E., Çelikkanat, H., Gökçe, F., and Şahin, E. (2008). Self-organized flocking in mobile robot swarms. *Swarm Intelligence*, 2(2):97–120.
- Vicsek, T., Czirok, A., Ben-Jacob, E., Cohen, I., and Shochet, O. (1995). Novel type of phase transition in a system of self-driven particles. *Physical Review Letters*, 75(6).