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A Note on the Effects of Enforcing Bound Constraints on Algorithm Comparisons using the IEEE CEC'05 Benchmark Function Suite

Tianjun Liao tliao@ulb.ac.be
Marco A. Montes de Oca mmontes@ulb.ac.be
Thomas Stützle stuetzle@ulb.ac.be

IRIDIA, CoDE, Université Libre de Bruxelles, Brussels, Belgium.

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Abstract

The functions proposed in the special session on real parameter optimization of the 2005 IEEE Congress on Evolutionary Computation (CEC'05) are playing a pivotal role in evaluation of the performance of continuous optimizers both for algorithm comparisons as well we for assessing the state of the art. However, the fact whether bound constraints are enforced for the final solutions returned by a continuous optimizer, can play a decisive role in this task. In this technical note we show that CMA-ES, the winner of the 2005 competition, is strongly impacted by the fact whether bound constraints are enforced or not. Our experimental results also indicate that claims about superior performanc over CMA-ES may be flawed due to possible problems with the enforcement of bound constraints.

1 Introduction

The special session on real parameter optimization of the 2005 IEEE Congress on Evolutionary Computation (CEC'05) has played an important role in evolutionary computation and other affine fields for two reasons. First, it provided a set of 25 benchmark functions that anyone can use to evaluate and compare the performance of new algorithms, standardizing, as a result, the data set researchers use to compare algorithms. Second, it served to assess the state of the art in continuous optimization. In particular, the winner of the 2005 competition, G-CMA-ES [2], is since then considered to be a representative of the state of the art in the field. Thus, it is reasonable to consider an algorithm that outperforms G-CMA-ES on the CEC'05 benchmark function suite to be state of the art in the field.

After six years that the aforementioned session was organized, it is not surprising that a number of authors have reported that their proposed algorithms outperform G-CMA-ES on (normally a subset of) the 25 benchmark functions designed for that session [4,6]. In this note, we want to call the attention of the community to the fact that at least some of these positive reports may in fact not be valid.

The source of the problem is the uncertainty surrounding how bound constraints should be handled in most of the special session's 25 benchmark functions. In the technical report describing the experimental protocol to follow, the authors write:

"All problems, except 7 and 25, have the global optimum within the given bounds and there is no need to perform search outside of the given bounds for these problems. 7 & 25 are exceptions without a search range and with the global optimum outside of the specified initialization range." [7] (p. 40).

Although for all functions (except 7 and 25) in their definition it is stated that the solution vector x is in each dimension an element of some subset of reals (by stating that $x \in [x_{\min}, x_{\max}]$, where $x_{\min} < x_{\max}$ are some constants), some authors possibly interpret these lines simply as a suggestion. As a result, it is unclear in many papers whether authors enforce bound constraints or not. In this note, we show data that demonstrate that a comparison of two algorithms in which one of them enforces bound constraints and the other does not, can lead to wrong conclusions.

In light of our experimental results, we advocate for an explicit statement in each paper that bound constraints are enforced; in addition, for each paper either in the paper or in a supplementary page to the paper, the explicit solution vectors should be reported to avoid misinterpretations. Given that the particular bound handling technique may have a significant impact on an algorithm's performance [2,3,5,8], in each article or its supplementary page, the bound handling mechanism used should be explicitly described. Note that it is perfectly valid to not enforce bound constraints, as long as all the compared algorithms do not enforce them. Without this basic piece of information, the resulting comparisons and the conclusions drawn from them are dubious at best.

2 Experimental Setup

We used the C version of G-CMA-ES from Hansen's website [1]. We followed the protocol described in [7], that is, we ran G-CMA-ES 25 times on each function and recorded the evolution of the objective function value with respect to the number of function evaluations used. The maximum number of function evaluations was $10000D$ where $D \in \{10, 30, 50\}$ is the dimensionality of a function. The algorithm was run until the maximum number of function evaluations were used.

To show the effect of enforcing bound constraints, we ran two version of G-CMA-ES, one in which bound constraints are enforced using a clamped bound handling [1] (G-CMA-ES-clampedbound), and one in which they are not (G-CMA-ES-nonbound).

To demonstrate that possibly wrong conclusions can be drawn if bound constraints are enforced in one algorithm but not in the other, we compare the results obtained with G-CMA-ES with those obtained with PS-CMA-ES [6].

[1] clamped-bound handling is a simple bound handling technique, which clamps all the generated outside-bound solutions on the nearest bound.

Incidentally, the authors of PS-CMA-ES conclude that their algorithm outperforms G-CMA-ES on IEEE CEC'05 benchmark functions. The results of PS-CMA-ES are the ones found in the original publication, in which the authors do not mention whether they enforce bound constraints or not. However, the data corresponding to G-CMA-ES were those reported by Auger and Hansen's paper [2], in which they enforced bound constraints.

3 Results

The results are analyzed along two dimensions. First, we look at the final objective function value obtained by each version of G-CMA-ES and by PS-CMA-ES. Then, we look at the evolution of the objective function value as a function of the number of function evaluations used. The whole set of solution quality development plots and the final solutions vectors of G-CMA-ES-clampedbound and G-CMA-ES-nonbound are reported in http://iridia.ulb.ac.be/supp/IridiaSupp2011-013/.

3.1 Average and Median Objective Function Values

Table 1 shows the performance of G-CMA-ES-clampedbound and G-CMA-ES-nonbound. In the clamped-bound column and the no-bound column, the mean and median errors obtained by G-CMA-ES-clampedbound and G-CMA-ES-nonbound are presented, respectively. The two-sided Wilcoxon matched-pairs signed-rank at 0.05 α-level was used for each function. We notice that in many functions, G-CMA-ES-nonbound obtains final solutions outside bounds. If this is the case, often all 25 independent solutions are outside the bounds, and only sometimes some of 25 independent solutions are outside bound. In a function where there is a significant difference between G-CMA-ES-clampedbound and G-CMA-ES-nonbound, almost all the solutions obtained by G-CMA-ES-nonbound are outside the bound. Moreover, some objective values that correspond to infeasible solutions are better than the objective values obtained by G-CMA-ES-clampedbound. With the increase of functions' dimensionality, the impact of ignoring bound constraints becomes also more serious. In particular, in 50 dimensional benchmark functions, G-CMA-ES-nonbound obtains in 74% of the functions infeasible solutions.

Table 2 shows the comparison of the distribution of average errors value between PS-CMA-ES [6], G-CMA-ES-nonbound and the published results of the original G-CMA-ES in CEC 2005. Notice that here we report values that are smaller than 1.00E−08 as 1.00E−08 because this was the limit defined in the CEC 2005 competition. There seems to be an interesting pattern, that is PS-CMA-ES and G-CMA-ES-nonbound are inferior to G-CMA-ES on 10 dimensions, but superior to G-CMA-ES on 30 and 50 dimensions. Taking the comparison between G-CMA-ES-nonbound and G-CMA-ES, for example, the infeasible solutions caused by ignoring bound constraints help a lot in the performance comparison. In the 50 dimensional case, G-CMA-ES-nonbound wins 13 times and loses 7 times; if we focus only on the functions where at least some solutions obtained by G-CMA-ES-nonbound are infeasible (which are 17), then it wins 11 times and loses 6 times. The published results of PS-CMA-ES are also presented. PS-CMA-ES obtained very similar, even often the exactly same average error values as G-CMA-ES-nonbound for which the latter obtained infeasible solutions; this trend is stronger for higher-dimensional functions. These average error values are highlighted using underline in Table 2. On the 30 and 50 dimensional functions f_{cec18}, f_{cec19}, f_{cec20}, they obtained the exactly same average error values, 8.16+02 and 8.36+02, respectively. However, all the corresponding solutions obtained by G-CMA-ES-nonbound are outside bound. On the 50 dimensional functions f_{cec21}-f_{cec24}, PS-CMA-ES also obtained very similar results to G-CMA-ES-nonbound and in those cases all the corresponding solutions obtained by G-CMA-ES-nonbound are infeasible. Whether the solution vectors of PS-CMA-ES are feasible is at least dubious.

Next, we compare the distribution of the average errors of PS-CMA-ES with G-CMA-ES-nonbound and G-CMA-ES-clampedbound. Table 3 shows that PS-CMA-ES is superior to G-CMA-ES-nonbound, while Table 4 shows that PS-CMA-ES is either worse or at least not better than G-CMA-ES-clampedbound. Therefore, the two different comparison results illustrate that indeed wrong conclusions can be drawn if bound constraints are enforced in one algorithm but not in the other.

3.2 Objective Function Value *vs* # Function Evaluations

For further understanding the effect of enforcing bound constraints, we looked into the development of the solution quality over the number of evaluations of G-CMA-ES-clampedbound and G-CMA-ES-nonbound on the CEC 2005 benchmark function suite. Figure 1 is drawn on some selected 30 dimensional functions. In f_{cec3} and f_{cec6}, enforcing bound constraints has a very small effect. The development of the solution quality G-CMA-ES-clampedbound and G-CMA-ES-nonbound is similar. There is no significant difference between their final average errors. Their corresponding solutions are also inside the bound, which can be seen from Table 1. In f_{cec18} and f_{cec19}, enforcing bound constraints results in a large effect. G-CMA-ES-nonbound obtained a better performance than G-CMA-ES-clampedbound for higher number of evaluations. However, we noted that all the solutions obtained by G-CMA-ES-nonbound are outside the bound. We found that a difference between G-CMA-ES-clampedbound and G-CMA-ES-nonbound exists even for a very small number of evaluations. The reason is that when the initial sampled solutions in G-CMA-ES are outside bound, the clamped-bound mechanism enforces them on the nearest bound.

f_{cec11} is an example that enforcing bound constraints improve the algorithm performance. In f_{cec25}, as the initialization range is declared instead of bound constraints, enforcing bound constraints did not be apply. So, the performance of G-CMA-ES-clampedbound and G-CMA-ES-nonbound are exactly the same over time.

4 Conclusions and Outlook

In this report, we show the large effect of enforcing or ignoring bound constraints based on G-CMA-ES on CEC 2005 benchmark functions. We also show that in algorithm comparisons a wrong conclusion can be drawn if bound constraints are enforced in one algorithm but not in the other. Enforcing bound constraints by clamping solutions to the bound in case they are outside is just one simple bound handing technique, which can guarantee that the generated solutions in the optimization process and therefore also the final solutions are inside bound. Other, more advanced bound handling techniques are also allowed, even if they also make an algorithm search outside bound for learning extra information. However, the obtained final solutions should be verified whether they are inside the domain bound to keep the solutions feasible no matter what bound handling techniques are used. Therefore, we recommend that every paper reporting results on the CEC 2005 benchmark set needs to check the feasibility of the obtained solutions and, ideally, the obtained solution vectors should be presented on pages supplementary to the published article.

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Table 1: The comparative results obtained by G-CMA-ES-clampedbound and G-CMA-ES-nonbound over 25 independent runs for CEC 2005 functions. Symbols $<$, $\approx$, and $>$ denote the performance of G-CMA-ES-nonbound is better than, similar to, worse than the G-CMA-ES-clampedbound by using two-sided Wilcoxon matched-pairs signed-rank at 0.05 α-level. The numbers in parenthesis represent the times of $<$, $\approx$, and $>$, respectively.

f_{cec}	10 dimensions					30 dimensions					50 dimensions				
	No-Bound Mean and Median			Clamped-Bound Mean and Median		No-Bound Mean and Median			Clamped-Bound Mean and Median		No-Bound Mean and Median			Clamped-Bound Mean and Median	
f_1	2.62E−16	2.50E−16	≈	2.30E−16	2.22E−16	1.13E−15	9.44E−16	≈	1.33E−15	1.30E−15	1.12E−15	1.11E−15	≈	1.09E−15	8.88E−16
f_2	3.03E−16	3.05E−16	≈	2.86E−16	2.78E−16	3.16E−15	3.00E−15	≈	3.17E−15	3.19E−15	8.23E−15	8.30E−15	≈	8.62E−15	8.69E−15
f_3	7.70E−16	7.77E−16	≈	7.54E−16	6.66E−16	1.59E−15	1.53E−15	≈	1.76E−15	1.72E−15	1.67E−15	1.67E−15	≈	1.74E−15	1.64E−15
f_4	3.61E−16	3.05E−16	≈	3.43E−16	3.33E−16	2.44E+03	1.18E+01	⊙ ≈	6.58E+02	1.75E+00	1.32E+05	1.45E+05	⊙ >	1.42E+04	1.27E+04
f_5	8.73E−12	8.58E−12	⊘ <	2.18E−11	2.33E−11	2.29E+01	1.09E−02	⊘ >	1.04E−10	1.05E−10	8.30E+02	7.18E+02	⊘ >	7.35E−02	5.51E−08
f_6	8.38E−16	8.60E−16	≈	9.39E−16	8.33E−16	4.20E−15	3.83E−15	≈	3.60E−15	3.41E−15	7.38E−15	7.69E−15	≈	8.23E−15	7.88E−15
f_7†	1.84E−16	1.53E−16	≈	1.85E−16	1.53E−16	3.39E−15	3.28E−15	≈	3.35E−15	3.23E−15	7.08E−15	7.08E−15	≈	7.19E−15	7.15E−15
f_8	2.01E+01	2.00E+01	⊘ ≈	2.00E+01	2.00E+01	2.07E+01	2.09E+01	⊘ ≈	2.05E+01	2.00E+01	2.11E+01	2.11E+01	⊘ ≈	2.10E+01	2.11E+01
f_9	1.59E−01	7.77E−16	≈	1.59E−01	2.50E−16	9.91E−01	9.95E−01	<	1.87E+00	1.99E+00	1.12E+00	9.95E−01	⊙ <	4.24E+00	3.98E+00
f_{10}	1.19E−01	5.27E−16	≈	3.18E−01	1.75E−15	1.39E+00	1.02E+00	≈	1.44E+00	9.95E−01	2.36E+00	1.99E+00	≈	2.81E+00	1.99E+00
f_{11}	6.44E−01	2.08E−10	⊘ >	5.19E−11	5.21E−11	6.17E+00	4.18E+00	⊘ >	7.17E−02	3.90E−10	1.64E+01	8.69E+00	⊘ >	9.94E−02	1.14E−09
f_{12}	6.77E+01	9.71E−16	⊘ <	4.07E+03	2.64E−15	1.38E+03	3.85E+02	⊘ <	1.19E+04	4.84E+03	7.38E+03	3.97E+03	⊘ <	4.25E+04	2.78E+04
f_{13}	6.78E−01	7.07E−01	≈	6.49E−01	6.37E−01	2.43E+00	2.47E+00	≈	2.63E+00	2.71E+00	4.31E+00	4.48E+00	≈	4.43E+00	4.37E+00
f_{14}	2.61E+00	2.70E+00	⊙ >	1.96E+00	1.98E+00	1.28E+01	1.30E+01	⊘ ≈	1.26E+01	1.26E+01	2.35E+01	2.35E+01	⊘ >	2.28E+01	2.30E+01
f_{15}	2.00E+02	2.00E+02	⊘ ≈	2.15E+02	2.00E+02	2.01E+02	2.00E+02	⊘ >	2.00E+02	2.00E+02	2.01E+02	2.00E+02	⊘ >	2.00E+02	2.00E+02
f_{16}	9.02E+01	9.11E+01	≈	9.04E+01	9.11E+01	7.95E+01	3.82E+01	⊙ >	1.46E+01	1.42E+01	1.36E+02	1.40E+02	⊘ >	1.14E+01	1.18E+01
f_{17}	1.33E+02	1.13E+02	⊙ ≈	1.17E+02	1.09E+02	4.31E+02	4.64E+02	⊘ >	2.52E+02	1.80E+02	7.73E+02	8.65E+02	⊘ >	1.91E+02	1.62E+02
f_{18}	7.48E+02	8.21E+02	⊙ >	3.12E+02	3.00E+02	8.16E+02	8.16E+02	⊘ <	9.04E+02	9.04E+02	8.36E+02	8.36E+02	⊘ <	9.13E+02	9.16E+02
f_{19}	7.75E+02	8.21E+02	⊙ >	3.20E+02	3.00E+02	8.16E+02	8.16E+02	⊘ <	9.04E+02	9.04E+02	8.36E+02	8.36E+02	⊘ <	9.13E+02	9.15E+02
f_{20}	7.62E+02	8.21E+02	⊙ >	3.20E+02	3.00E+02	8.16E+02	8.16E+02	⊘ <	9.04E+02	9.04E+02	8.36E+02	8.36E+02	⊘ <	9.15E+02	9.15E+02
f_{21}	1.06E+03	1.07E+03	⊘ >	5.00E+02	5.00E+02	8.57E+02	8.57E+02	⊘ >	5.00E+02	5.00E+02	7.15E+02	7.15E+02	⊘ ≈	6.64E+02	5.00E+02
f_{22}	6.38E+02	5.26E+02	⊘ <	7.28E+02	7.28E+02	6.13E+02	5.00E+02	⊘ <	8.10E+02	8.11E+02	5.00E+02	5.00E+02	⊘ <	8.20E+02	8.19E+02
f_{23}	1.09E+03	1.09E+03	⊘ >	5.86E+02	5.59E+02	8.69E+02	8.67E+02	⊘ >	5.34E+02	5.34E+02	7.27E+02	7.27E+02	⊘ ≈	6.78E+02	5.40E+02
f_{24}	4.05E+02	4.04E+02	⊘ >	2.33E+02	2.00E+02	2.10E+02	2.10E+02	⊘ >	2.00E+02	2.00E+02	2.14E+02	2.14E+02	⊘ >	2.00E+02	2.00E+02
f_{25}†	4.34E+02	4.04E+02	≈	4.34E+02	4.04E+02	2.10E+02	2.10E+02	≈	2.10E+02	2.10E+02	2.14E+02	2.14E+02	≈	2.14E+02	2.14E+02
	f_1-f_{25} (<, ≈, >): (3, 14, 8)					f_1-f_{25}(<, ≈, >): (6, 11, 8)					f_1-f_{25}(<, ≈, >): (6, 11, 8)				
	f_1-f_6, f_8-f_{24} (<, ≈, >): (3, 12, 8)					f_1-f_6, f_8-f_{24} (<, ≈, >): (6, 9, 8)					f_1-f_6, f_8-f_{24} (<, ≈, >): (6, 9, 8)				
	Num of funcs with solutions$^{\oslash or \odot}$: 14/23 (61%)					Num of funcs with solutions$^{\oslash or \odot}$: 16/23 (70%)					Num of funcs with solutions$^{\oslash or \odot}$: 17/23 (74%)				

⊘ denotes that all 25 solutions are outside bounds.
⊙ denotes some of the 25 solutions are outside bounds.
† denotes that the specialized initialization ranges are applied instead of bounds constraints according to CEC 2005's protocal. Therefore, there is no difference between nonbound and clampedbound handling.

Table 2: The average errors obtained by PS-CMA-ES, G-CMA-ES-nonbound and original G-CMA-ES over 25 independent runs for CEC 2005 functions. The numbers in parenthesis represent the times of win, draw, and lose, respectively, when the corresponding algorithms are compared with G-CMA-ES on the average errors. $<, =, >$ denote the infeasible average error obtained by G-CMA-ES-nonbound is better, equal and worse than original G-CMA-ES, respectively. The times of $<, =, >$ are presented in parenthesis with $^{\oslash or \odot}$. The highlighted values in underline indicate that the corresponding average error value is the same or very close to the infeasible average error value obtained by G-CMA-ES-nonbound

f_{cec}	10 dimensions			30 dimensions			50 dimensions		
	Mean Errors			Mean Errors			Mean Errors		
	PS-CMA-ES	No-Bound[1]	G-CMA-ES	PS-CMA-ES	No-Bound[1]	G-CMA-ES	PS-CMA-ES	No-Bound[1]	G-CMA-ES
f_1	1.00E−08	1.00E−08	1.00E−08	1.00E−08	1.00E−08	1.00E−08	1.00E−08	1.00E−08	1.00E−08
f_2	1.00E−08	1.00E−08	1.00E−08	1.00E−08	1.00E−08	1.00E−08	9.79E−04	1.00E−08	1.00E−08
f_3	1.59E+00	1.00E−08	1.00E−08	8.00E+04	1.00E−08	1.00E−08	3.28E+05	1.00E−08	1.00E−08
f_4	1.00E−08	1.00E−08	1.00E−08	8.47E−04	2.44E+03 $^{\odot}$ <	1.11E+04	1.58E+03	1.32E+05 $^{\odot}$ <	4.68E+05
f_5	3.73E−05	1.00E−08 $^{\oslash}$ =	1.00E−08	3.98E+02	2.29E+01 $^{\oslash}$ >	1.00E−08	1.18E+03	8.30E+02 $^{\oslash}$ >	2.85E+00
f_6	1.93E−05	1.00E−08	1.00E−08	1.35E+01	1.00E−08	1.00E−08	2.98E+01	1.00E−08	1.00E−08
f_7	1.00E−08	1.00E−08	1.00E−08	1.00E−08	1.00E−08	1.00E−08	1.00E−08	1.00E−08	1.00E−08
f_8	2.03E+01	2.01E+01 $^{\oslash}$ >	2.00E+01	2.10E+01	2.07E+01 $^{\oslash}$ >	2.01E+01	2.11E+01	2.11E+01 $^{\oslash}$ >	2.01E+01
f_9	1.00E−08	1.59E−01	2.39E−01	1.00E−08	9.91E−01	9.38E−01	1.00E−08	1.12E+00 $^{\odot}$ <	1.39E+00
f_{10}	1.00E−08	1.19E−01	7.96E−02	1.00E−08	1.39E+00	1.65E+00	1.00E−08	2.36E+00	1.72E+00
f_{11}	1.33E−01	6.44E−01 $^{\oslash}$ <	9.34E−01	3.91E+00	6.17E+00 $^{\oslash}$ >	5.48E+00	1.22E+01	1.64E+01 $^{\oslash}$ >	1.17E+01
f_{12}	1.00E−08	6.77E+01 $^{\oslash}$ >	2.93E+01	7.89E+01	1.38E+03 $^{\oslash}$ <	4.43E+04	2.36E+03	7.38E+03 $^{\oslash}$ <	2.27E+05
f_{13}	3.95E−01	6.78E−01	6.96E−01	2.11E+00	2.43E+00	2.49E+00	4.00E+00	4.31E+00	4.59E+00
f_{14}	3.47E+00	2.61E+00 $^{\odot}$ <	3.01E+00	1.29E+01	1.28E+01 $^{\oslash}$ <	1.29E+01	2.25E+01	2.35E+01 $^{\oslash}$ >	2.29E+01
f_{15}	8.12E+01	2.00E+02 $^{\oslash}$ <	2.28E+02	2.10E+02	2.01E+02 $^{\oslash}$ <	2.08E+02	2.64E+02	2.01E+02 $^{\oslash}$ <	2.04E+02
f_{16}	8.97E+01	9.02E+01	9.13E+01	2.61E+01	7.95E+01 $^{\odot}$ >	3.50E+01	2.27E+01	1.36E+02 $^{\oslash}$ >	3.09E+01
f_{17}	1.03E+02	1.33E+02 $^{\odot}$ >	1.23E+02	5.17E+01	4.31E+02 $^{\oslash}$ >	2.91E+02	6.16E+01	7.73E+02 $^{\oslash}$ >	2.34E+02
f_{18}	4.72E+02	7.48E+02 $^{\odot}$ >	3.32E+02	8.16E+02	8.16E+02 $^{\oslash}$ <	9.04E+02	8.36E+02	8.36E+02 $^{\oslash}$ <	9.13E+02
f_{19}	4.67E+02	7.75E+02 $^{\odot}$ >	3.26E+02	8.16E+02	8.16E+02 $^{\oslash}$ <	9.04E+02	8.36E+02	8.36E+02 $^{\oslash}$ <	9.12E+02
f_{20}	4.94E+02	7.62E+02 $^{\odot}$ >	3.00E+02	8.16E+02	8.16E+02 $^{\oslash}$ <	9.04E+02	8.36E+02	8.36E+02 $^{\oslash}$ <	9.12E+02
f_{21}	5.57E+02	1.06E+03 $^{\oslash}$ >	5.00E+02	7.11E+02	8.57E+02 $^{\oslash}$ >	5.00E+02	7.18E+02	7.15E+02 $^{\oslash}$ <	1.00E+03
f_{22}	5.87E+02	6.38E+02 $^{\oslash}$ <	7.29E+02	5.00E+02	6.13E+02 $^{\oslash}$ <	8.03E+02	5.00E+02	5.00E+02 $^{\oslash}$ <	8.05E+02
f_{23}	6.43E+02	1.09E+03 $^{\oslash}$ >	5.59E+02	7.99E+02	8.69E+02 $^{\oslash}$ >	5.34E+02	7.24E+02	7.27E+02 $^{\oslash}$ <	1.01E+03
f_{24}	4.03E+02	4.05E+02 $^{\oslash}$ >	2.00E+02	2.10E+02	2.10E+02 $^{\oslash}$ <	9.10E+02	2.14E+02	2.14E+02 $^{\oslash}$ <	9.55E+02
f_{25}	4.03E+02	4.34E+02	3.74E+02	2.10E+02	2.10E+02	2.11E+02	2.14E+02	2.14E+02	2.15E+02
	(9, 4, 12)	(7, 7, 11) $\ni (4,1,9)^{\oslash or \odot}$		(14, 4, 7)	(12, 5, 8) $\ni (9,0,7)^{\oslash or \odot}$		(16, 2, 7)†	(13, 5, 7) $\ni (11,0,6)^{\oslash or \odot}$	

[1] No-Bound is the abbreviation of G-CMA-ES-nonbound in this table.
$^{\oslash}$ denotes that all 25 solutions are outside bounds.
$^{\odot}$ denotes some of the 25 solutions are outside bounds.
† denotes there is a significant difference over the distribution of average errors between the corresponding algorithm with G-CMA-ES by a two-sided Wilcoxon matched-pairs signed-ranks test at 0,05 α-level.

Table 3: The average errors obtained by PS-CMA-ES, G-CMA-ES-nonbound over 25 independent runs for CEC 2005 functions. The numbers in parenthesis represent the times of win, draw, and lose, respectively, when the corresponding algorithms are compared with G-CMA-ES-nonbound on the average errors.

f_{cec}	10 dimensions		30 dimensions		50 dimensions	
	Mean Errors		Mean Errors		Mean Errors	
	PS-CMA-ES	No-Bound[1]	PS-CMA-ES	No-Bound[1]	PS-CMA-ES	No-Bound[1]
f_1	1.00E−08	1.00E−08	1.00E−08	1.00E−08	1.00E−08	1.00E−08
f_2	1.00E−08	1.00E−08	1.00E−08	1.00E−08	9.79E−04	1.00E−08
f_3	1.59E+00	1.00E−08	8.00E+04	1.00E−08	3.28E+05	1.00E−08
f_4	1.00E−08	1.00E−08	8.47E−04	2.44E+03	1.58E+03	1.32E+05
f_5	3.73E−05	1.00E−08	3.98E+02	2.29E+01	1.18E+03	8.30E+02
f_6	1.93E−05	1.00E−08	1.35E+01	1.00E−08	2.98E+01	1.00E−08
f_7	1.00E−08	1.00E−08	1.00E−08	1.00E−08	1.00E−08	1.00E−08
f_8	2.03E+01	2.01E+01	2.10E+01	2.07E+01	2.11E+01	2.11E+01
f_9	1.00E−08	1.59E−01	1.00E−08	9.91E−01	1.00E−08	1.12E+00
f_{10}	1.00E−08	1.19E−01	1.00E−08	1.39E+00	1.00E−08	2.36E+00
f_{11}	1.33E−01	6.44E−01	3.91E+00	6.17E+00	1.22E+01	1.64E+01
f_{12}	1.00E−08	6.77E+01	7.89E+01	1.38E+03	2.36E+03	7.38E+03
f_{13}	3.95E−01	6.78E−01	2.11E+00	2.43E+00	4.00E+00	4.31E+00
f_{14}	3.47E+00	2.61E+00	1.29E+01	1.28E+01	2.25E+01	2.35E+01
f_{15}	8.12E+01	2.00E+02	2.10E+02	2.01E+02	2.64E+02	2.01E+02
f_{16}	8.97E+01	9.02E+01	2.61E+01	7.95E+01	2.27E+01	1.36E+02
f_{17}	1.03E+02	1.33E+02	5.17E+01	4.31E+02	6.16E+01	7.73E+02
f_{18}	4.72E+02	7.48E+02	8.16E+02	8.16E+02	8.36E+02	8.36E+02
f_{19}	4.67E+02	7.75E+02	8.16E+02	8.16E+02	8.36E+02	8.36E+02
f_{20}	4.94E+02	7.62E+02	8.16E+02	8.16E+02	8.36E+02	8.36E+02
f_{21}	5.57E+02	1.06E+03	7.11E+02	8.57E+02	7.18E+02	7.15E+02
f_{22}	5.87E+02	6.38E+02	5.00E+02	6.13E+02	5.00E+02	5.00E+02
f_{23}	6.43E+02	1.09E+03	7.99E+02	8.69E+02	7.24E+02	7.27E+02
f_{24}	4.03E+02	4.05E+02	2.10E+02	2.10E+02	2.14E+02	2.14E+02
f_{25}	4.03E+02	4.34E+02	2.10E+02	2.10E+02	2.14E+02	2.14E+02
	(16, 4, 5)†		(11, 8, 6)		(10, 9, 6)	

[1] No-Bound is the abbreviation of G-CMA-ES-nonbound in this table.
⊘ denotes that all 25 solutions are outside bounds.
⊙ denotes some of the 25 solutions are outside bounds.
† denotes there is a significant difference over the distribution of average errors between the corresponding algorithm with G-CMA-ES-nonbound by a two-sided Wilcoxon matched-pairs signed-ranks test at 0,05 α-level.

Table 4: The average errors obtained by PS-CMA-ES, G-CMA-ES-clampedbound over 25 independent runs for CEC 2005 functions. The numbers in parenthesis represent the times of win, draw, and lose, respectively, when the corresponding algorithms are compared with G-CMA-ES-clampedbound on the average errors.

f_{cec}	10 dimensions		30 dimensions		50 dimensions	
	Mean Errors		Mean Errors		Mean Errors	
	PS-CMA-ES	Clamped-Bound[1]	PS-CMA-ES	Clamped-Bound[1]	PS-CMA-ES	Clamped-Bound[1]
f_1	1.00E−08	1.00E−08	1.00E−08	1.00E−08	1.00E−08	1.00E−08
f_2	1.00E−08	1.00E−08	1.00E−08	1.00E−08	9.79E−04	1.00E−08
f_3	1.59E+00	1.00E−08	8.00E+04	1.00E−08	3.28E+05	1.00E−08
f_4	1.00E−08	1.00E−08	8.47E−04	6.58E+02	1.58E+03	1.42E+04
f_5	3.73E−05	1.00E−08	3.98E+02	1.00E−08	1.18E+03	7.35E−02
f_6	1.93E−05	1.00E−08	1.35E+01	1.00E−08	2.98E+01	1.00E−08
f_7	1.00E−08	1.00E−08	1.00E−08	1.00E−08	1.00E−08	1.00E−08
f_8	2.03E+01	2.00E+01	2.10E+01	2.05E+01	2.11E+01	2.10E+01
f_9	1.00E−08	1.59E−01	1.00E−08	1.87E+00	1.00E−08	4.24E+00
f_{10}	1.00E−08	3.18E−01	1.00E−08	1.44E+00	1.00E−08	2.81E+00
f_{11}	1.33E−01	1.00E−08	3.91E+00	7.17E−02	1.22E+01	9.94E−02
f_{12}	1.00E−08	4.07E+03	7.89E+01	1.19E+04	2.36E+03	4.25E+04
f_{13}	3.95E−01	6.49E−01	2.11E+00	2.63E+00	4.00E+00	4.43E+00
f_{14}	3.47E+00	1.96E+00	1.29E+01	1.26E+01	2.25E+01	2.28E+01
f_{15}	8.12E+01	2.15E+02	2.10E+02	2.00E+02	2.64E+02	2.00E+02
f_{16}	8.97E+01	9.04E+01	2.61E+01	1.46E+01	2.27E+01	1.14E+01
f_{17}	1.03E+02	1.17E+02	5.17E+01	2.52E+02	6.16E+01	1.91E+02
f_{18}	4.72E+02	3.12E+02	8.16E+02	9.04E+02	8.36E+02	9.13E+02
f_{19}	4.67E+02	3.20E+02	8.16E+02	9.04E+02	8.36E+02	9.13E+02
f_{20}	4.94E+02	3.20E+02	8.16E+02	9.04E+02	8.36E+02	9.15E+02
f_{21}	5.57E+02	5.00E+02	7.11E+02	5.00E+02	7.18E+02	6.64E+02
f_{22}	5.87E+02	7.28E+02	5.00E+02	8.10E+02	5.00E+02	8.20E+02
f_{23}	6.43E+02	5.86E+02	7.99E+02	5.34E+02	7.24E+02	6.78E+02
f_{24}	4.03E+02	2.33E+02	2.10E+02	2.00E+02	2.14E+02	2.00E+02
f_{25}	4.03E+02	4.34E+02	2.10E+02	2.10E+02	2.14E+02	2.14E+02
	(9, 4, 12)		(10, 4, 11)		(11, 3, 11)	

[1] Clamped-Bound is the abbreviation of G-CMA-ES-clampedbound in this table.
$^{\oslash}$ denotes that all 25 solutions are outside bounds.
$^{\odot}$ denotes some of the 25 solutions are outside bounds.
† denotes there is a significant difference over the distribution of average errors between the corresponding algorithm with G-CMA-ES-clampedbound by a two-sided Wilcoxon matched-pairs signed-ranks test at 0,05 α-level.

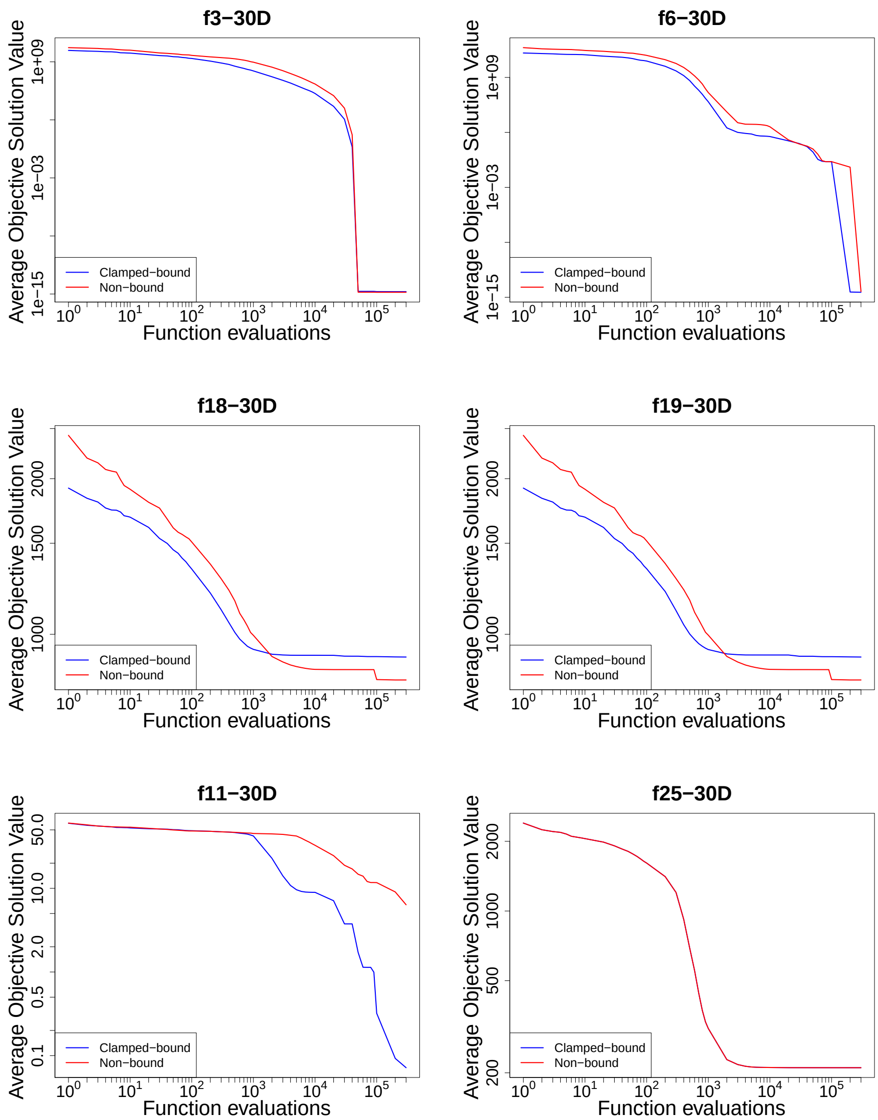

Figure 1: The development of the solution quality over evaluation numbers for G-CMA-ES-clampedbound and G-CMA-ES-nonbound on selected 30 dimensional CEC 2005 functions