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A Note on the Effects of Enforcing Bound Constraints on Algorithm Comparisons using the IEEE CEC'05 Benchmark Function Suite

Tianjun Liao tliao@ulb.ac.be
IRIDIA, CoDE, Université Libre de Bruxelles, Brussels, Belgium

Daniel Molina daniel.molina@uca.es
Dept. of Computer Engineering, University of Cadiz, Cadiz, Spain

Marco A. Montes de Oca mmontes@math.udel.edu
Dept. of Mathematical Sciences, University of Delaware, Newark, DE, USA

Thomas Stützle stuetzle@ulb.ac.be
IRIDIA, CoDE, Université Libre de Bruxelles, Brussels, Belgium

Abstract

The benchmark functions proposed in the special session on real parameter optimization of the 2005 IEEE Congress on Evolutionary Computation (CEC'05) are playing an important role in performance comparisons of continuous optimizers and, consequently, in the assessment of the state of the art in continuous optimization. Unfortunately, the original wording for describing the experimental protocol used for the aforementioned session is rather vague about how bound constraints should be handled. During experiments with bound handling mechanisms in IPOP-CMA-ES, the algorithm with the best performance in the aforementioned session, and MA-LSch-CMA, a recent memetic algorithm, we noticed that the performance of these algorithms substantially varies depending on whether bound constraints are enforced or not. We found that if bound constraints are not enforced, the solutions generated may be associated with lower error values than if bounds are enforced. However, the solutions generated when bound constraints are not enforced are often infeasible. These results point toward a major problem in many published experimental evaluations. In particular, several claims about superior performance of an algorithm over IPOP-CMA-ES (or any other algorithm) may be flawed due to possible problems with the enforcement of bound constraints. Therefore, we advocate for reporting explicitly the bound handling mechanism used and the final solutions found in experimental studies involving the CEC'05 benchmark functions.

Keywords

Continuous optimization, bound constraints, feasible solutions, IPOP-CMA-ES, MA-LSch-CMA.

1 Introduction

The special session on real parameter optimization of the 2005 IEEE Congress on Evolutionary Computation (CEC'05) has played an important role in evolutionary computation and other affine fields for two reasons. First, it provided a set of 25 freely scalable benchmark functions that anyone can use to evaluate the performance of new algorithms. Those 25 functions have become a standard benchmark set that researchers use to compare algorithms. The central role that this benchmark function set currently plays is also illustrated by the more than 375 citations[1] to (Suganthan et al., 2005), which is the original source that introduced the benchmark function set. Second, it served to assess the state of the art in continuous optimization. In particular, the best performing algorithm of the session, IPOP-CMA-ES (Auger and Hansen, 2005), is since then considered to be a representative of the state of the art in continuous optimization. Thus, it is reasonable to consider an algorithm that outperforms IPOP-CMA-ES on the CEC'05 benchmark function suite to be state of the art.

Six years after the aforementioned session was organized, it is not surprising that a number of authors have reported that their proposed algorithms outperform IPOP-CMA-ES on (normally a subset of) the 25 benchmark functions designed for that session (Muller et al., 2009; Vrugt et al., 2009; Peng et al., 2010; Molina et al., 2010). In this work, we want to call the attention of the research community to the fact that at least some of these positive reports may in fact not be valid.

The source of the problem is the uncertainty surrounding how bound constraints should be handled in most of the special session's 25 benchmark functions. In the technical report describing the experimental protocol to follow, the authors write: *"All problems, except 7 and 25, have the global optimum within the given bounds and there is no need to perform search outside of the given bounds for these problems. 7 & 25 are exceptions without a search range and with the global optimum outside of the specified initialization range."* (Suganthan et al., 2005) (p. 40).

Although for all functions (except functions f_7 and f_{25}) in their definition it is stated that the solution vector x is in each dimension an element of some subset of reals (by stating that $x \in [x_{\min}, x_{\max}]$, where $x_{\min} < x_{\max}$ are some constants), some authors possibly interpret these lines as a suggestion. In fact, in many papers authors did not report whether they enforce bound constraints.

In this work, we show that the way bound constraints are handled has a strong impact on the performance of IPOP-CMA-ES on the CEC'05 benchmark set. We do so using the `C` version of IPOP-CMA-ES from Hansen's website. Interestingly, this `C` version of IPOP-CMA-ES does not use an explicit bound handling mechanism in its original version. Therefore, we modified the code to include two ways of bound handling. In variant one, bound constraints are enforced by always clamping generated solutions outside the bounds to the nearest solution on the bounds (*acb* for "always clamp bounds"). In variant two, the bound constraints are enforced by clamping the final solution s at the end of an algorithm's run to the nearest solution on the bounds, resulting in solution s', if s violates some bound constraints (*fcb* for "final clamp bounds"). As a third variant, we use directly the original code where the bound constraints are never enforced (*ncb* for "never clamp bounds"). These same three variants are tested using a memetic algorithm, MA-LSch-CMA (Molina et al., 2010), which is a recent memetic algorithm that uses CMA-ES as a local search and has been reported to improve over IPOP-CMA-ES performance.

Our experiments confirm that the bound handling mechanism has a significant impact also on the results of these two algorithms on the CEC'05 benchmark set. Interestingly, for several functions the algorithm versions without bound handling obtain infeasible solutions that have, however, significantly better evaluation function values than the best feasible solutions found by the methods with bound handling. Comparing the evaluation function values of the versions without enforcement of bound constraints to those of algorithms that are claimed to outperform IPOP-CMA-ES, we find

[1] According to Google Scholar as of September 2011.

evidence that these algorithms apparently obtain infeasible solutions, thus invalidating the authors' conclusions.

2 Experiments on enforcing bound constraints

In this first set of experiments, we examine the impact of bound handling on the performance of the C version of IPOP-CMA-ES from Hansen's website `http://www.lri.fr/~hansen/cmaesintro.html` and MA-LSch-CMA (Molina et al., 2010), available from the website `http://sci2s.ugr.es/EAMHCO/#Software`. For the experiments on the CEC'05 benchmark functions, we followed the protocol described in (Suganthan et al., 2005), that is, we ran IPOP-CMA-ES and MA-LSch-CMA 25 times on each function and recorded the evolution of the objective function value with respect to the number of function evaluations used. The maximum number of function evaluations was $10000 \cdot D$, where $D \in \{10, 30, 50\}$ is the dimensionality of a function. The algorithm could stop when the maximum number of evaluations is reached or the error obtained is lower than 10^{-8}. Error values lower than this threshold value are considered equal to the optimum.

As a first illustration of the impact of bound handling, we plot the development of the average errors of the fitness values (the optima are known) over the number of function evaluations for the three versions of IPOP-CMA-ES and MA-LSch-CMA. Figure 1 gives an example of the obtained data for functions f_{14} and f_{18} from the CEC'05 benchmark function suite for 50 dimensions. It can be observed that for both functions, *ncb* actually searches outside the bounds. This can be seen by the fact that version *fcb* at the final step worsens strongly the average results of version *ncb*, which happens because in this final step the solutions are modified by clamping the bounds. Interestingly, for both functions all 25 solutions obtained by version *ncb* are outside the bounds (see also Tables 1 and 2). When comparing versions *acb* and *ncb*, it is noteworthy that on function f_{14} version *acb* gives a lower average error than version *ncb*, while the opposite happens on function f_{18}.

We now examine how the difference in performance due to the way bound constraints are handled generalizes to the whole CEC'05 benchmark set. When comparing *ncb* and *fcb*, we found that *fcb* never improves upon *ncb*. In fact, in all functions, where IPOP-CMA-ES and MA-LSch-CMA have all their final solutions outside the bounds, *fcb* is statistically significantly worse than *ncb*. The numerical results that confirm this statement are given in the supplementary page `http://iridia.ulb.ac.be/supp/IridiaSupp2011-013`.

The comparison of versions *ncb* and *acb* is more interesting. Table 1 shows the average errors of *ncb* and *acb* for IPOP-CMA-ES. The two-sided Wilcoxon matched-pairs signed-rank test at the 0.05 α-level was used to check for statistical difference on each function. A first important observation is that on many functions, that is in 14 to 17 functions depending on the dimensionality, IPOP-CMA-ES-*ncb* obtains final solutions outside the bounds. In most of these cases, all 25 final solutions are outside the bounds. Typically, the differences between IPOP-CMA-ES-*ncb* and IPOP-CMA-ES-*acb* are also statistically significant if the final solutions of IPOP-CMA-ES-*ncb* are outside the bounds. Moreover, all functions where IPOP-CMA-ES-*ncb* outperforms IPOP-CMA-ES-*acb* correspond to functions where all solutions obtained by IPOP-CMA-ES-*ncb* are outside the bounds (e.g. f_9, f_{12}, f_{18}, f_{19}, f_{20} and f_{22} over 30 and 50 dimension).

Analogously, Table 2 shows the performance of *ncb* and *acb* for MA-LSch-CMA. Again, version *ncb* obtains many final solutions outside the bounds, for dimensions 30 and 50 this is the case 18 and 19 times, respectively, that is, even more often than for IPOP-CMA-ES. Taking the 50 dimensional benchmark functions as an example, all functions where MA-LSch-CMA-*ncb* outperforms MA-LSch-CMA-*acb* are cases where all solutions obtained by MA-LSch-CMA-*ncb* are outside the bounds (e.g. f_5, f_{11}, f_{12}, f_{15}, f_{18}, f_{19}, f_{20} and f_{22}).

In summary, it remains a noteworthy result that the *ncb* version of the algorithms can obtain better results than with the bound constraints, but in all the cases, these better solutions were infeasibles.

3 The impact of bound handling on algorithm comparisons

As a next step, we provide evidence that the decision of whether bound constraints are enforced or not have a significant impact on algorithm comparisons. We take the reported average errors of the results reported for IPOP-CMA-ES in the CEC'05 special session as a reference and refer to these results as IPOP-CMA-ES-05. We first focus on the comparison of different versions of IPOP-CMA-ES in Table 3. Interestingly, IPOP-CMA-ES-*ncb* gives for both, more 30 and 50 dimensional problems, on more functions lower average errors than IPOP-CMA-ES-05 than vice versa; IPOP-CMA-ES-*fcb* is statistically significantly worse than IPOP-CMA-ES-05; and, IPOP-CMA-ES-*acb* gives statistically significantly better performance than IPOP-CMA-ES-05.[2] As such, the way bound constraints are handled leads to different conclusions in algorithm comparisons. A similar impact of the way bound constraints are handled is observed for MA-LSch-CMA.

Next, we focus on the comparison of the average errors between PS-CMA-ES (Muller et al., 2009), IPOP-CMA-ES-*ncb* and IPOP-CMA-ES-05 in Table 4. PS-CMA-ES was chosen as an example of an algorithm reported to have outperformed IPOP-CMA-ES-05. Both, PS-CMA-ES and IPOP-CMA-ES-*ncb*, are superior to IPOP-CMA-ES-05 on 30 and 50 dimensions. However, there is an interesting pattern related to the fact whether IPOP-CMA-ES-*ncb* has the final solutions outside the bounds or not. Let us focus on the cases where IPOP-CMA-ES-*ncb* obtains all solutions outside the bounds and statistically significantly improves over IPOP-CMA-ES-05. In many such cases, PS-CMA-ES does obtain the same average errors (see, for example, functions f_{18}–f_{20} and f_{24} for both, 30 and 50 dimensions and function f_{22} for 50 dimensions), or very similar values (see, for example, function f_{14} for 30 dimensions and functions f_{22} and f_{23} for 50 dimensions). Such cases are underlined in Table 4. Note that a similar pattern also arises for the published results of the MA-LSch-CMA algorithm in (Molina et al., 2010), where actually the bound constraints have not been checked in the applied CMA-ES local search algorithm. This knowledge together with the similar pattern puts at least serious doubts on the fact whether the reported solutions in (Muller et al., 2009) are actually inside the bounds.

4 Conclusions

In this note, we show the large effect the enforcement or not of bound constraints has on IPOP-CMA-ES and MA-LSch-CMA on the CEC'05 benchmark function suite. Without enforcement of bound constraints, often infeasible solutions are obtained that in many cases are better than the best feasible solutions found if bound constraints are enforced.

[2] Note that two reasons may explain this latter result. First, the bound handling methods adopted in the two cases are different: IPOP-CMA-ES-*acb* simply clamps infeasible solutions to the variable bounds, while IPOP-CMA-ES-05 uses a more complex bound handling mechanism, which is described in (Hansen et al., 2009). Second, there may be minor differences between the `C` implementation of IPOP-CMA-ES-*acb* used here and the Matlab implemented used for IPOP-CMA-ES-05.

Interestingly, many such "infeasible" solutions are of better quality that the ones obtained by IPOP-CMA-ES-05, the best performing algorithm on the CEC'05 benchmark set. This result has also an effect on the conclusions of several algorithm comparisons. We gave evidence that in several cases where improved performance over IPOP-CMA-ES was reported, the fact whether bound constraints are enforced or not plays a decisive role. To avoid possible doubts about the feasibility of the final solutions, we recommend that in the future every paper that reports results on the CEC'05 benchmark function suite but also on other benchmark suites should (i) explicitly describe the used bound handling mechanism, (ii) explicitly check the feasibility of the final solutions, and (iii) present the final solutions at least in supplementary pages to the paper to avoid misinterpretations. Note that it is perfectly valid to not enforce bound constraints, as long as all the compared algorithms do not enforce them. Without this basic piece of information, however, the resulting comparisons and the conclusions drawn from them are dubious at best.

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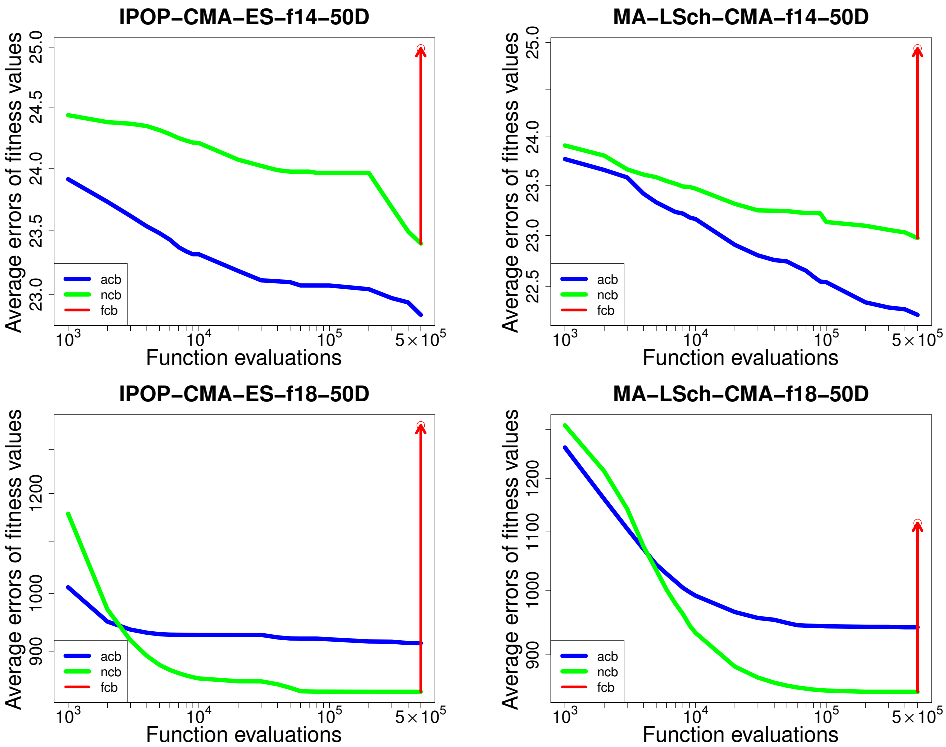

Figure 1: The development of the average error of the fitness values for IPOP-CMA-ES and MA-LSch-CMA over the number of function evaluations on functions f_{14} and f_{18} of 50 dimensions.

Table 1: The comparison between IPOP-CMA-ES-*ncb* and IPOP-CMA-ES-*acb* over 25 independent runs for each of the 25 CEC'05 functions based on the average errors obtained. 23 of these 25 functions actually have bound constraints; for functions f_7 and f_{25} only a bounded initialization range is specified. "$\oslash$" denotes that all 25 final solutions are outside the bounds. "$\odot$" denotes that some but not all of the 25 solutions are outside the bounds. Symbols $<$, $\approx$, and $>$ denote whether the performance of IPOP-CMA-ES-*ncb* is statistically better, indifferent, or worse than that of IPOP-CMA-ES-*acb* according to a two-sided Wilcoxon matched-pairs signed-rank test at the 0.05 α-level. The average errors that correspond to a statistically better result are highlighted. The numbers in parenthesis at the bottom of the table represent the frequency of $<$, $\approx$, and $>$, respectively.

	10 dimensions			30 dimensions			50 dimensions		
f_{cec}	*ncb*		*acb*	*ncb*		*acb*	*ncb*		*acb*
f_1	2.62E−16	$\approx$	2.30E−16	1.13E−15	$\approx$	1.33E−15	1.11E−15	$\approx$	1.08E−15
f_2	3.03E−16	$\approx$	2.86E−16	3.09E−15	$\approx$	3.20E−15	8.18E−15	$\approx$	8.92E−15
f_3	7.70E−16	$\approx$	7.54E−16	1.52E−15	$\approx$	1.75E−15	1.60E−15	$\approx$	1.78E−15
f_4	3.61E−16	$\approx$	3.43E−16	2.44E+03 $\odot$	$\approx$	6.58E+02	1.32E+05 $\odot$	$>$	1.43E+04
f_5	7.76E−12 $\oslash$	$<$	2.17E−11	2.30E+01 $\oslash$	$>$	1.02E−10	7.91E+02 $\oslash$	$>$	7.41E−02
f_6	8.38E−16	$\approx$	9.36E−16	4.28E−15	$\approx$	3.52E−15	7.18E−15	$\approx$	7.91E−15
f_7†	1.85E−16	$\approx$	1.85E−16	3.33E−15	$\approx$	3.32E−15	7.05E−15	$\approx$	7.05E−15
f_8	2.01E+01 $\oslash$	$\approx$	2.00E+01	2.07E+01 $\oslash$	$>$	2.04E+01	2.11E+01 $\oslash$	$>$	2.09E+01
f_9	1.59E−01	$\approx$	1.59E−01	1.01E+00	$<$	1.87E+00	1.12E+00 $\odot$	$<$	4.36E+00
f_{10}	1.19E−01	$\approx$	3.18E−01	1.37E+00	$\approx$	1.44E+00	2.36E+00	$\approx$	2.89E+00
f_{11}	6.44E−01 $\oslash$	$>$	5.31E−11	6.36E+00 $\oslash$	$>$	7.17E−02	1.49E+01 $\oslash$	$>$	9.94E−02
f_{12}	6.77E+01 $\oslash$	$<$	4.07E+03	1.38E+03 $\oslash$	$<$	1.19E+04	7.38E+03 $\oslash$	$<$	4.25E+04
f_{13}	6.78E−01	$\approx$	6.49E−01	2.47E+00	$\approx$	2.63E+00	4.31E+00	$\approx$	4.44E+00
f_{14}	2.61E+00 $\odot$	$>$	1.96E+00	1.28E+01 $\oslash$	$\approx$	1.26E+01	2.34E+01 $\oslash$	$>$	2.28E+01
f_{15}	2.00E+02 $\oslash$	$\approx$	2.15E+02	2.01E+02 $\oslash$	$>$	2.00E+02	2.01E+02 $\oslash$	$>$	2.00E+02
f_{16}	9.02E+01	$\approx$	9.04E+01	7.95E+01 $\odot$	$>$	1.48E+01	1.36E+02 $\oslash$	$>$	1.10E+01
f_{17}	1.33E+02 $\odot$	$\approx$	1.17E+02	4.31E+02 $\oslash$	$>$	2.52E+02	7.69E+02 $\oslash$	$>$	1.91E+02
f_{18}	7.48E+02 $\odot$	$>$	3.16E+02	8.16E+02 $\oslash$	$<$	9.04E+02	8.36E+02 $\oslash$	$<$	9.13E+02
f_{19}	7.75E+02 $\odot$	$>$	3.20E+02	8.16E+02 $\oslash$	$<$	9.04E+02	8.36E+02 $\oslash$	$<$	9.13E+02
f_{20}	7.62E+02 $\odot$	$>$	3.20E+02	8.16E+02 $\oslash$	$<$	9.04E+02	8.36E+02 $\oslash$	$<$	9.15E+02
f_{21}	1.06E+03 $\oslash$	$>$	5.00E+02	8.57E+02 $\oslash$	$>$	5.00E+02	7.15E+02 $\oslash$	$\approx$	6.64E+02
f_{22}	6.38E+02 $\oslash$	$<$	7.28E+02	5.98E+02 $\oslash$	$<$	8.10E+02	5.00E+02 $\oslash$	$<$	8.19E+02
f_{23}	1.09E+03 $\oslash$	$>$	5.86E+02	8.69E+02 $\oslash$	$>$	5.34E+02	7.27E+02 $\oslash$	$\approx$	6.97E+02
f_{24}	4.05E+02 $\oslash$	$>$	2.33E+02	2.10E+02 $\oslash$	$>$	2.00E+02	2.14E+02 $\oslash$	$>$	2.00E+02
f_{25}†	4.34E+02	$\approx$	4.34E+02	2.10E+02	$\approx$	2.10E+02	2.14E+02	$\approx$	2.14E+02
	f_1-f_{25} ($<, \approx, >$): (3, 14, 8)			f_1-f_{25}($<, \approx, >$): (6, 10, 9)			f_1-f_{25}($<, \approx, >$): (6, 10, 9)		
	$\overline{f_7 or f_{25}}$ ($<, \approx, >$): (3, 12, 8)			$\overline{f_7 or f_{25}}$ ($<, \approx, >$): (6, 8, 9)			$\overline{f_7 or f_{25}}$ ($<, \approx, >$): (6, 8, 9)		
	$< or >$: 11/23 (48%)			$< or >$: 15/23 (65%)			$< or >$: 15/23 (65%)		
	solutions$^{\oslash}$ or $\odot$: 14/23 (61%)			solutions$^{\oslash}$ or $\odot$: 16/23 (70%)			solutions$^{\oslash}$ or $\odot$: 17/23 (74%)		

† denotes that the specialized initialization ranges are applied instead of bound constraints according to CEC'05's protocol.

Table 2: The comparison between MA-LSch-CMA-*ncb* and MA-LSch-CMA-*acb* over 25 independent runs for each of the 25 CEC'05 functions based on the average errors obtained. 23 of these 25 functions actually have bound constraints; for functions f_7 and f_{25} only a bounded initialization range is specified. "$^{\oslash}$" denotes that all 25 solutions are outside the bounds. "$^{\odot}$" denotes that some but not all of the 25 solutions are outside the bounds. Symbols $<$, $\approx$, and $>$ denote whether the performance of MA-LSch-CMA-*ncb* is statistically better, indifferent, or worse than that of MA-LSch-CMA-*acb* according to a two-sided Wilcoxon matched-pairs signed-rank test at the 0.05 α-level. The average errors that correspond to a statistically better result are highlighted. The numbers in parenthesis at the bottom of the table represent the frequency of $<$, $\approx$, and $>$, respectively.

	10 dimensions		30 dimensions		50 dimensions	
f_{cec}	*ncb*	*acb*	*ncb*	*acb*	*ncb*	*acb*
f_1	8.33E−09 $\approx$	7.87E−09	8.99E−09 $\approx$	8.95E−09	9.22E−09 $\approx$	9.60E−09
f_2	8.23E−09 $\approx$	8.31E−09	2.51E−08 $\approx$	9.09E−09	8.99E−01 $\approx$	3.06E−02
f_3	3.68E+02 $>$	**7.58E−09**	4.41E+03 $^{\odot}$ $\approx$	2.75E+04	8.11E+04 $^{\odot}$ $>$	**3.21E+04**
f_4	7.60E−09 $\approx$	5.54E−03	**1.28E+02** $<$	3.02E+02	5.38E+03 $^{\odot}$ $>$	**3.23E+03**
f_5	7.78E+01 $^{\oslash}$ $>$	**6.75E−07**	**6.12E+02** $^{\oslash}$ $<$	1.26E+03	**2.08E+03** $^{\oslash}$ $<$	2.69E+03
f_6	8.06E−09 $\approx$	3.19E−01	2.31E+02 $^{\odot}$ $>$	**1.12E+00**	5.58E+02 $^{\odot}$ $>$	**4.10E+00**
f_7 †	1.65E−01 $\approx$	1.43E−01	1.57E−02 $\approx$	1.75E−02	4.23E−03 $\approx$	5.40E−03
f_8	2.00E+01 $^{\oslash}$ $\approx$	2.00E+01	2.00E+01 $^{\oslash}$ $\approx$	2.00E+01	2.00E+01 $^{\oslash}$ $\approx$	2.00E+01
f_9	8.20E−09 $>$	**7.53E−09**	8.79E−09 $\approx$	8.54E−09	8.98E−09 $\approx$	9.14E−09
f_{10}	3.14E+00 $\approx$	2.67E+00	2.00E+01 $^{\odot}$ $\approx$	2.25E+01	4.80E+01 $^{\odot}$ $\approx$	5.01E+01
f_{11}	4.53E+00 $^{\oslash}$ $>$	**2.43E+00**	2.20E+01 $^{\oslash}$ $\approx$	2.15E+01	**3.95E+01** $^{\oslash}$ $<$	4.13E+01
f_{12}	2.95E+02 $^{\odot}$ $\approx$	1.14E+02	**7.52E+02** $^{\oslash}$ $<$	1.67E+03	**4.56E+03** $^{\oslash}$ $<$	1.39E+04
f_{13}	5.03E−01 $\approx$	5.45E−01	2.04E+00 $\approx$	2.03E+00	3.67E+00 $>$	**3.15E+00**
f_{14}	2.87E+00 $^{\odot}$ $>$	**2.25E+00**	1.32E+01 $^{\oslash}$ $>$	**1.25E+01**	2.30E+01 $^{\oslash}$ $>$	**2.22E+01**
f_{15}	2.27E+02 $^{\odot}$ $\approx$	2.24E+02	**2.59E+02** $^{\oslash}$ $<$	3.00E+02	**2.29E+02** $^{\oslash}$ $<$	3.72E+02
f_{16}	9.45E+01 $^{\odot}$ $\approx$	9.18E+01	1.06E+02 $^{\odot}$ $\approx$	1.26E+02	5.91E+01 $^{\odot}$ $>$	**6.90E+01**
f_{17}	1.04E+02 $\approx$	1.01E+02	1.66E+02 $^{\odot}$ $\approx$	1.83E+02	1.41E+02 $^{\oslash}$ $\approx$	1.47E+02
f_{18}	**8.20E+02** $^{\odot}$ $<$	8.84E+02	**8.22E+02** $^{\oslash}$ $<$	8.98E+02	**8.47E+02** $^{\oslash}$ $<$	9.41E+02
f_{19}	8.17E+02 $^{\odot}$ $\approx$	8.78E+02	**8.22E+02** $^{\oslash}$ $<$	9.01E+02	**8.48E+02** $^{\oslash}$ $<$	9.38E+02
f_{20}	7.69E+02 $^{\odot}$ $\approx$	8.63E+02	**8.23E+02** $^{\oslash}$ $<$	8.96E+02	**8.48E+02** $^{\oslash}$ $<$	9.28E+02
f_{21}	8.57E+02 $^{\odot}$ $\approx$	7.94E+02	8.47E+02 $^{\oslash}$ $>$	**5.12E+02**	7.23E+02 $^{\oslash}$ $>$	**5.00E+02**
f_{22}	7.63E+02 $^{\odot}$ $>$	**7.53E+02**	**5.34E+02** $^{\oslash}$ $<$	8.80E+02	**5.00E+02** $^{\oslash}$ $<$	9.14E+02
f_{23}	8.74E+02 $\approx$	8.88E+02	8.40E+02 $^{\oslash}$ $>$	**5.34E+02**	7.26E+02 $^{\oslash}$ $>$	**5.39E+02**
f_{24}	3.94E+02 $^{\oslash}$ $>$	**2.28E+02**	2.14E+02 $^{\oslash}$ $>$	**2.00E+02**	2.21E+02 $^{\oslash}$ $>$	**2.00E+02**
f_{25} †	4.88E+02 $\approx$	4.55E+02	2.13E+02 $\approx$	2.14E+02	2.21E+02 $\approx$	2.21E+02
	$\underline{f_1\text{-}f_{25}}$ ($<, \approx, >$): (1, 17, 7)		$\underline{f_1\text{-}f_{25}}$($<, \approx, >$): (8, 12, 5)		$\underline{f_1\text{-}f_{25}}$($<, \approx, >$): (8, 8, 9)	
	$\overline{f_7 or f_{25}}$ ($<, \approx, >$): (1, 15, 7)		$\overline{f_7 or f_{25}}$ ($<, \approx, >$): (8, 10, 5)		$\overline{f_7 or f_{25}}$ ($<, \approx, >$): (8, 5, 9)	
	$< or >$: 8/23 (35%)		$< or >$: 13/23 (57%)		$< or >$: 17/23 (74%)	
	solutions$^{\oslash}$ or $\odot$: 14/23 (61%)		solutions$^{\oslash}$ or $\odot$: 18/23 (79%)		solutions$^{\oslash}$ or $\odot$: 19/23 (83%)	

† denotes that the specialized initialization ranges are applied instead of bound constraints according to CEC'05's protocal.

Table 3: Summary of the comparison between the three versions of IPOP-CMA-ES and the three versions of MA-LSch-CMA when compared to the average errors of IPOP-CMA-ES-05 on 30 and 50 dimensions. Error values lower than 10^{-8} are approximated to 10^{-8}.

Algorithms vs. IPOP-CMA-ES-05 w.r.t. average error values: (better, equal, worse)			
30 Dim	IPOP-CMA-ES-*ncb*: (12, 5, 8)	IPOP-CMA-ES-*fcb*: (4, 5, 16)†	IPOP-CMA-ES-*acb*: (10, 11, 4)†
50 Dim	IPOP-CMA-ES-*ncb*: (13, 5, 7)	IPOP-CMA-ES-*fcb*: (2, 5, 18)†	IPOP-CMA-ES-*acb*: (13, 6, 6)†
30 Dim	MA-LSch-CMA-*ncb*: (11, 1, 13)	MA-LSch-CMA-*fcb*: (4, 1, 20)†	MA-LSch-CMA-*acb*: (11, 3, 11)
50 Dim	MA-LSch-CMA-*ncb*: (13, 1, 11)	MA-LSch-CMA-*fcb*: (3, 1, 21)†	MA-LSch-CMA-*acb*: (10, 1, 14)

† denotes there is a significant difference over the distribution of average errors between the corresponding algorithm and IPOP-CMA-ES-05 according to a two-sided Wilcoxon matched-pairs signed-rank test at the 0,05 α-level.

Table 4: The average errors obtained by PS-CMA-ES, IPOP-CMA-ES-*ncb* and IPOP-CMA-ES-05 over 25 independent runs for each CEC'05 function. The numbers in parenthesis represent the number of times an algorithm is better, equal or worse, respectively, compared to IPOP-CMA-ES-05. Error values lower than 10^{-8} are approximated to 10^{-8}. The underlined values indicate that the corresponding average error values of PS-CMA-ES are the same or very close to the infeasible average error values obtained by IPOP-CMA-ES-*ncb*.

	30 dimensions			50 dimensions		
f_{cec}	PS-CMA-ES	IPOP-CMA-ES-*ncb*	IPOP-CMA-ES-05	PS-CMA-ES	IPOP-CMA-ES-*ncb*	IPOP-CMA-ES-05
f_1	1.00E−08	1.00E−08	1.00E−08	1.00E−08	1.00E−08	1.00E−08
f_2	1.00E−08	1.00E−08	1.00E−08	9.79E−04	1.00E−08	1.00E−08
f_3	8.00E+04	1.00E−08	1.00E−08	3.28E+05	1.00E−08	1.00E−08
f_4	8.47E−04	2.44E+03 $\odot$ $<$	1.11E+04	1.58E+03	1.32E+05 $\odot$ $<$	4.68E+05
f_5	3.98E+02	2.30E+01 $\oslash$ $>$	1.00E−08	1.18E+03	7.91E+02 $\oslash$ $>$	2.85E+00
f_6	1.35E+01	1.00E−08	1.00E−08	2.98E+01	1.00E−08	1.00E−08
f_7	1.00E−08	1.00E−08	1.00E−08	1.00E−08	1.00E−08	1.00E−08
f_8	2.10E+01	2.07E+01 $\oslash$ $>$	2.01E+01	2.11E+01	2.11E+01 $\oslash$ $>$	2.01E+01
f_9	1.00E−08	1.01E+00	9.38E−01	1.00E−08	1.12E+00 $\odot$ $<$	1.39E+00
f_{10}	1.00E−08	1.37E+00	1.65E+00	1.00E−08	2.36E+00	1.72E+00
f_{11}	3.91E+00	6.36E+00 $\oslash$ $>$	5.48E+00	1.22E+01	1.49E+01 $\oslash$ $>$	1.17E+01
f_{12}	7.89E+01	1.38E+03 $\oslash$ $<$	4.43E+04	2.36E+03	7.38E+03 $\oslash$ $<$	2.27E+05
f_{13}	2.11E+00	2.47E+00	2.49E+00	4.00E+00	4.31E+00	4.59E+00
f_{14}	1.29E+01	1.28E+01 $\oslash$ $<$	1.29E+01	2.25E+01	2.34E+01 $\oslash$ $>$	2.29E+01
f_{15}	2.10E+02	2.01E+02 $\oslash$ $<$	2.08E+02	2.64E+02	2.01E+02 $\oslash$ $<$	2.04E+02
f_{16}	2.61E+01	7.95E+01 $\odot$ $>$	3.50E+01	2.27E+01	1.36E+02 $\oslash$ $>$	3.09E+01
f_{17}	5.17E+01	4.31E+02 $\oslash$ $>$	2.91E+02	6.16E+01	7.69E+02 $\oslash$ $>$	2.34E+02
f_{18}	8.16E+02	8.16E+02 $\oslash$ $<$	9.04E+02	8.36E+02	8.36E+02 $\oslash$ $<$	9.13E+02
f_{19}	8.16E+02	8.16E+02 $\oslash$ $<$	9.04E+02	8.36E+02	8.36E+02 $\oslash$ $<$	9.12E+02
f_{20}	8.16E+02	8.16E+02 $\oslash$ $<$	9.04E+02	8.36E+02	8.36E+02 $\oslash$ $<$	9.12E+02
f_{21}	7.11E+02	8.57E+02 $\oslash$ $>$	5.00E+02	7.18E+02	7.15E+02 $\oslash$ $<$	1.00E+03
f_{22}	5.00E+02	5.98E+02 $\oslash$ $<$	8.03E+02	5.00E+02	5.00E+02 $\oslash$ $<$	8.05E+02
f_{23}	7.99E+02	8.69E+02 $\oslash$ $>$	5.34E+02	7.24E+02	7.27E+02 $\oslash$ $<$	1.01E+03
f_{24}	2.10E+02	2.10E+02 $\oslash$ $<$	9.10E+02	2.14E+02	2.14E+02 $\oslash$ $<$	9.55E+02
f_{25}	2.10E+02	2.10E+02	2.11E+02	2.14E+02	2.14E+02	2.15E+02
	(14, 4, 7)	(12, 5, 8)		(16, 2, 7)†	(13, 5, 7)	

$\oslash$ denotes that all 25 solutions of IPOP-CMA-ES-*ncb* are outside the bounds. $\odot$ denotes some of the 25 solutions of IPOP-CMA-ES-*ncb* are outside the bounds.

† denotes there is a significant difference over the distribution of average errors between PS-CMA-ES and IPOP-CMA-ES-05 according to a two-sided Wilcoxon matched-pairs signed-rank test at the 0,05 α-level.