An application of Iterated Local Search to the Graph Coloring Problem

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Abstract

Graph coloring is a well known problem from graph theory that, when solving it with local search algorithms, is typically treated as a series of constraint satisfaction problems: for a given number of colors k, one has to find a feasible coloring; once such a coloring is found, the number of colors is decreased and the local search starts again. Here we explore the application of Iterated Local Search to the graph coloring problem. Iterated Local Search is a simple and powerful metaheuristic that has shown very good results for a variety of optimization problems. In our research we investigate different perturbation schemes and present computational results on some hard instances from the DIMACS benchmark suite.

1 Introduction

The Graph Coloring Problem (GCP) is a well known combinatorial problem defined as follows: given a directed graph $G = (V, E)$, where V is the set of $|V| = n$ vertices and $E \subseteq V \times V$ is the set of edges, and an integer k (number of colors), find a mapping $\Psi : V \mapsto 1, 2, ..., k$ such that for each $[u, v] \in E$ we have $\Psi(v) \neq \Psi(u)$. In fact, this problem statement corresponds to the decision version, where we look for a solution that satisfies all constraints. In the optimization counterpart, the GCP consist of finding the minimum k satisfying the constraints. Such k is called the chromatic number of G and is denoted by χ_G.

The GCP is an interesting problem for theory and practice. In fact, several classes of real-life problems such as examination timetabling [3] and frequency assignment [4] can be modeled as GCP extensions. Yet, it cannot be expected that an algorithm can find, in polynomial time, a solution to an arbitrary GCP instance, because the GCP is $\mathcal{NP}$-hard [15]. In fact, exact algorithms can solve only small size instances [22]. For larger instances approximate algorithms have to be used and a large number of such algorithms has been proposed [10, 14, 16, 17, 20].

In this work we explore the application of Iterated Local Search (ILS) [21] to the GCP. ILS consists in the iterative application of a local search procedure to starting solutions that are obtained from the previous local optimum through a solution perturbation. The optimization variant of GCP can be stated as a sequence of constraint satisfaction problems, where k is being decremented sequentially by one until no admissible mapping exists, meaning that $\chi_G = k + 1$. In this case, the problem is attacked by ILS as a constraint satisfaction problem.

The article is structured as follows. Section 2 presents a review of approximation algorithms for the GCP and show which one we use. Section 3 introduces available benchmark sets. Section 4 presents

ILS and describes some details of the ILS implementation for the GCP. Section 5 gives experimental results. Conclusions are presented in Section 6.

2 Approximate algorithms for graph coloring

Approximate algorithms for the GCP fall into two main classes, construction heuristics and local search algorithms.

Construction heuristics start from an empty solution and successively augment a partial coloring until the full graph is colored. During the solution construction these algorithms maintain feasibility that is, they return a conflict free coloring. Well known construction heuristics are the Brelaz heuristic [2], the Recursive Largest First (RLF) heuristic [20] or iterated, randomized construction heuristics like the Iterated Greedy algorithm [6].

Local search for the GCP starts with some initial, infeasible color assignment and iteratively moves to neighboring solutions, trying to reduce the number of conflicts until a feasible solution is found or a stopping criteria is met. Two of vertices i, j are in conflict, if both are assigned the same color and $(i, j) \in E$. In fact, local search applied to the GCP iteratively tries to *repair* the current color assignment guided by an evaluation function that counts the total number of conflicts. In case a candidate solution with zero conflicts is encountered, this candidate solution corresponds to a feasible coloring of the graph.

Often, in articles, only results for obtaining an a priori determined number of colors are given. This a priori number of color often corresponds to a very good one. But in real applications, one won't solve the GCP by first guessing a very good coloring and then running the algorithm; rather, one determines a good initial feasible coloring and then tries to reduce the number of colors. In our work we follow this latter approach. In particular, we can distinguish three different phases; a different kind of algorithm of one of the two classes previously mentioned is applied in each of them:

- *Initialization phase*: we use the exact coloring algorithm implemented by Trick[1] [22] based on DSATUR [2]. We stop the algorithm when coloring the graph with a lower number of colors becomes difficult (in the sense of CPU time required). The output of this phase is a feasible coloring.

- *Color number decreasing phase*: once DSATUR or local search (described in the next point) provided a feasible coloring, one color is removed and the nodes that were assigned to the removed color are reassigned to another available color with a heuristic (*dsat*) which colours first the node most constrained, i.e. the node with the highest number of different colors used in its neighborhood and the highest degree (number of edges incident) [2].

- *Local Search phase*: it takes place when the coloring returned by the decreasing phase is not feasible; local search based algorithms are used for finding a feasible coloring. In this sense local search is used for solving the decision version of the problem.

The whole procedure ends when local search is not able to solve feasibility for the given number of colors.

The main aim of this work is to improve the local search phase. In the simplest case, local search algorithms accept only improving moves and they terminate in local optima. Metaheuristics are techniques which are intended to avoid the disadvantages of early termination in local optima. Among the first metaheuristic approaches to the GCP were Simulated Annealing implementations. Simulated Annealing was first applied to the GCP by Chams et al. [5] and was intensively tested by Johnson et al. [17] on random graphs. Among the most widely applied metaheuristics to the GCP are Tabu Search algorithms. The first implementations of Tabu Search are due to Hertz and de Werra [16]. The best peak performance is apparently obtained by the variant proposed by Hao and Dorne [8, 14]. Solving the GCP

[1] Available via http://mat.gsia.cmu.edu/COLOR/color.html, June 2002

by Evolutionary Algorithms was proposed by Davis [7], who reported several crossover operators combined with several ordering of vertices. Eiben et al. [9] applied an Adaptive Evolutionary Algorithm to the GCP; this algorithm changes periodically the evaluation function to avoid local optima. More recently, Laguna and Martí [19] proposed an application of GRASP to the GCP and presented good results for sparse graphs. Finally, several hybrid approaches were proposed. These typically combine Evolutionary Algorithms with Tabu Search implementations. The first such approach for the GCP was proposed by Fleurent and Ferland [11, 12] and the currently most performing of these approaches seems to be the one of Galinier and Hao [13]. The first application of Iterated Local Search to the graph coloring is that of Paquete and Stützle [25], for which interesting performance results were obtained, though significant room for further investigation remains.

3 Benchmark Problems

We tested our approaches on some of the benchmark instances proposed for the Symposium at `http://mat.gsia.cmu.edu/COLOR02`. We first divided the instances into two groups: those which are solved to optimum by the DSATUR algorithm in less than 8 seconds on our machine with a Pentium III 700 MHz processor with 256 KB cache and 512 MB RAM and those for which the DSATUR algorithm requires more longer time. We then divided this latter in three groups: (i) the instances for which the exact optimum is known, (ii) the instances for which the optimum is not known and (iii) the instances which are large (more than 1000 nodes). A table with this classification is given in Appendix. For the experimental analysis of our algorithms we took a sample from each of these three groups. More specifically we used the following graphs:

- *Random graphs*. These are instances from Johnson [17] in which for a given set of vertices the graph is obtained by including each possible edge $[u, v]$ with a probability p for each pair. The chromatic number for these instances is unknown.
- *Leighton graphs*. They are structured graphs generated by a procedure that uses the number of vertices, the desired chromatic number, the average vertex degree and a random vector of integers to generate a certain number of cliques [20]. The chromatic number of these instances is known.
- *Queen graphs*. From Donald Knuths Stanford GraphBase. Given an $n \times n$ chessboard, a queen graph is a graph with n^2 nodes, each corresponding to a square of the board. Two nodes are connected by an edge if the corresponding squares are in the same row, column or diagonal. The chromatic number is not known, but since the maximum clique in the graph is no more than n the chromatic number is not less than n.
- *Full Insertion graphs*. They are obtained by a generalization of Mycielski transformation [24] with inserted nodes to increase graph size but not density.
- *Latin squares graphs.* They are deducted from Latin Squares.

4 Iterated Local Search for coloring graphs

The essence of ILS [1, 26] is to build a biased randomized walk in the space of the local optima (local optima with respect to some local search algorithm). This walk is built by iteratively perturbing a locally optimal solution, next applying a local search algorithm to obtain a new locally optimal solution, and finally using an acceptance criterion for deciding from which of these solutions to continue the search. The perturbation must be sufficiently strong to allow the local search to effectively escape from local optima and to explore different solutions, but also weak enough to prevent the algorithm from reducing to a simple random restart algorithm, which is known to typically perform poorly. ILS is appealing both for its simplicity and for the very good results it provided, for example, in the Traveling Salesman Problem [18] or Scheduling Problems [21].

procedure *Iterated Local Search*
  s_0 = GenerateInitialSolution()
  s = LocalSearch(s_0)
  repeat
    s' = Perturbation(s, *history*)
    s'' = LocalSearch(s')
    s = AcceptanceCriterion(s, s'', *history*)
  until termination condition met
end

Figure 1: Pseudocode of an iterated local search procedure (ILS).

To apply an ILS algorithm, four components have to be specified. These are a procedure GenerateInitialSolution() that generates an initial solution s_0, a procedure Perturbation, that modifies the current solution s leading to some intermediate solution s', a procedure LocalSearch that returns an improved solution s'', and an AcceptanceCriterion that decides to which solution the next perturbation is applied. An algorithmic scheme for ILS is given in Figure 1.

In principle, any local search algorithm can be used, but the performance of the ILS algorithm with respect to solution quality and computation speed depends strongly on the one chosen. An iterated descent algorithm is often used, but it is also possible to apply more sophisticated local search algorithms like Tabu Search, as we do in our case.

The perturbation mechanism should be chosen *strong enough* to allow to leave the current local optimum and to allow the local search to explore different solutions. At the same time, the modification should be *weak enough* to keep enough characteristics of the current local optimum.

The procedure AcceptanceCriterion is used to decide from which solution the search is continued by applying the next pertubation. One important aspect of the acceptance criterion and the perturbation is to introduce a bias between intensification and diversification of the search. Intensification of the search around the best found solution is achieved, for example, by applying the perturbation always to the best found solution and using small perturbations. Diversification is achieved, in the extreme case, by accepting every new solution s'' and applying large perturbations.

4.1 Iterated Local Search operators

4.1.1 Local Search

The commonly used neighborhood when treating the GCP as a decision problem is the 1-opt neighborhood that in each step changes the color assignment of exactly one vertex. For searching the 1-opt neighborhood, two different local search architectures are commonly applied.

1. The first one is based on the *min-conflicts* heuristic, which was proposed to solve constraint satisfaction problems of which graph coloring is a particular case. In each local search step, a vertex that is in conflict is chosen at random; then the color that minimizes the number of conflicts [23] is assigned to this vertex [23].

2. The second scheme examines all possible combinations of vertices and colors (i, c), where i is a node and c is a color, to discover the maximal reduction of the number of conflicts; if several such pairs i, c exist, one is chosen randomly [8, 10]. In [8] this neighborhood is further reduced by considering only moves that affect vertices that are currently involved in a conflict.

The second architecture is greedier than the first, because at each step a larger set of candidate moves is examined. Since a straighforward implementation of a local search using the second neighborhood examination scheme is quite costly, we speed-up the neighborhood evaluation by means of a two-dimensional table T of size $n \cdot k$, where each entry $t_{i,j}$ stores the effect on the evaluation function incurred by changing the color of vertex i to color j. Each time a move is performed, only the part of the table that is affected by the move is updated. This table has to be initialized in $\mathcal{O}(n^2 \cdot k)$, but each update of the matrix then only takes $\mathcal{O}(n \cdot k)$ in the worst case (for sparse graphs the update is much faster) [12].

In addition to this we enhanced both local search architectures with the use of a Tabu Search meta-heuristic in order to avoid to get stucked in a local optimum. For the setting of the tabu list length we adopted two different schemes as suggested in the respective literature. Thus for the first local search architecture we used a fixed value, named *tabu length* (tl), and keep it as the parameter to tune; for the second architecture we followed,instead, the scheme in [14] and [8] where the length of tabu list is taken as $Random(A) + \delta \times |n_c|$, with n_c the set of conflicting vertices and δ the parameter to tune.

Our stopping criterion is the maximum number of iterations without improvement for the lowest number of constraint violations between nodes reached so far. More specifically, this number of maximum iteration is given by $|V| \times k \times f_{end}$ where f_{end} is a parameter to be decided.

4.1.2 Perturbation

The main part of our research concerns the examination of different ways of perturbing solutions. Here, we considered three different possibilities:

P1, **directed diversification**: we perform p_{iter} moves using the Tabu Search procedure with long tabu list settings and mix this strategy with random moves: At each step a random choice is made whether we execute a random search step (such a step is done with a probability w_p) or a Tabu Search step (with probability 1-w_p).

P1, **random recoloring**: a certain number of colors is removed and nodes are recolored with a randomly chosen color different from the one they had previously.

P3, **dsat recoloring**: we use the same type of perturbation as in the random recoloring except that the recoloring of the nodes affected directly by the perturbation is done using the *dsat* heuristic.

The first kind of perturbation was already studied in [25] were it performed when compared to three other schemes which were (i) to assign to some vertices randomly chosen colors, (ii) to add a certain number of edges during some number of iterations, leading to a modification of the instance definition, and (iii) to assign randomly chosen colors to vertices that are in conflict.

Let us note that for all the perturbations the modifications introduced by the solution perturbation are recorded in the tabu list to prevent the search to immediately undo the perturbation.

4.1.3 Initial solution

As stated in Section 2, the initial solution is the solution provided by DSATUR (with CPU time limit of 8 seconds on our machine) and modified by *dsat* for reducing the number of colors.

4.1.4 Acceptance criteria

The acceptance criterion uses one of the following rules: accept every new solution or always apply the perturbation to best solution found so far in the search. This corresponds to strategy for the diversification and the intensification of the search, respectively.

5 Experimental results

5.1 Local search

In a preliminary analysis we compare the two local search architectures. In Table 1 we show the results obtained by using only the two different local searches extended by the tabu search. For each instance, values of $tl \in \{1, 2, 3, 4, 5\}$ and of $\delta \in \{0.5, 1, 1.5, 2, 2.5, 3, 3.5, 4, 4.5, 5\}$ were tested, while as stopping criteria we used $f_{end} = 100$. We report in Table 1 the best results obtained among the different

	Local Search 1					Local Search 2				
inst.	tl	k	% succ.	iter.	time	δ	k	% succ.	iter.	time
1-FullIns-4	-	5	-	-	-	-	5	-	-	-
1-Insertions-5	-	6	-	-	-	-	6	-	-	-
2-FullIns-4	-	6	-	-	-	-	6	-	-	-
2-Insertions-4	-	5	-	-	-	-	5	-	-	-
3-FullIns-4	-	7	-	-	-	-	7	-	-	-
3-Insertions-4	-	5	-	-	-	-	5	-	-	-
4-FullIns-4	-	8	-	-	-	-	8	-	-	-
4-Insertions-4	-	5	-	-	-	-	5	-	-	-
5-FullIns-3	-	8	-	-	-	-	8	-	-	-
DSJC250.5	1	31	60	369,228	15.3	0.5	28	20	153,608	7.2
DSJC500.1	1	14	100	11,307	1.5	1	12	10	564,769	17.5
DSJC500.9	3	132	10	2,343	80.7	1	127	10	3,573	37.5
DSJR500.1	-	12	-	-	-	-	12	-	-	-
DSJR500.5	1	126	40	204,312	31.1	0.5	124	20	1,322,263	74.4
le450-15a	-	17	-	-	-	0.5	15	100	69,575	2.6
le450-15b	-	16	-	-	-	1	15	100	55,241	2.3
le450-15c	2	17	20	2,570	9.3	3	15	60	28,034	3.7
le450-15d	5	18	50	3,911	5.9	2	15	20	65,606	4.9
le450-25c	-	29	-	-	-	0.5	26	100	100,595	3.8
le450-25d	-	28	-	-	-	0.5	26	100	85,563	3.3
le450-5a	3	6	40	2,605	1.4	1	5	100	43,976	2.4
le450-5b	2	6	30	5,713	1.4	0.5	5	60	39,755	2.1
le450-5d	3	6	100	14,511	1.9	0.5	5	100	2,383	3,0
queen13-13	-	16	-	-	-	0.5	14	50	104,607	2.4
queen16-16	-	19	-	-	-	0.5	18	100	1,267	1.0

Table 1: Comparision between two schemes of Local Search. Results are obtained on 10 runs. We report for each instance: the value of delta which gave best results, the best k-coloring found, the percentage of successful runs, the number of iterations needed for solving only the best k-coloring and the time.

values for the parameters. We run both algorithms 10 times on each instance and we report the lowest number of colors found, the percentage of success (indicating the number of runs in which the instance was solved), the mean number of iterations for solving only the best k-coloring found and the mean time. For some instances the algorithms were unable to find a better coloring than the one found by DSATUR (which may be already optimal). In this case only the minimum k-coloring found is reported.

The results clearly indicate that the second architecture for the local search appears to outperform the first one: typically the number of colors found is less than for the first one.

For the following experiments we only use the second local search and use the values of δ for which the best solutions were obtained.

5.2 Perturbations

For the first kind of perturbation proposed we refer to [25] and we use the parameters settings and results presented there. The second and the third kind of perturbation have as the only parameter the number of colors to be removed. We link this value to the size of the instance assuming that the number of color to be removed is a fraction of the current k-coloring to solve ($\gamma \times k$). We tested $\gamma \in \{0.05, 0.15, 0.25\}$.

Further parameters to be considered are the number of local search iterations before the next perturbation and which acceptance criterion should be applied. For the former we use $ls_{iter} \times |V|$ and we tested $ls_{iter} = \{1, 2, 4, 8\}$ and for the latter we considered a percentage of new solutions accepted indicated by Div and we tested: $Div \in \{80\%, 100\%\}$ where $Div = 100\%$ corresponds to the case in which every new solution is accepted.

Table 2 shows the results obtained in 25 runs for the best combination of parameters on different instances. In the table we report the starting k-coloring which is the last one solved by DSATUR, the best k-coloring found at the end of the search, the percentage of successful runs, the mean of the total number of iteration needed by the algorithm to solve to feasibility all the k-coloring (from the starting one down to the best), the mean number of iterations for solving the best k-coloring found and the mean time for the whole run. Last columns indicate the combination of parameters which performed best.

From the experimental results we can observe the following results:

- on Leighton and Queens instances the third type of perturbation can reach the same or higher success rates in slightly less iterations than the second kind.

- on Leighton and Queens instances the second and third type of perturbation shows better performance than the first one, because they are able to reach the same success rate in much less iterations.

- the results on Random Instances (DSJC250.5 and DSJC500.1) seem to indicate that the first kind of perturbation is better than the second and third one. It has to be considered however that in [25] a number of ten millions iterations was used as stopping criteria while in our case the number used is much lower.

In general, there seems to be a slight advantage for the third type of perturbation. The goodness of this third approach has to be seen, in our opinion, in the possibility of combining two different kinds of guidance of the search, each of them lead by different information. That is, the dsat heuristic reconstructs part of a solution according to informations which are different from those used by local search mechanisms. Concerning the strength of perturbation needed, some information can be drawn considering the best combinations of parameters. In particular, the number of colors to remove before applying the *heuristic* seems to be low. Hence, the part of solution to be reconstructed is small and the strength of the perturbation is far from the extreme case of a random restart. Only instances le450-15c and le450-15d seem to require much disruptive perturbations.

When comparing the results of the ILS with those of he simple local search in Table 1, we notice that, with the only exception of instance DSJC500.1, the percentage of success increases. This indicates that, under equal conditions, the application of ILS helps to improve the performance of local search and it gains further robustness.

5.3 Comparison with available benchmarks

We report the peak performance of ILS in Table 3 and we compare the results with the best results published so far which are those obtained by the Hybrid Evolutionary Algorithm (HEA) of Galinier and Hao [14]. They also present peak performance, intended as consequence of tuning efforts. In their approach, after recombination of two elements randomly taken in a population of solutions, they improve the new solution obtained by applying the same Tabu Search algorithm we used here. Therefore comparison can be made properly comparing the number of iterations of Tabu Search. In the table we report the best k-coloring found and the number of Tabu Search iterations needed for solving only that specific number of colors.

For structured graphs we just report the results of Table 2. Indeed, already with $f_{end} = 100$ ILS requires less iterations for finding feasibility and clearly appears more robust. Here, times reported are the times relative only to the last coloring, as done in [14].

For random graphs, instead, results in table 2 are worst than those of HEA but the conditions in which they were generated were different. We tried, therefore, to reproduce the conditions as equal as possible so we re-run the experiments solving only the given number of colors and with $f_{end} = 300$. Results improve but remain slightly worse than those of HEA.

P1										
inst.	start k	best k	% succ.	tot. iter	iter.	time	ls_{iter}	Div	p_{iter}	w_p
DSJC250.5	-	28	76	-	3,550,914	-	30	100	1	0.5
DSJC500.1	-	-	-	-	-	-	-	-	-	-
le450-15a	-	15	100	-	95,246	-	30	100	0.5	0.5
le450-15b	-	15	100	-	63,920	-	30	100	0.2	0.5
le450-15c	-	15	100	-	451,714	-	10	100	2	0.25
le450-15d	-	15	100	-	2,128,483	-	10	100	2	0.25
le450-25c	-	26	100	-	108,312	-	20	100	0.2	0.5
le450-25d	-	26	100	-	99,785	-	30	100	0.2	0.25
queen13-13	-	-	-	-	-	-	-	-	-	-

P2									
inst.	start k	best k	s% succ.	tot. iter	iter.	time	ls_{iter}	Div	γ
DSJC250.5	36	28	40	439,151	367,213	27.9	4	100	0.05
DSJC500.1	15	12	8	389,887	385,389	17.2	4	80	0.15
le450-15a	17	15	100	60,375	60,302	4.24	8	100	0.05
le450-15b	16	15	100	72,287	72,287	4.9	8	80	0.05
le450-15c	29	15	100	262,250	107,173	41.3	4	80	0.25
le450-15d	23	15	100	237,346	168,355	27.4	2	80	0.05
le450-25c	29	26	96	109,264	103,636	13.5	2	100	0.05
le450-25d	28	26	100	85,889	85,374	6.4	4	100	0.05
queen13-13	16	14	64	70,711	70,410	2.9	4	80	0.05

P3									
inst.	start k	best k	s% succ.	tot. iter	iter.	time	ls_{iter}	Div	γ
DSJC250.5	36	28	40	390,408	324,157	39.6	2	100	0.05
DSJC500.1	15	12	4	285,815	274,567	11.3	8	100	0.15
le450-15a	17	15	100	57,918	57,821	4.0	8	100	0.05
le450-15b	16	15	100	67,492	67,492	6.1	4	80	0.15
le450-15c	29	15	100	180,764	84,967	31.6	4	80	0.25
le450-15d	23	15	100	355,683	95,958	77.1	1	80	0.25
le450-25c	29	26	100	100,107	94,839	8.2	4	100	0.05
le450-25d	28	26	100	102,457	101,887	5.5	8	100	0.05
queen13-13	16	14	88	72,887	72,623	2.6	4	80	0.15

Table 2: Comparison between three schemes of perturbations. Results are obtained on 25 runs with $f_{end} = 100$. We report for each instance: the starting k-coloring corresponding to the best k-coloring solved by DSATUR in 8 seconds, the best k-coloring found by the ILS algorithm, the percentage of successful runs, the mean number of iterations for solving all k-colorings, the mean number of iterations for solving only the best k-coloring, the mean CPU time needed, and the winning parameters characterizing the strength of perturbation (percentage of diversification, Div, number of local search iterations before perturbing the solution, ls_{iter}, number of perturbing moves of kind P1, p_{iter}, walk probability w_p, and fraction of current k colors to remove, γ. Results of the comparison of different perturbation schemes (see text for more details).

	ILS				HEA		
inst.	best k	% succ.	iter.	time	% succ.	iter.	time
DSJC250.5	28	80	557,949	29.0	90	490,000	79
le450-15c	15	100	84,967	12.1	60	194,000	47
le450-25c	26	100	94,839	6.7	100	800,000	327

Table 3: Experimental results of ILS compared with those of Hybrid Evolutionary Algorithm (HEA) presented in [14]. ILS was run 10 times on each instance. For the instance DSJC250.5 f_{end} was set equal to 300 while for the other instances was 100. The best k-coloring found, the percentage of successful run, the mean number of iterations for solving best k and the mean time for solving best k are reported.

6 Conclusions

In this work we studied the application of ILS to graph coloring problem focusing on large or hard instances, where "hard" has here the meaning of not fast solvable with DSATUR exact method. The main contribution of this abstract is the analysis of the performance of different kinds of perturbations. The comparison was made after a phase of tuning so that the best configuration of parameters were considered. Among the perturbations tested the most robust approach seems to be to remove a few number of colors and reconstructing the solution by means of some heuristic information. Compared to the perturbation applied in [25], this perturbation also has the advantage of requiring only one parameters instead of three.

In this extended abstract we compared algorithms mainly based on peak performance, that is, the performance of the algorithms when the parameters are reasonably fine-tuned (experiments on the larger instances in the benchmark suite are currently done and the results will be presented at the Symposium). In this process, however, we noted that parameter settings may be sensitive to the particular instance under solution. It is our intention to extend our algorithms by some type of reactive search, where parameters are tuned automatically during the algorithm run. To this aim, we are currently analyzing the relationship between distance of solutions, diversification and search performance in order to gain more insight into the problem and promising ways to automatically adjust the free parameters.

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A Instances classification

Our instances classification. We first divided the instances into four groups: (i) those which are solved to optimum by the DSATUR algorithm in less than 8 seconds on our machine with a Pentium III 700 MHz processor with 256 KB cache and 512 MB RAM; (ii) the instances for which the exact optimum is known but it can not be found by DSATUR in short time; (iii) the instances for which the optimum is not known and (iv) the instances which are large (more than 1000 nodes).

solved by DSATUR	opt-known	opt-unknown	large
inithx.i.1.col	1-Insertions-4.col	1-FullIns-3.col	5-FullIns-4.col
2-Insertions-3.col	2-Insertions-4.col	1-FullIns-4.col	3-FullIns-5.col
3-Insertions-3.col	4-Insertions-3.col	1-FullIns-5.col	3-Insertions-5.col
anna.col	le450-15a.col	1-Insertions-5.col	4-FullIns-5.col
david.col	le450-15b.col	1-Insertions-6.col	abb313GPIA.col
fpsol2.i.1.col	le450-15c.col	2-FullIns-3.col	ash608GPIA.col
fpsol2.i.2.col	le450-15d.col	2-FullIns-4.col	ash958GPIA.col
fpsol2.i.3.col	le450-25c.col	2-FullIns-5.col	qg.order100.col
games120.col	le450-25d.col	2-Insertions-5.col	qg.order40.col
homer.col	le450-5a.col	3-FullIns-3.col	qg.order60.col
huck.col	le450-5b.col	3-FullIns-4.col	wap01.col
inithx.i.2.col	le450-5d.col	3-Insertions-4.col	wap02.col
inithx.i.2.col	queen11-11.col	4-FullIns-3.col	wap03.col
inithx.i.3.col	queen13-13.col	4-FullIns-4.col	wap04.col
inithx.i.3.col	queen8-8.col	4-Insertions-4.col	wap05.col
jean.col	queen9-9.col	5-FullIns-3.col	wap06.col
le450-25a.col		ash331GPIA.col	wap07.col
le450-25b.col		DSJC1000.1.col	wap08.col
miles1000.col		DSJC1000.5.col	
miles1500.col		DSJC1000.9.col	
miles250.col		DSJC125.1.col	
miles500.col		DSJC125.5.col	
miles750.col		DSJC125.9.col	
mug100-1.col		DSJC250.1.col	
mug100-25.col		DSJC250.5.col	
mug88-1.col		DSJC250.9.col	
mug88-25.col		DSJC500.1.col	
mulsol.i.1.col		DSJC500.5.col	
mulsol.i.2.col		DSJC500.9.col	
mulsol.i.3.col		DSJR500.1.col	
mulsol.i.4.col		DSJR500.1c.col	
mulsol.i.5.col		DSJR500.5.col	
myciel3.col		latin-square-10.col	
myciel6.col		queen10-10.col	
myciel7.col		queen12-12.col	
queen5-5.col		queen14-14.col	
queen8-12.col		queen15-15.col	
zeroin.i.1.col		queen16-16.col	
zeroin.i.2.col		school1-nsh.col	
zeroin.i.3.col		school1.col	
myciel4.col		will199GPIA.col	
le450-5c.col			
myciel5.col			
qg.order30.col			
queen6-6.col			
queen7-7.col			

B Benchmark code

We report the results obtained by running the benchmark code available from `http://mat.gsia.cmu.edu/COLOR/color.html` on our machine.

DFMAX(r100.5.b)
0.01 (user) 0.00 (sys) 0.00 (real)
Best: 4 57 35 5 61 34 3 62 90

DFMAX(r200.5.b)
0.13 (user) 0.00 (sys) 0.00 (real)
Best: 113 86 147 66 14 134 32 127 161 186 70

DFMAX(r300.5.b)
1.16 (user) 0.00 (sys) 1.00 (real)
Best: 279 222 116 17 39 127 190 158 196 288 263 54

DFMAX(r400.5.b)
7.15 (user) 0.00 (sys) 7.00 (real)
Best: 370 108 27 50 87 275 145 222 355 88 306 335 379

DFMAX(r500.5.b)
27.26 (user) 0.01 (sys) 28.00 (real)
Best: 345 204 148 480 16 336 76 223 260 403 141 382 289