Data association in multi-target detection using the transferable belief model.

André Ayoun
THOMSON-CSF ISR
3 rue Ampre,
91349 Massy Cedex, France
andre.ayoun@isr.thomson-csf.com

and
Philippe Smets
IRIDIA
Université Libre de Bruxelles
50 av. Roosevelt, CP 194-6, 1050 Bruxelles, Belgium
psmets@ulb.ac.be
http://iridia.ulb.ac.be/~psmets

May 22, 2000

Abstract

In the transferable belief model, a model for the quantified representation of beliefs, some masses can be allocated to the empty set. It reflects the conflict between the sources of information. This quantified conflict can be used in order to solve the problem of data association in a multi-target detection problem. We present and illustrate the procedure by studying an example based on the detection of sub-marines. Their number and the association of each sensor to a particular source are determined by the procedure.
Keywords: Transferable belief model, belief functions, data association, Dempster-Shafer theory, conflict of beliefs, fusion.

1 Introduction.

Multi-sensor data fusion is the data processing function that combines data collected from systems comprising several sensors. These multi-sensor systems are characterized by the following features that must be taken into account:

- the different sensors observe the same scene, or at least partially (overlapping fields of view);
- they may have different resolutions, accuracies and points of view.

The usual functions requested from multisensor systems are detection, localization, and recognition of the objects that may be present in the observed area.

After presenting the generic architecture and usual terminology for sensor fusion, this paper focuses on the problem of associating symbolic data produced by the different sensors, possibly conflicting. A simplified "submarine detection" problem illustrates the association problem. This problem had been introduced by Schubert [11] and our solution is strongly influenced by his solution.

The problem can be summarized as follows: given a collection of sensor declarations of the form "I see a target" and knowing roughly the capabilities of the different sensors in terms of reliability and detection areas. The system shall make decisions about the number of targets and the areas where they should be, by associating and fusing the declarations. For the sake of simplicity, the problem is purely static (no temporal aspect is considered). In the same way, the sensors are supposed to output only symbolic declarations and not to provide with any attribute, which could be useful for association.

Classical solutions for this problem are a) purely probabilistic,([3], [4], [7], [17]) based on probability models for a priori knowledge and for measurement errors, b) possibilistic models [6]. Probabilistic approach is faced with the problem of conflict elimination. On the other hand, possibilistic framework offers a solution to conflict suppression but looses information contained in the probabilistic model. The proposed solution is based on the transferable belief model, a numerical model to represent quantified beliefs based on belief functions [12,13,14,15].

The choice of the TBM is based on the next arguments. The TBM is mathematically more general, thus theoretically more flexible, than its competitors based on probability and possibility functions. It is endowed by well justified rules for combining pieces of evidence and for weighting them. Still more important is the fact that the TBM allows us to handle explicitly the conflict that can appear between the pieces of information thanks to the non-normalization of the belief functions when combining pieces of evidence. This measure of conflict is the core of the method developed here. These properties explain why the TBM is an appropriate tool to solve the problem of data association in a context of multi-target detection.

Section 2 recalls the usual problems of multisensor fusion and indicates where the association takes place within an generic processing architecture. Section 3 describes the original problem of Schubert and details its resolution using the TBM. The given example is fully developed and an operational interpretation of intersensors and internal conflicts is proposed.

2 Generic sensor fusion architecture.

2.1 Terminology.

The following terminology is currently used within the sensor fusion community ([17], [3], [4], [7]). In [17], Waltz and Llinas define:

- *Detection.* A binary decision to determine the presence of a signal (or a target) based upon sensor measurements. In the sequel we use the word detection for the decision or declaration performed by a single sensor assessing the presence of a target (object to be detected).
- *Association.* Cross correlation of measurements and m-ary decisions to partition all measurements into sets of common origin. One can distin-

guish between associating a set of measurements (partitioning) and associating a measurement (or a set of measurements) to a given object. In the latter case, the association is much easier if an observation model is available, which allows to derive the expected measurements, knowing the state of the objet. In dynamic situations, the 'racked' object state is estimated thanks to the fusion of past information correlated to the corresponding object.

- *Classification.* m-ary decision, using associated sensor measurements to assign the measurements to one of m classes. Classification refers to the assignment of the associated measurements to the predefined classes. A more operational concept is ***recognition*** which is nothing else than classification among domain-application classes (such as target types).

Other definitions we require are:

- *Sensor.* A device or system that observes a scene and outputs pieces of information related to the objects present in the scene.

- *Measurement.* Numerical data delivered by a sensor in relation to an object of the scene. The sensor output can be composed of both measurement and data characterizing the measurement such as accuracy, quality or confidence. Measurements can be related to a variety of attributes, for example the position or color of the object.

- *Symbolic declarations.* Sensors may output either numeric measurements such as position, speed, spectrum,... , or symbolic declarations such as presence/absence, type of the detected object,.... Along with numeric measurements, the concept of accuracy is used, whilst confidence is rather used to characterize the symbolic declarations. The various uncertainty theoretical frameworks (probabilities, evidence theory, fuzzy sets,...) apply to model accuracy as well as confidence.

- *Localization.* It consists in estimating the location (position) of the detected objet from the measurements provided by the sensors. When the object evolves in time, the sequel of measurements are used, in relation with dynamic models of the object behavior to track these objects. In other words, tracking consists both in association and position estimation.

- *Target.* Objects that if present in the scene should be detected, or localized or classified.

- *False alarm.* An erroneous detection: an object is declared to be seen when in fact there is none.

- *Resolution.* If two objects in the observation space are too close to each others, they cannot be observed as two different objects by a given sensor. The minimum distance under which two objects are not separated is called resolution.

- *Prediction.* When temporal aspects are taken into account, temporal continuity of the observation must be achieved. Prediction of the current

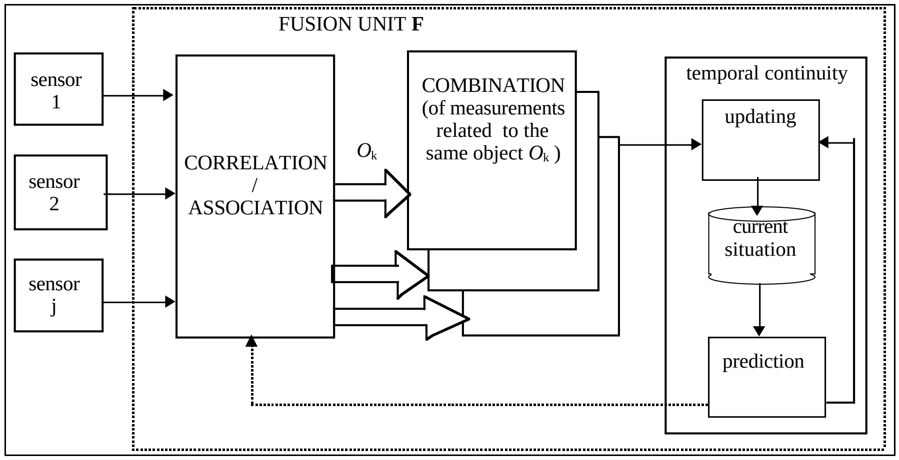

Figure 1: Generic functional architecture.

scene state, based on past observations, must be done before correlating the present observations to the known state.

- *Updating.* Refinement of the current scene state based on the current observation.

Prediction and updating both refer to an estimation of the state : updating occurs immediately after a measurement (or a set of measurements) has been received, while prediction is the extrapolation of the estimation based on behavior models

2.2 Generic functional architecture.

Figure 1 illustrates the sequence of operations performed in sensor fusion. The first step consists of correlating (or associating) the measurements delivered by the different sensors which correspond to a same object. Then the set of measurements associated to object k are "fused" in order to provide the expected information (such as localization or classification).

Finally, the temporal continuity is maintained through prediction and updating. In the remainder of this paper, for the sake of clarity, temporal continuity aspects are not dealt with. However, the correlation of sensor measurements to existing objects (maintained in the "current situation") can be performed.

Another problem, which is not addressed here, is the problem of detection and recognition of entities that are composed of several elementary objects. Such entities are called aggregates, and the corresponding problem refers more to *scene intelligence* rather than to *sensor fusion*.

3 The submarine detection example.

In most surveillance applications, the sensors are spread inside or around the area to be observed. For this reason, the association problem is complex and may result in a highly combinatorial problem. To illustrate this complexity, we propose a surveillance example, called the submarine detection problem, which could be easily translated into a battlefield surveillance problem.

The interest of this problem is to focus on the association of sensor observations. In this simplified example, we do not make any assumption on similarity between measurements originated from a same target (or source). Use of similarity models between observations is introduced by Appriou [1], [2], Grabisch [6] and Nimier [10].

3.1 The problem.

Originally, the problem is expressed as follows:

Having five sensors that can locate submarines in a given observed scene, how many submarines are there, which sensor is associated with which submarine and where are the submarines?

We assume that the five sensors, denoted A, B, C, D and E, respectively, observe the same sea area (observed scene). For convenience, we assume that the scene is composed of resolution cells. Let Ω be the set of resolution cells. Each sensor is looking at all resolution cells (the whole sea area). Once it detects one target, it reports the detection and the position of the resolution cell where the detection was made.

The sensors are characterized by their measurement errors:

- they can locate at most one target, whatever the real number of targets is. (This corresponds to partial detection in case of multiple targets, as some (all but one) targets are undetected). It could reflect the fact the sensor first localizes a target and then measures its distance. So either the sensor is 'blind' and misses all targets, or it detects one target and provides its location. We do not consider a more elaborated system where the sensor could then move to another target.

- the location precision is much better than the size of the resolution cell so that in this example there is no localization error.

Besides, we know the confidence or reliability of each sensor, which we define as the 'probability that the sensor is in working condition', assuming that when the sensor is in working condition, what it states is true. Table 1 lists the confidence values for the five sensors. These confidence values result from the user's opinion about the sensors' reliabilities. Suppose sensor A states: there is an object in α. The .7 value is the probability that the sensor is reliable, in which case there is indeed an object in α. The complementary .3 value is the probability that the sensor is not reliable, in which case we cannot accept anything of what it states: there might be an object, it might be anywhere, even in α by sheer luck. Suppose now that the sensor A states: I don't detect any object in the observed scene. If the sensor is in working condition, what happens with probability .7, then it means that all detection cells are empty. If the sensor is not in working condition, what happens with probability .3, it means that I know nothing about the fact that there is or not a submarine in any detection cells of the observed scene. In our example, we consider a case where all sensors report a detection.

The .7 probability value could be obtained

Sensor	A	B	C	D	E
Confidence	0.7	0.8	0.6	0.6	0.9

Table 1: Confidence values of the five sensors.

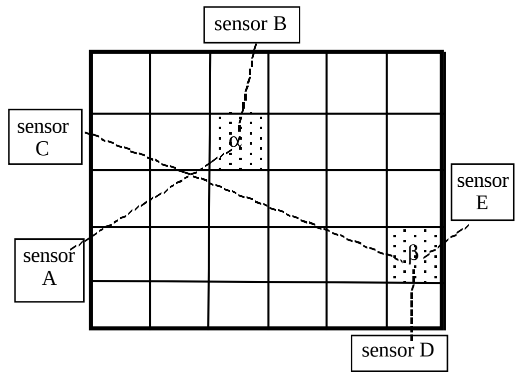

Figure 2: Measurements of sensors A, B, C, D, E, in areas α and β.

- either because in the past the sensor has been properly working in 70% of the cases (and past experience is large enough and unbiased so that observed frequencies can be equated to probabilities) or
- because if the user had to bet on the sensor being in working condition or not, he/she would bet with a probability .7 that the sensor is reliable, and .3 that it is not reliable.

Both cases lead to the same analysis.

Let us consider the very simple case where sensors A and B locate a target in resolution cell α, while sensors C, D and E locate a target in cell β, and cells α and β are not overlapping.

The questions are therefore:

1. how many submarines are we detecting? (a detection problem)
2. where are the submarines located ? (a localization problem).
3. which sensor has detected which target? (an association problem).

This problem is illustrated in figure 2.

If the sensors were perfect, we would conclude that there are two targets, one in a and one in b, that sensors A and B report on target a, whereas sensors B, C and D report on target b. We consider how these conclusions could be reached once uncertainty is introduced. This example is only illustrative, but the method can be extend to less obvious cases when common sense would be at lost.

3.2 Solution using the Transferable Belief Model and a sensor clustering method.

3.2.1 The TBM.

The TBM is a model to represent quantified beliefs based on belief functions. It generalized the classical bayesian model, in that it covers exactly the same problem, the representation of beliefs, but instead of assuming that beliefs are additive, like in the probability model, it considers that degrees of beliefs are super-additive. One of its major properties is that the belief given in the union of two mutually exclusive events can be larger or equal to the sum of the beliefs given to the individual events. This model is endowed with rules for combining belief functions generated by distinct pieces of evidence and for weighting the collected belief functions, what is called in this context a discounting. An up to date presentation of the TBM and the belief functions can be found in [13].

The TBM considers that belief holding and decision making are distinct processes. Beliefs can be held outside any decision context. But once decisions must be made, it is assumed that they must be somehow induced by the underlying beliefs. So even though beliefs are represented by belief functions, when decision must be made, a probability function is generated which will be used to make optimal decisions as classically advocated. The function that builds the needed probability function is called the pignistic transformation, and the resulting probability function is called the pignistic probability function, denoted $BetP$ [15].

3.2.2 Notation.

We use the following notation $m^{\Omega}_{\text{agent}}[K]$ to represent basic belief assignments and their related functions. The notation made explicit what are the domain of the beliefs (also called the the frame of discernment, and denoted Ω), the origin of the beliefs (the background knowledge K), and who is the belief holder (also called the agent). So $m^{\Omega}_{Y}[K]$ is the basic belief assignment that represents the beliefs held by an agent Y on the frame of discernment Ω given the background knowledge K. The term $m^{\Omega}_{Y}[K](x)$ is the value taken by this basic belief assignment at $x \subseteq \Omega$. For the related belief functions, plausibility functions..., the m symbol is replaced by bel, pl...

The symbol $BetP^{\Omega}[m^{\Omega}]$ denotes the pignistic probability function induced on the betting frame Ω by a basic belief assignment m^{Ω}. Its values are given by:

$$BetP^{\Omega}[m^{\Omega}](A) = \sum_{\emptyset \neq X \subseteq \Omega} \frac{m^{\Omega}(X)}{1 - m^{\Omega}(\emptyset)} \frac{|X \cap A|}{|X|}$$

$BetP^{\Omega}[m^{\Omega}]$ is a classical probability function to be used to compute expected utilities when decisions must be made and beliefs about which element of Ω prevails are represented by the basic belief assignment m^{Ω} ([15], [13]).

To combine n belief functions, we use Dempster's rule of combination, denoted by the $\oplus$ symbol. Given n basic belief assignments $m^{\Omega}[E_i], i = 1...n$, based on n distinct pieces of evidence $E_1, ...E_n$, the basic belief assignment that results from the combination of these basic belief assignments is given, for every $A \subseteq \Omega$, by:

$$m^{\Omega}[E_1, E_2, ...E_n](A) = \sum_{A_1 \cap A_2 \cap \ldots \cap A_n = A} m^{\Omega}[E_1](A_1) m^{\Omega}[E_2](A_2) \ldots m^{\Omega}[E_n](A_n).$$

For simplicity sake, we will omit those indices that are not necessary.

In the Transferable Belief Model, we do not impose that $m(\emptyset) = 0$. So when several belief functions are combined by Dempster's rule of combination, the basic belief mass $m^{\Omega}[E_1, E_2, ...E_n](\emptyset)$ can be positive.

One interpretation of $m(\emptyset) > 0$ is that there is some underlying conflict between the sources that are combined in order to produce the bba m, where bba is short for 'basic belief assignment'. Consider for example a bba m_0 defined on Ω with $m_0(\emptyset) = 0$ and $bel_0(\overline{A}) > 0$, where $\overline{A}$ is the complement of A relative to Ω. Suppose you collect another piece of evidence, a conditioning one, that just states that A is true for sure. Its related bba is given by m_A with $m_A(A) = 1$. You had some belief given to $\overline{A}$ and now you learn that no belief should have been given to $\overline{A}$. So a conflict appears between the first belief bel_0 and the new one m_A. The largest $bel_0(\overline{A})$, the largest the conflict. The worst conflict between two pieces of evidence would be encountered if $bel_0(\overline{A}) = 1$, as it means that you were sure that $\overline{A}$ holds and now you learn for sure that it is false. This leads to a contradiction, the conflict encountered in classical logic. The best case would be $bel_0(\overline{A}) = 0$, in which case there is no conflict between bel_0 and the new piece of information. After combining the two pieces of information, we get a new bba $m = m_0 \oplus m_A$ with $m(\emptyset) = bel_0(\overline{A})$. So $m(\emptyset)$ can be understood as the amount of conflict between m_0 and the conditioning evidence represented by m_A. This can be generalized to any pair of belief functions, and we can understand $m(\emptyset)$ as the amount of conflict present in m, and that results from the pieces of evidence that were taken into account when building m

The measure of conflict is at the core of our data association method. We will try to create an association between the sensors and the targets that 'minimize' the conflict observed after combining the pieces of evidence collected from the various sensors, or at least to keep it at an 'acceptable' level.

3.2.3 Solution.

Each sensor produces a (very simple) belief function with a mass 1 allocated to the resolution cells α (sensors A and B) or β (sensors C, D and E) (see figure 2). So, we have: $m^{\Omega}_A(\alpha) = m^{\Omega}_B(\alpha) = m^{\Omega}_C(\beta) = m^{\Omega}_D(\beta) = m^{\Omega}_E(\beta) = 1$. We will omit the Ω superscript for simplicity sake.

These basic belief assignments translate the sensors' claims. For example, sensor A claims that there is an object in α and sensor D claims there is one in β. The fusion unit F, which function is to integrate the data, collects each of these five basic belief assignments and discounts them with the confidence that F gives to each sensor (those of table 1). For instance, the result of discounting sensor A basic belief assignment is the basic belief assignment $m_F[A](\alpha) = .7$ and $m_F[A](\Omega) = .3$. This basic belief assignment expresses the belief held by F that there is an object in α and this belief results from what sensor A states and F's opinion about the reliability of sensor A. The resulting discounted belief functions are given in the table 2.

sensor	A	B	C	D	E
$m_F[S](\alpha)$	.7	.8			
$m_F[S](\beta)$			.6	.6	.9
$m_F[S](\Omega)$	.3	.2	.4	.4	.1

Table 2: Basic belief assignments induced by the five sensors and discounted by fusion unit F.

We will now consider subsequently the hypotheses that these measurements resulted from either one submarine or two submarines.

Case 1: suppose there is **ONE** submarine. This means that all the declarations refer to the same event. Therefore, the fusion unit F combines the five belief functions using Dempster's rule of combination, denoted by $\oplus$.

$$m_F[ABCDE] = \oplus_{S \in \{A,B,C,D,E\}} m_F[S].$$

The result is:

$$\begin{aligned} m_F[ABCDE](\emptyset) &= 0.925 \\ m_F[ABCDE](\alpha) &= 0.015 \\ m_F[ABCDE](\beta) &= 0.059 \\ m_F[ABCDE](\Omega) &= 0.001 \end{aligned}$$

The value 0.925 obtained for the empty set reflects a high degree of conflict between the sensors' measurements.

Case 2: suppose there are **TWO** submarines. Now, let us assume that there are two submarines, so that some sensor measurements may refer to one submarine and the others to the other submarine. Schubert's idea [11] is to cluster the sensors whose measurements are compatible, that is refer to the same target. As the hypothesis is that there are two submarines, the set of five measurements is to be partitioned into two clusters, denoted χ_1 and χ_2.

Table 3 presents the masses given to the conflicts when the five sensors are grouped in two clusters.

The least conflicting solution is AB, CDE (see table 3) which has the smallest internal conflict. The conflict is null. We accept the heuristic that we should try to keep the number of submarines as small as possible. The presence of two submarines is sufficient to explain the data. Of course there might be three or more submarines. As far as the data can be explained by the presence of two submarines, that hypothesis is accepted.

We can therefore conclude that :

- there are two submarines, without conflict;
- one is in α with belief .94 and plausibility 1, and it is observed by sensors A and B;
- one is in β with belief .984 and plausibility 1 and it is observed by sensors C, D and E.

Cluster χ_1	Cluster χ_2	$m_F[\chi_1](\emptyset)$	$m_F[\chi_1](\emptyset)$
A	BCDE	0	0.787
B	ACDE	0	0.689
C	ABDE	0	0.902
D	ABCE	0	0. 902
E	ABCD	0	0.79
AB	CDE	0	0
AC	BDE	0.42	0.768
AD	BCE	0.42	0.768
AE	BCD	0.63	0.672
BC	ADE	.48	.672
BD	ACE	.48	.672
BE	ACD	.72	.588
CD	ABE	0	.846
CE	ABD	0	.564
DE	ABC	0	.564

Table 3: Values of the basic belief masses given to $\emptyset$ by F after combining the sensors' basic belief assignments within each cluster χ_1 and χ_2.

3.3 Generalization.

3.3.1 Sensors exhibiting internal conflict.

The previous example can be extended to sensors exhibiting internal conflict, that is conflict present before combining the sensor belief functions. Such "logical sensors" could themselves be built as a collection of individual sub-sensors (figure 3). We further assume that these sensors are so built that all sub-sensors of a given sensor look at the same submarine. So sensor A is built out of sub-sensors A1 and A2. By construction sub-sensors A1 and A2 observe the same object. Similarly with sub-sensors B1, B2 and B3 that look also to the same object, but not necessarily the one observed by A1 and A2. Each sub-sensor opinion is described by a belief function and their combination could generate some conflict.

To illustrate this case, the previous example has been modified. Let Ω be the set of three resolution cells and let α, β and γ be these three resolution cells. The basic belief masses generated by each sensor are allocated to the subsets of the power set of Ω and not only to singletons of Ω. Table 4 presents the basic belief assignments generated by five sensors A,B,C,D and E on Ω.

The values of these basic belief assignments can be interpreted as follows:

- all sensors have a small amount of internal conflict (0.05),
- sensors A and B are moderately confident in the presence of a submarine in α but discrepancies appear in the confidence given to $\{\alpha, \beta\}$ or to $\{\alpha, \gamma\}$,
- sensors C, D and E are support essentially the fact that a submarine is present in β.

Using the same clustering technique as in section 3.2, the best partition is AB,CDE, with an average conflict of 0.15. Cluster AB supports the presence of a submarine in α ($bel[A, B](\alpha) = .79$, $pl[A, B](\alpha) = .81$, $BetP(\alpha) = .93$) whereas

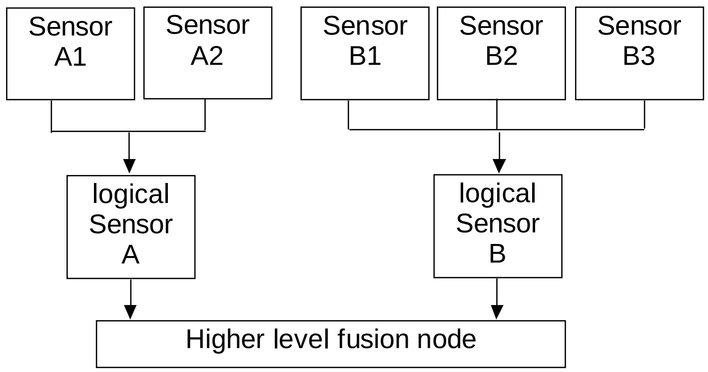

Figure 3: Logical sensor exhibiting internal conflict.

$\Omega \setminus$ Sensors	A	B	C	D	E	A,B	C,D,E
$\emptyset$	0.05	0.05	0.05	0.05	0.05	0.14	0.163
α	0.6	0.7	0	0.1	0	0.79	0.015
β	0	0.1	0.5	0.5	0.5	0.03	0.75
γ	0.1	0	0	0	0.1	0.02	0.012
$\alpha\beta$	0.1	0	0.1	0	0	0	0.012
$\alpha\gamma$	0	0.1	0	0	0	0.02	0
$\beta\gamma$	0	0	0	0.1	0.05	0	0.03
$\alpha\beta\gamma$	0.15	0.05	0.35	0.25	0.3	0	0.018

Table 4: The five basic belief assignments collected from the five sensors A to E, and those obtained after combining sensors A and B, and sensors C,D and E. Subsets of Ω are denoted by their elements, so $\alpha\beta$ denotes the set $\{\alpha,\beta\}$

cluster CDE supports the presence of a submarine in β ($bel[C,D,E](\beta) = .75$, $pl[C,D,E](\beta) = .92$, $BetP(\beta) = .92$).

This clustering method can be easily extended to the case of any number M of sensors. For high values of M, the search for the appropriate number of submarines can be computationally unmanageable. Stepwise procedures similar to the procedure described in stepwise regression analysis and other approximations are required.

Note: To perform such stepwise computation, the next equations are useful. Suppose n+1 basic belief assignments $m_0, m_1...m_n$. Let $m_{1...n} = m_1 \oplus m_2 \oplus ... \oplus m_n$, and $m_{0...n} = m_0 \oplus m_{1...n}$. Then the computation of $m_{0...n}(\emptyset)$ can be done without computing the whole basic belief assignment $m_{0...n}$. Indeed,

$$m_{0...n}(\emptyset) = \sum_{A \subseteq \Omega} m_0[A](\emptyset) m_{1...n}(A).$$

The inverse equation that might be useful for backward elimination of the basic belief assignment m_0 from $m_{0...n}$ is

$$m_{1...n}(\emptyset) = \sum_{A \subseteq \Omega} c(A) m_{0...n}(A)$$

where

$$c(A) = \sum_{X \subseteq A} \frac{(-1)^{|X|}}{q_0(X)}$$

.

The value $m_0[A](\emptyset)$ is the mass given to $\emptyset$ after conditioning m_0 on $A \subseteq \Omega$ by Dempster's rule of conditioning (without normalization) [13]. The coefficients $c(A)$ can be computed by the same Möbius transform that links the b function to the m function, replacing $b(X)$ by $1/q(X)$ and multiplying the resulting terms by $(-1)^{|X|}$ [9].

These relations simplify the computation for forward-backward stepwise selection of a new association of the sensors. Suppose sensors 1 to n have already been associated to a given submarine. We can then check what would become the conflict when associating one sensor among sensors $n+1, n+2...n+m$ to the same submarine. We only need to compute $m_{1...n,n+j}(\emptyset)$ for $j = 1...m$, and chose the sensor $n+j$ that minimize the increase in conflict. Similarly we can judge how much the conflict would be reduced by eliminating from one association the belief function m_i, by computing $m_{1...i-1,i+1,...n}(\emptyset)$ using the backward elimination formula.

3.3.2 Logical sensors composed of subsensors.

The following example illustrates how to obtain internal conflict within logical sensors composed of physical subsensors.

Suppose we have five sensors A to E and that sensors A to D are composed each from two sub-sensors (indexed 1 and 2), and that sensor E is composed from three sub-sensors (denoted E_1, E_2 and E_3). We also assume that each sensor is looking at one and only one target, hence its sub-sensors are all observing the same target. Suppose now that there are five resolution cells, denoted α, β, γ, δ and ϵ.

Sensor	A		B		C		D		E		
Sub-sensor	A_1	A_2	B_1	B_2	C_1	C_2	D_1	D_2	E_1	E_2	E_3
α	.2	.1									.05
β			.2								
$\alpha\beta$	.1			.5							
γ							.4		.3	.3	.3
$\alpha\gamma$		.2						.5	.4		.2
$\beta\gamma$								.4		.4	
$\alpha\beta\gamma$							.2				
δ						.1					
$\gamma\delta$					.5	.4					
$\alpha\beta\gamma\delta\epsilon$	.7	.7	.8	.5	.5	.5	.4	.1	.3	.3	.45

Table 5: Basic belief assignments for each sub-sensors (missing values are null).

Sensor	A	B	C	D	E
$\emptyset$					.04
α	.3				.01
β		.2			
$\alpha\beta$	.07	.4			
γ				.4	.76
$\alpha\gamma$	.14			.3	.1
$\beta\gamma$				.24	.05
$\alpha\beta\gamma$				.02	
δ			.1		
$\gamma\delta$			.65		
$\alpha\beta\gamma\delta\epsilon$	.49	.4	.25	.04	.04

Table 6: Basic belief assignments for each sensor obtained by combining (by Dempster's rule of combination) the basic belief assignments generated by their sub-sensors, respectively.

Table 5 presents the basic belief assignments produce by each sub-sensor. Table 6 presents the basic belief assignments computed for each sensor by combining (by Dempster's rule of combination) the basic belief assignments generated by their sub-sensors, respectively.

Table 7 presents the conflict (the mass given to $\emptyset$) when all five sensors are assumed to report on the same target (total conflict = 0.59), and the conflicts computed when the five sensors are clustered into two groups, those in each group reporting to the same target (case with two targets). The solution resulting in the lowest level of conflict is obtained when A and B are in one cluster, denoted χ_1, and C, D and E are in the second cluster, denoted χ_2.

To proceed, we need to introduce the notion of tolerable conflict (see further note on tolerable conflict). There is no absolute meaningful threshold. But we can consider that if a sensor that we accept as a potential source of information exhibits an internal conflict value v, a combination of such n sensors may show a conflict of $1-(1-v)^n$ (as explained hereafter). As a consequence, the total conflict generated by clustering the n sensors into subsets of k_1 and k_2 sensors

Cluster χ_1	Cluster χ_2	$m_F[\chi_1](\emptyset)$	$m_F[\chi_2](\emptyset)$	sum
$\emptyset$	ABCDE	0	.59	.59
A	BCDE	0	.47	.47
B	ACDE	0	.30	.30
C	ABDE	0	.45	.45
D	ABCE	.04	.50	.54
AB	**CDE**	**.06**	**.08**	**.14**
AC	BDE	.16	.33	.49
AD	BCE	.11	.42	.53
AE	BCD	.14	.38	.52
BC	ADE	.28	.20	.48
BD	ACE	.16	.26	.42
BE	ACD	.23	.24	.47
CD	ABE	.04	.33	.37
CE	ABD	.05	.27	.32
DE	ABC	.02	.39	.41

Table 7: Values of conflict ($m(\emptyset)$) observed when the five sensors are assumed to report on the same target (case with one target) and when the five sensors are clustered into two groups, those in each group reporting to the same target (case with two targets). The best clustering is indicated in bold.

(with $k_1 + k_2 = n$) should not exceed $1 - (1 - v)^{k_1} + 1 - (1 - v)^{k_2}$. This value is selected as a tolerance threshold.

With the preferred cluster, the sum of the conflict is 0.14. This value is to be compared to the calculated threshold, with $k_1 = 2$ and $k_2 = 3$, which is 0.19. Therefore, we can consider that the obtained level of conflict agrees with what could be expected with the five sensors and may be tolerated.

With five sensors, the tolerable conflict is $1 - (1 - 0.04)^5 = 0.18$, hence the observed value of 0.59 is not tolerable, and the hypothesis of one target should be rejected.

We can thus accept that there are two targets, that sensors A and B report on one target, and that sensors C, D and E report on a second target.

Table 8 presents the pignistic probabilities obtained on $\{\alpha,\ \beta,\ \gamma,\ \delta,\ \epsilon\}$ from sensors in clusters χ_1 and χ_2 . If a decision is based on the most probable location, χ_1 reports that one target is in α and χ_2 reports that the other target in is γ.

For illustrative purpose, we also present in Table 8 the pignistic probabilities computed from the basic belief assignments observed for each sensor individually.

- Sensor A supports the presence of a target in location α,
- Sensor B supports the presence of a target in location β ,
- Sensor C supports the presence of a target in location δ, and
- Sensors D and E support the presence of a target in location γ.

Combining these decisions would be unclear in this case, and crude analysis might conclude in the presence of four targets (when the solution with two

$BetP$	χ_1 AB	χ_2 CDE	A	B	C	D	E
α	**.48**	.06	**.50**	.28	.05	.16	.07
β	.32	.05	.13	**.48**	.05	.13	.04
γ	.08	**.78**	.17	.08	.38	**.68**	**.88**
δ	.06	.09	.10	.08	**.48**	.01	.01
ϵ	.06	.02	.10	.08	.05	.01	.01

Table 8: Values of $BetP$ on Ω for sensor clusters χ_1 and χ_2, where χ_1={A,B} and χ_2={C,D,E} are the best clusters, and for each sensor individually. In bold, the most probable resolution cell.

targets seems adequate). This just illustrates the danger of making decisions at intermediate level and trying to combine them, instead of making a decision at the last step of the computation, after all belief functions have been combined.

Note on tolerable conflict. Suppose two bba m_1 and m_2 on Ω whose non-null basic belief masses are $m_i(\emptyset)$, $m_i(A)$ and $m_i(\Omega)$ for $i = 1, 2$. Both belief functions show some internal conflict as both $m_i(\emptyset) > 0$. These two belief functions are not conflicting between them as they both point to the same subset. Nevertheless, $m_{12}(\emptyset) = m_1(\emptyset) + m_2(\emptyset) - m_1(\emptyset).m_2(\emptyset) = 1 - (1 - m_1(\emptyset)).(1 - m_2(\emptyset))$. This conflict $m_{12}(\emptyset)$ does not translate a conflict between m_1 and m_2, but the presence of some internal conflicts in each basic belief assignment that is translated by the non zero masses given to $\emptyset$. As far as we had tolerated conflicts like $m_1(\emptyset)$ and $m_2(\emptyset)$, the conflict $m_{12}(\emptyset)$ must also be tolerated.

Suppose now a set of n basic belief assignments $m_i : i = 1...n$. If we accept that each belief function is reporting on a single target, it means that we tolerate a conflict c equal to the maximum of the individual conflicts: $c = \max_{i=1...n} m_i(\emptyset)$. After combining these n basic belief assignments, the expected and thus tolerated - conflict is $1 - (1 - c)^n$. This empirical rule just acknowledges that once you tolerate some conflict, and c is a conflict you tolerate as you accept to use the m_i's, then you should tolerate the conflict that would be create by combining them. This last conflict is $1 - (1 - c)^n$.

Therefore, this tolerable conflict provides a threshold when it is compared with the computed conflict. If the computed conflict is smaller or equal to the tolerable conflict, then the belief functions can be accepted as being not conflicting, thus, in our submarine example, as reporting on the same event. If the computed conflict is larger ('much larger') than the tolerable conflict, it means the computed conflict cannot be explained by the internal conflicts, and, in our submarine example, one can then seriously doubt that the belief functions are all reporting on the same target. Of course what is meant by 'much larger' is fuzzy, but such fuzziness is not unusual. In statistical hypothesis testing, what is meant really by 'statistically significant', the 5% level is arbitrary, why not 4%? Similarly at what level of R^2 one should stop including new variables in a stepwise regression? Only experience could tell what is a 'tolerable conflict', the proposed threshold providing just a reference point good for assessing an 'order of magnitude'.

4 Conclusion.

We have presented a method that permits us to assess how many sources of information are present and observed by a set of sensors. The method is based on the transferable belief model. It uses the basic belief mass $m(\emptyset)$ as a measure of conflict and the sensors are clustered so that the conflict is minimized.

Such a method seems convenient for dealing with conflicting information. It seems powerful for helping decision-makers who are confronted with conflicting declarations collected from multiple sensors when the conflict might reflect the fact the sensors are not observing the same object.

As it is presented, this method does not tackle with the problem of inaccuracy representation. However, combining conventional filtering techniques (based on a probabilistic framework and Bayes' rule) with the proposed one is fully possible. Such mixed approaches as developed by Appriou [2] and by Sossai et al. [16] are very promising.

Description of more complete problems, representative of multisensor data association and fusion can be found in [5] and [8]. In such real-life problems, use of temporal continuity is helpful, both for association and state estimation maintenance, raising other theoretic difficulties such as updating and knowledge refined, deliberately left apart in this article.

Bibliography

[1] Appriou A., Probabilités et incertitude en fusion de données multisenseur, Revue Scientifique et Technique de la Défense, 1er trimestre 1991.
[2] Appriou A., Situation Assessment based on spatially ambiguous multisensor measurements, this Journal.
[3] Bar Shalom Y. and Li X. R. . Multi-target multi-sensor tracking : principles and techniques, YBS Publishing, 1995.
[4] Blackman S. S., Multiple Target Tracking with Radar Applications. Artech House, Norwood, Ma, 1986
[5] Colloquium on Surveillance Sensor Tracking, International research center for telecommunications transmission and radar, preprint of Delft University of technology - department of electrical engineering, June 1997.
[6] Grabisch M. and Prade H. The correlation problem in sensor fusion in a possibilistic framework. This Journal.
[7] Hall David L., Mathematical Techniques in Multi-sensor Data Fusion, Artech House, Norwood, Ma, 1991
[8] Hewish M., Sensor fusion for air defense, International Defense Review 4/1993
[9] Kennes R., Computational aspects of the Möbius transform of a graph. IEEE-SMC, 22: 201-223, 1992.
[10] Nimier V., Introduction d'informations contextuelles dans des algorithmes de fusion multicapteurs, Traitement du Signal: volume 14 numro 5, spcial 1997.
[11] Schubert J., Cluster-based Specification Techniques in Dempster-Shafer Theory, Proc. European Conf. Symbolic and Quantitative Approaches to Reasoning and Uncertainty, (ECSQARU '95), Springer-Verlag (LNCS), Berlin, 1995.
[12]Smets Ph., The application of the transferable belief model to diagnostic problems. Int. J. Intelligent Systems 13:127-157, 1998.

[13] Smets Ph., The Transferable Belief Model for Quantified Belief Representation. In Handbook of defeasible reasoning and uncertainty management systems. Gabbay D. M. and Smets Ph. Eds. Vol. 1, Kluwer, Doordrecht, 1998, pg. 267-301.
[14] Smets Ph., Practical use of belief functions. in Uncertainty in AI 99, Laskey K. B. and Prade H. (eds.) Morgan Kaufmann, San Francisco, pg. 612-621, 1999.
[15] Smets Ph. and Kennes R. The transferable belief model. Artificial Intelligence 66:191-234, 1994.
[16] Sossai C., Bison P. and Chemello G., Fusion of abstract knowledge and uncertain information in robotics, this Journal, 2000.
[17] Waltz E. and Linas J., Multisensor data fusion. Artech House, Norwood, Ma, 1990.