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Self-Organizing and Scalable Shape Formation for a Swarm of Pico Satellites

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Abstract

We present a scalable and distributed control strategy for swarms of satellites to autonomously form an hexagon lattice in space around a predefined meeting point. The control strategy is modeled as an artificial potential field. Such potential field is split in two main terms: a contribution to form locally hexagonal lattices based on the well known Lennard-Jones potential, and a global potential used to join the lattices into a single one. The control strategy uses only simple local information about few neighbouring satellites and assumes that each satellite can estimate its position with respect to the meeting point. Experiments show the results of the method with up to 500 satellites. The proposed method is general and could be adapted to build different kinds of lattices and shapes.

1 Introduction

According to many, the capabilities of future space exploration is severely limited by the physical impossibility to obtain the needed improvement in our space propulsion technologies. According to this opinion, the amount of mass we can afford to put into orbit will increase only marginally, over the next decades, shifting the research and technology development efforts toward the miniaturization of spacecraft systems. Thus, the satellites of the future could be much more similar to a CubeSat [1] than to one large and massive system. In this scenario, the cooperation between many small spacecraft is essential and needs to be completely automated (operating simultaneously a large number of satellites form ground is just unthinkable). To this aim, new scientific disciplines such as collective robotics and swarm intelligence provide a number of interesting solutions that can help in the automation of satellite swarm operations [2]. In particular, swarm intelligence [3] is a research field that aims at understanding the principles of decentralized control of a swarm of agents, taking inspiration from the behavior of social insects like ants and bees. Coordinated observation, planet exploration, and on-orbit self-assembly [4] are only some future applications where swarm intelligence could provide important contributions. As examples, some recent concepts, such as ESA's APIES [5] and NASA's ANTS [6], consider the use of swarms of satellites to achieve better observations of the asteroid belt and enhance fault tolerance. The decentralized control of a swarm of satellites is one the main challenges in such missions.

Previous work has been already carried out in this area and distributed control strategies have been proposed to deal with multi-satellites systems, but as soon as the number of satellites becomes high, the controller either needs the introduction of hierarchical levels among the agents [2], or is simply not able to construct regular formations.

In this paper, we focus on the problem of building a perfectly hexagonal lattice in a circular orbit. Such a configuration could be particularly important for applications such as autonomous self-assembly of solar powered satellites [7], large antennas and large reflectors in space. To this aim we will consider large swarms of small and simple satellites (up to 500 in number) that we will refer to as *pico satellites*.

2 Problem statement

Consider N identical satellites randomly distributed in space under the gravitational influence of a near planet, and a point $\vec{p}$ located on a certain orbit around the planet. Such point defines the origin of a moving reference frame with angular velocity ω. The aim of this work is to devise a control strategy $\vec{u}$ for the swarm of satellites to form a flat hexagonal lattice on the xy plane of this reference frame. The satellites must respect a mutual target distance σ that is a control parameter decided at design time.

Scalability is the main issue of this work. In Section 4 we show simulated experimental runs with up to 500 spacecraft. The control strategy does not depend (either explicitly or implicitly) on the number of satellites N forming the swarm. For this to happen, $\vec{u}$ is allowed to make use only of information on a limited number M of neighbouring satellites.

We assume that each spacecraft can estimate its position with respect to the meeting point $\vec{p}$. Therefore, the satellites of the swarm take decisions using mainly local information, and with a limited form of global knowledge about the origin of the reference frame.

Furthermore, the control strategy must respect some safety constraints, such as the absence of collisions among the agents and no satellite lost in space. In addition, the initial random distribution of agents should not play a destructive role for convergence to the final structure.

Finally, the control strategy must cope with the limited thrusting capabilities of the pico satellites, both in terms of thrusting power and propellant consumption.

From the mathematical point of view, each spacecraft has been modeled as a point mass whose motion is described by the classical system of Hill's equations [8]:

$$\begin{cases} \ddot{q}_x - 2\omega\dot{q}_y - 3\omega^2 q_x = u_x \\ \ddot{q}_y + 2\omega\dot{q}_x = u_y \\ \ddot{q}_z + \omega^2 q_z = u_z \end{cases}$$

where $\vec{q} = [q_x, q_y, q_z]$ is the position of the i-th satellite with respect to $\vec{p}$, and $\vec{u} = [u_x, u_y, u_z]$ is its control strategy. By using the given equations, we assume that the orbit of $\vec{p}$ is circular. A simple Runge-Kutta integration scheme has been used for all the experiments described in this paper. As a test case we consider our pico satellites to have a mass $m = 100$ kg, a thrusting capability of $T_{max} = 0.05$N. We also consider a number of different orbital environments, and in particular geostationary orbits (GEO), low Earth orbits (LEO) and Jupiter orbits close to the ones of its satellites Amalthea, Metis and Io. In Table 1 we have reported the numbers used to characterise these orbital environments. The pico satellites maximum thrust T_{max} needs to be able, in the final

Orbit	ω (rad/s)	R (km)	T (s)
LEO	$1\ 10^{-3}$	7,000	6,283
GEO	$7.3\ 10^{-5}$	42,000	86,071
Amalthea	$1.5\ 10^{-4}$	181,000	41,888
Metis	$2.5\ 10^{-4}$	129,000	25,133
Io	$4.1\ 10^{-5}$	421,600	153,248

Table 1: Different types of orbital environments considered

swarm configuration, to counteract the tidal gravity as to avoid to be carried indefinitely away.

This puts a limit to the radius of the final assembled lattice, a limit we may relate to the satellite number, to σ and to T_{max}/m. The maximum value of the tidal acceleration acts along the x axis and has a value of $\tilde{a} = 3\omega^2 q_x$. If we approximate the final lattice configuration to a circle of radius r we have that $A = \pi r^2$ is the final area. In a perfect hexagonal lattice we have:

$$A = \sum_{i=1}^{6} \frac{iN_i}{3} S < 2NS = N\sigma^2 \frac{\sqrt{3}}{2}$$

where S is the surface of the equilateral triangle with side σ and N_i is the number of satellites connected as to create i equilateral triangles. Thus we can write:

$$\tilde{a} = 3\omega^2 q_x \approx 3\omega^2 r < 3\omega^2 \sqrt{\frac{\sqrt{3}}{2\pi} N\sigma}$$

and obtain the condition:

$$T_{max} > m\tilde{a} > 3m\omega^2 \sqrt{\frac{\sqrt{3}}{2\pi} N\sigma}$$

The above equation is very useful to determine, given the pico satellite design, i.e. T_{max} and m, the possible dimensions of the final lattice we can build in the xy plane.

3 The control strategy

The control strategy $\vec{u}$ studied in this work follows the artificial potential approach [9] [10]. Unlike previous approaches, in which the potential field is defined as the composition of an attractive and repulsive field, the first leading the agents to the goal position and the second taking care of obstacle avoidance, here the artificial potential is a superposition of a local and a global contribution, plus a dissipative term. As the following discussion will show, this way of defining the artificial potential enables to separate the problems of controlling the external shape of the swarm from the way the satellites interact locally with each other.

The task of forming a flat hexagonal lattice in space can be decomposed in three distinct problems:

1. flattening the distribution of satellites on the xy plane;
2. creating the lattice on that plane while avoiding collisions;
3. preventing satellites from getting lost in space.

As explained, $\vec{u}$ can be expressed as the superposition of three contributions:

$$\vec{u} = \vec{g} + \vec{l} + \vec{d} \tag{1}$$

where

- $\vec{g}$ is a potential that attracts each satellite towards the origin of the common reference frame (i.e. the meeting point) and flattens the distribution on the xy plane. Hence, $\vec{g}$ tackles Problems 1 and 3;
- $\vec{l}$ is a potential that creates local clusters with the neighbouring satellites (Problem 2);
- $\vec{d}$ is a damping factor, similar to viscosity, used to stabilize the behaviour of the swarm and to ensure convergence.

The pico satellites we are considering for this study possess limited thrusting capabilities. More specifically, the magnitude of $\vec{u}$ cannot exceed the threshold value u_{MAX}. Similarly, the change of thrusting direction between two successive control actions is bound by $\Delta\theta_{\mathrm{MAX}}$.

In the following, we present the details of each term individually. We conclude this section explaining the stabilization mechanism of the formation after the swarm has converged to the final structure.

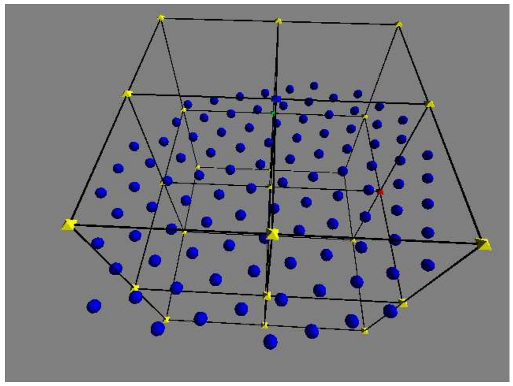

Figure 1: An hexagonal lattice obtained with 100 satellites in the xy plane of the LHLV frame of the Chief satellite.

3.1 Global attraction to the origin

As explained in Section 2, we assume that all the satellites in the swarm know their own position in the global reference frame (relative to the meeting point $\vec{p}$). $\vec{g}$ defines a global potential that attracts the satellites to the xy plane around the origin. Recalling that $\vec{q} = [q_x, q_y, q_z]$ is the position of a satellite with respect to $\vec{p}$, and defining the normalized vector

$$\hat{q} = [\hat{q}_x, \hat{q}_y, \hat{q}_z] = \frac{\vec{q}}{\|\vec{q}\|},$$

then

$$\vec{g} = \begin{bmatrix} -\eta_{xy}\|\vec{q}\|^2\hat{q}_x \\ -\eta_{xy}\|\vec{q}\|^2\hat{q}_y \\ -\eta_z q_z \end{bmatrix} \tag{2}$$

where η_{xy} is a design parameter that accounts for the attraction to the origin (i.e. $\vec{p}$) on the xy plane, and η_z plays the same role for the attraction to the xy play parallel to the z axis.

It is important to notice that the outer shape of the lattice is controlled by this potential. The components of $\vec{g}$ on the xy plane in Equation 2 define a paraboloid. Its sections cut parallel to the xy plane are circles, therefore the outer shape is circular (see Figure 1). If we substitute the paraboloid with another function, while keeping g_z the same, we will find other shapes.

3.2 Local lattice formation

To find a rule that makes it possible to create the wanted lattice, we took inspiration from a well known model of intermolecular interaction, the Lennard-Jones potential [11]:

$$V(r) = \epsilon\left[\left(\frac{\sigma}{r}\right)^{12} - 2\left(\frac{\sigma}{r}\right)^{6}\right]$$

As the graph in Figure 2 shows, two molecules (i.e. satellites) experience an attractive force when their distance r is larger than the target distance σ. Instead, when $r < \sigma$, the force is repulsive. The force is null when $r = \sigma$. Therefore, when two molecules are close enough, their stable arrangement is such that their mutual distance is exactly σ. When more molecules are considered, as depicted in Figure 3, the stable arrangement on a plane is an hexagon.

Besides its behaviour, the Lennard-Jones potential is interesting also because its parameters are very intuitive from the point of view of controller design: σ is the target distance and ϵ is the depth of the potential well, which accounts for the attractiveness and stability of the minimum located at σ.

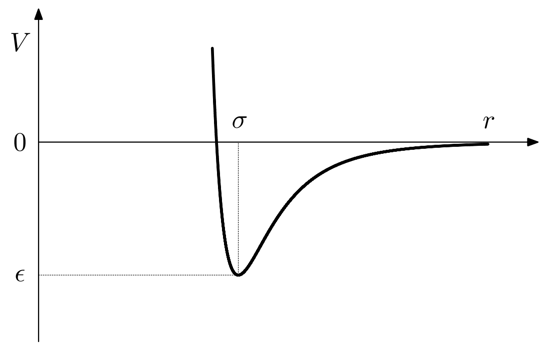

Figure 2: The Lennard-Jones potential.

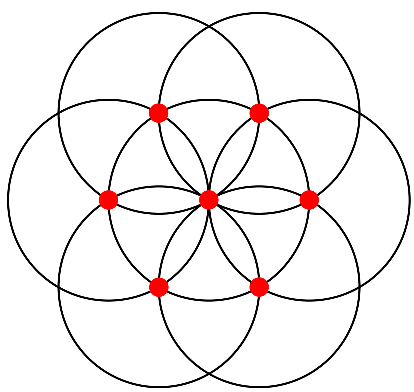

Figure 3: The point of minimum energy of the Lennard-Jones potential define an hexagonal lattice.

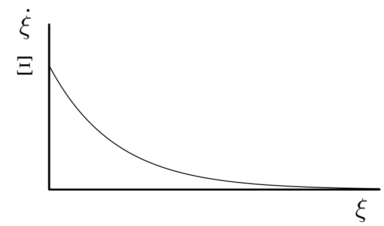

Figure 4: Rate of change virtual viscosity ξ to stabilize the final formation.

The magnitude of the artificial force of interaction $\vec{l_i}$ between a satellite and its i-th neighbour is given by

$$l_i = -\frac{dV}{dr} = \frac{12\epsilon}{r}\left[\left(\frac{\sigma}{r}\right)^{12} - \left(\frac{\sigma}{r}\right)^{6}\right]$$

Since the global potential g already attracts the satellites to the xy plane, it is enough that the direction of $\vec{l_i}$ be parallel to this plane, so

$$\vec{l_i} = \begin{bmatrix} l_i \hat{q}_x \\ l_i \hat{q}_y \\ 0 \end{bmatrix}$$

Eventually, $\vec{l}$ is defined as the average of the artificial forces due to the M closest neighbours:

$$\vec{l} = \frac{1}{M}\sum_{i=1}^{M} \vec{l_i}$$

Without averaging, the magnitude of $\vec{l}$ would be strongly dependent on M. Since $\vec{l}$ and $\vec{g}$ are summed, this in turn would make the choice of η_{xy} and η_z dependent from M. Instead, averaging removes this unnecessary dependence.

3.3 Ensuring convergence

The two potentials $\vec{g}$ and $\vec{l}$ alone are not enough to ensure convergence. In fact, both define conservative fields. Without a further dissipative term, convergence would be impossible.

The role of term $\vec{d}$ in Equation 1 is to dissipate the artificial energy, thus letting the swarm converge to the desired hexagonal lattice. The expression of $\vec{d}$ derives from these simple physics considerations and it is analogous to a virtual viscosity:

$$\vec{d} = -\xi\dot{\vec{q}}$$

where ξ is a design parameter usually < 0.5.

3.4 Formation stabilization after convergence

When the swarm has converged to the final structure, residual oscillations around the equilibrium point are present. Such oscillations lead to a waste of ΔV for the satellites.

To solve this problem and damp the oscillations, the $\vec{d}$ term is again useful. In fact, increasing the ξ parameter means increasing the virtual viscosity in the potential field. If viscosity reaches a sufficiently high value, then the residual speed of the satellites is not enough to let them move apart or oscillate and therefore the satellites remain trapped in the virtual equilibrium points. Stabilization around the equilibrium point is then obtained increasing the virtual viscosity ξ according to the following rule (see also Figure 4):

$$\dot{\xi} = \begin{cases} \Xi e^{-\xi/2} & \text{if } \xi < 2, \\ 0 & \text{otherwise} \end{cases}$$

Another separate problem is that of *when* to trigger the stabilization. In the current status of the work, we have devised a simple time-based criteria. Each satellite individually measures the time elapsed since the beginning of the shape formation process. After a certain time threshold T, which is a further design parameter, stabilization is triggered. A more elegant method would be to trigger the stabilization with a distributed consensus algorithm, such as those in [12].

4 Results

To assess the results of the proposed controller we use two main parameters. The first is the fuel consumption of each satellite and its statistical distribution along the swarm, the second is the quality of the hexagonal lattice obtained. We use Tsiolkovsky formula and thus assume that the fuel consumption is related to the ΔV_i of each satellite i evaluated by the simple expression:

$$\Delta V_i = \int_0^{t_f} \|\vec{u}_i\| dt$$

where t_f is the final lattice acquisition time.

The evaluation of the quality of the final acquired lattice is defined as:

$$\chi = \sum_i^N \sum_{j \in \mathcal{N}_i} \frac{|\sigma - r_{ij}|}{\sigma}$$

where $\mathcal{N}_i$ is the set containing the closest neighbours of the satellite i, r_{ij} is the relative distance between the satellites i and j at the final lattice acquisition time.

Our experiments show that χ depends on the amount of tidal gravity present and on the shape of the global potential (that can anyway be removed once the lattice has been assembled). Values of the order of $\chi = 0.006$ can be achieved assuming LEO orbits and a maximum thrust level of 0.01N on a spacecraft mass of 100kg.

5 Conclusions

We have presented a scalable and decentralized control strategy for large swarms of satellites to form bidimensional lattices in circular orbits.

The method consists in defining the controller as an artificial potential field composed by the superposition of a global field and a local field. The global field attracts the satellites to a predefined meeting point and flattens their spatial distribution. Sections of $\vec{g}$ parallel to the xy plane define the outer shape of the formation. In this work we focused on the construction of hexagonal lattices but different $\vec{g}$ functions can be equally used to obtain different other shapes.

The local potential takes care of the interactions of a satellite with its neighbours. In this work we used the Lennard-Jones potential, whose parameters are particularly intuitive to set. In fact, the mutual distance between the satellites can be chosen by the designer, along with the number of neighbours each satellite must consider to form the lattice. Other lattices could be constructed just by changing the Lennard-Jones potential. This work, therefore, sets a possible conjunction between lattice formation with satellites and crystallography. We plan to further study such conjunction by trying other known potentials in literature.

Results show that lattice formation is very accurate and that arrangements errors due to local minima are seldom present, although the parameters of the control strategy have been chosen only manually. Optimization of the parameters to minimize ΔV consumption is a foreseen development of this work.

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