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Abstract

In recent years, much attention has been devoted to the development of metaheuristics and local search algorithms for tackling stochastic combinatorial optimization problems. In this paper, we propose an effective local search algorithm that makes use of *empirical estimation* techniques for a class of stochastic combinatorial optimization problems. We illustrate our approach and assess its performance on the probabilistic traveling salesman problem. Experimental results show that our approach is very competitive.

1 Introduction

The probabilistic traveling salesman problem (PTSP) [4] is a paradigmatic example of a stochastic combinatorial optimization problem. It is similar to the TSP with the difference that each node has a probability of requiring a visit. The *a priori* optimization [1] approach for the PTSP consists in finding an *a priori* solution that visits all the nodes such that the expected cost of *a posteriori* solutions is minimized: The *a priori* solution must be found prior to knowing which nodes are to be visited; the associated *a posteriori* solution is computed *after* knowing which nodes need to be visited and it is obtained by skipping the nodes that do not require to be visited and visiting others in the order in which they appear in the *a priori* solution. This paper focuses on an iterative improvement algorithm, that is, an algorithm that starts from some initial solution and then iteratively moves to an improving neighboring one until a local optimum is found. Essential for designing and implementing an effective iterative improvement algorithm is that the cost differences among neighboring solutions are computed efficiently. Currently, the state-of-the-art iterative improvement algorithms for the PTSP, namely, `2-p-opt` and `1-shift` use for this task closed-form expressions based on heavy mathematical derivations [2]. Recently, we introduced a new algorithm called `2.5-opt-ACs` that also uses closed-form expressions and moreover adopts the classical TSP speedup techniques [3]. We showed that this algorithm is more effective than `2-p-opt` and `1-shift` with respect to both solution quality and computation time [3]. In this paper, we propose an effective iterative improvement algorithm that makes use of *empirical estimation* and variance reduction techniques.

2 Estimation-based iterative improvement algorithm for the PTSP

The PTSP is a stochastic combinatorial optimization problem that can be described as: Minimize $F(x) = E\big[f(x, \Omega)\big]$, subject to $x \in S$, where x is an *a priori* solution, S is the set of feasible solutions, the operator E denotes the mathematical expectation, and $f(x, \Omega)$ is the cost of the *a posteriori* solution that depends on a random variable Ω, which is an n-variate Bernoulli distribution; a realization ω of Ω prescribes which nodes need being visited. An unbiased estimator of $F(x)$ of a PTSP solution x can be computed on the basis of a sample of costs of *a posteriori* solutions obtained from M independent realizations of the random variable Ω.

In iterative improvement algorithms for the PTSP, we need to compare two neighboring solutions x and x' to select the one of lower cost. For x', an *unbiased* estimator of $F(x')$ can be estimated analogously to $F(x)$ by using a different set of M' independent realizations of Ω. However, in order to increase the accuracy of this estimator, the well-known variance-reduction technique *"common random numbers"* can be adopted. In the context of PTSP, this technique consists in using the same set of realizations of Ω for estimating the costs $F(x')$ and $F(x)$. Consequently, we have $M' = M$ and the estimator $\hat{F}_M(x') - \hat{F}_M(x)$ of the cost difference is given by: $\hat{F}_M(x') - \hat{F}_M(x) = \frac{1}{M}\sum_{r=1}^{M}\Big(f(x', \omega_r) - f(x, \omega_r)\Big)$. We implemented iterative improvement algorithms that use this way of estimating cost differences exploiting a neighborhood structure that uses the *node-insertion neighborhood* on top of the *2-exchange neighborhood* structure, that is, the well-known *2.5-exchange neighborhood*. To make the computation of the cost differences as efficient as possible, given two neighboring *a priori* solutions and a realization ω, the algorithm needs to identify the edges that are not common to the two *a posteriori* solutions. This is realized as follows: for every edge $\langle i, j\rangle$ that is deleted from x, one needs to find the corresponding edge $\langle i^*, j^*\rangle$ that is deleted in the *a posteriori* solution of x. We call this edge the *a posteriori edge* and it is obtained as follows: If node i requires visit, then $i^* = i$, otherwise, i^* is the first predecessor of i in x such that $\omega[i^*] = 1$, that is, the first predecessor for which the realization is one, indicating it requires visit. If node j requires visit, then $j^* = j$, otherwise, j^* is the first successor of j such that $\omega[j^*] = 1$. Recall that in a *2-exchange* move, the edges $\langle a, b\rangle$ and $\langle c, d\rangle$ are deleted from x and replaced by $\langle a, c\rangle$ and $\langle b, d\rangle$. For a given realization ω and the corresponding *a posteriori edges*, $\langle a^*, b^*\rangle$, $\langle c^*, d^*\rangle$, the cost difference between the two *a posteriori* solutions is given by $c_{a^*,c^*} + c_{b^*,d^*} - c_{a^*,b^*} - c_{c^*,d^*}$, where $c_{i,j}$ is the cost of edge $\langle i, j\rangle$. The procedure described can be directly extended to *node-insertion* moves. Furthermore, the proposed algorithm adopts neighborhood reduction techniques such as *fixed-radius search*, *candidate lists* and *don't look bits*. This algorithm is called `2.5-opt-EEs`. For further reference, see [3].

Intuitively, the variance of the cost difference estimator depends on the probability associated with each node. The smaller the probability values, the higher the variance. In this case, the usage of a large number of realizations reduces the variance of the estimator. Nevertheless, using a large number of realizations for high probability values is simply a waste of time. In order to address this issue, we adopt an adaptive sampling procedure that saves computational time by selecting the most appropriate number of realizations with respect to the variance of the cost difference estimator. This procedure is realized using *Student's t-test* in the following way: Given two neighboring *a priori* solutions, the cost difference between their corresponding *a posteriori* solutions is sequentially computed on a number of realizations. As soon as the *t-test* rejects the null hypothesis that the cost difference estimator is equal to zero, the computation is stopped. If no statistical evidence is gathered, then the computation is continued until a maximum number

Table 1: Mean and standard deviation (s.d.) of final solution cost and computation time in seconds.

	Algorithm	Solution Cost		Computation Time	
		mean	s.d.	mean	s.d.
$p = 0.050$	`2.5-opt-EEais`	3995478	366491	13.47	2.29
	`2.5-opt-EEs`	4012670	377854	41.95	6.41
	`2.5-opt-ACs`	3993213	372801	780.85	115.84
$p = 0.075$	`2.5-opt-EEais`	4576135	403363	6.90	0.98
	`2.5-opt-EEs`	4579572	381368	22.39	3.35
	`2.5-opt-ACs`	4579831	399972	581.56	77.68
$p = 0.100$	`2.5-opt-EEais`	5073047	414194	4.52	0.53
	`2.5-opt-EEs`	5078611	400207	14.57	1.94
	`2.5-opt-ACs`	5088197	400986	454.79	64.91
$p = 0.125$	`2.5-opt-EEais`	5524534	424238	3.39	0.40
	`2.5-opt-EEs`	5537658	427805	10.81	1.33
	`2.5-opt-ACs`	5555043	411029	367.22	45.81
$p = 0.150$	`2.5-opt-EEais`	5952696	432452	2.71	0.25
	`2.5-opt-EEs`	5963539	439965	8.51	1.00
	`2.5-opt-ACs`	5978640	431100	309.45	41.62
$p = 0.175$	`2.5-opt-EEais`	6349469	444421	2.23	0.21
	`2.5-opt-EEs`	6357512	443292	7.09	0.81
	`2.5-opt-ACs`	6380038	446660	258.70	36.76
$p = 0.200$	`2.5-opt-EEais`	6707241	476088	1.92	0.18
	`2.5-opt-EEs`	6715865	470162	6.01	0.64
	`2.5-opt-ACs`	6690302	454250	226.89	27.66

of realizations—a parameter—has been considered. The sign of the estimator is determines the solution of lower cost.

In order to reduce the high variance of the cost difference estimator for low probability values, we use the variance reduction technique *"importance sampling"*. Given two neighboring *a priori* solutions, this technique, instead of using realizations from the given distribution Ω, considers realizations from another distribution Ω^*—the so-called biased distribution—that forces the nodes involved in the cost difference computation to occur more frequently. The resulting cost difference between two *a posteriori* solutions for each realization is corrected for the adoption of the biased distribution and the correction is given by the likelihood ratio of the original distribution with respect to the biased distribution. We denote the proposed algorithm `2.5-opt-EEais`.

Here we report some example results obtained on *clustered homogeneous* PTSP instances of 1000 nodes, which are arranged in a $10^6 \times 10^6$ square and where each node has a same probability p of appearing in a realization. We considered a probability range in $[0.050, 0.200]$ with a step size of 0.025; 100 instances were generated for each probability level. For the hardware setting and implementation specific details, we refer the reader to [3]. Each iterative improvement algorithm is run until it reaches a local optimum. In order to compare the cost of the *a priori* solutions reached by each algorithms, we used the closed-form expression that computes the exact cost [4]. The results, measured across the 100 instances, are shown in Table 1.

Regarding the time required to reach local optima, irrespective of the value of p, `2.5-opt-EEais` is approximately 1.5 to 2 orders of magnitude faster than `2.5-opt-ACs` and it is faster than `2.5-opt-EEs` by a factor of 3. The average cost of local optima obtained by `2.5-opt-EEais` is comparable to one of `2.5-opt-EEs` and `2.5-opt-ACs`: the paired Wilcoxon test (α=0.05) does not reject the null hypothesis that the algorithms produce equivalent results.

3 Conclusion and Future Work

The main novelty of our approach consists of using the *empirical estimation* techniques and variance reduction techniques in the *delta evaluation* procedure. The proposed approach is conceptually simple, easy to implement, scalable to large instance sizes and can be applied to problems in which the cost difference cannot be expressed in a closed-form. We will devote our further research to assess the behavior of the proposed approach when used as an embedded heuristic in metaheuristics such as iterated local search, ant colony optimization and genetic algorithms. From the application perspective, the *estimation-based* iterative improvement algorithms will be applied to more complex problems such as stochastic vehicle routing, stochastic scheduling, and TSP with time windows and stochastic service time.

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