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Abstract

For about three decades, artificial intelligence has been concerned with a debate on the adequacy of probability for treating uncertainty. The transferable belief model is an alternative framework that resulted from that debate. The peculiarity of the transferable belief model is its dichotomical structure in which a non-Bayesian knowledge representation co-exists with a Bayesian decision making.

The reason of this structure is twofold: theoretical and practical. On the one hand, the structure was intended to overcome the difficulties pointed out by many practitioners in using probability for knowledge representation. On the other hand, it aimed at responding to the pragmatic concern of guaranteeing optimal decision making.

1 Introduction

For centuries, philosophers have proposed epistemological theories to explain what knowledge is, how it is acquired, and how human beings use it to make decisions. These theories differ even much one from the other, but they all eventually deal with the same issue: uncertainty. Indeed, every epistemological investigation starts from the observation that humans have a partial knowledge of reality and are therefore only able to obtain an imperfect image of reality itself. Uncertainty has always been perceived as a central issue by philosophers, especially for the role it plays in the dichotomy knowledge/action.

With the development of artificial intelligence, uncertainty gained an even more central role. The goal of artificial intelligence is to develop machines that are able to acquire knowledge and to use it to make decisions and act. In the first phase of its development, artificial intelligence was not concerned with uncertainty because machines were designed to perform intellectual activities, like playing chess [15] or proving theorems [14]. In other words, these first machines were living in an *abstract* world of which a complete model is given and in which every action leads deterministically to the desired result. Only in the 70's, the attention of researchers shifted to real-world problems and uncertainty had to be explicitly dealt with. At that moment, probability appeared as the natural candidate for the task and was adopted in a number of applications. Nevertheless, some difficulties were encountered with the practical use of probability and a number of criticisms were raised against its adequacy. As a consequence, part of the artificial intelligence community committed itself to the development of alternative methods and had to face a number of central issues including the representation of partial knowledge and the optimality in decision making.

In this paper, we discuss how the transferable belief model proposed by Philippe Smets addressed these issues. Smets devised an original way to build a model with a flexible knowledge representation apparatus and which guarantees a coherent decision making. The key idea introduced by Philippe Smets is to treat knowledge representation and decision making as two distinct issues: The transferable belief model is a two-level model in which knowledge representation is non-Bayesian while decision making is Bayesian. At what Smets calls the *credal level*, beliefs are represented and updated in a non-Bayesian fashion; at what he calls the *pignistic level* (from the Latin *pignus*, the bet), beliefs are casted into a structure that obeys the axioms of probability theory. By this dichotomical structure, Smets managed to give an answer to the doubts raised against probability for knowledge representation while responding to the pragmatic concern of guaranteeing optimal decision making.

The contribution of Philippe Smets to the understanding of the relationship between knowledge representation and decision making under uncertainty can be analyzed in three different ways: through a study of his theoretical contributions, through an investigation of the practical applications of the transferable belief model, and through an appreciation of the epistemological implications of his approach. With this paper we intend to explore the latter way. In our view, the conceptual framework of the transferable belief model should be seen as a further important step forward in the long lasting attempt to formalize and to employ empirical knowledge. We regard therefore the transferable belief model with respect to the historical context in which it emerged. This historical perspective allows to disclose the motivation laying behind the dichotomical structure proposed by Philippe Smets, which was intended precisely to overcome the difficulties pointed out in the probabilistic framework without renouncing to its well founded decision making apparatus.

The paper is structured as follows. In Section 2 we present an overview of the criticisms raised against probability and of some alternative models that have been proposed. In Section 3 we analyze the dichotomical structure of the transferable belief model and we discuss its motivations and epistemological implications. In Section 4 we conclude the paper.

2 The debate on probability and on alternative approaches

When in the 70's, techniques of artificial intelligence started being applied to real world problems, uncertainty emerged as a major issue. Probability appeared at first as the natural choice for building models of partial knowledge and for supporting decision making under uncertainty. Probability was a well-established 300-year old framework with a respectable record of successes in treating practical problems in many scientific domain ranging from statistical mechanics to information theory. Nevertheless, some researchers started to criticize the use of probability in artificial intelligence and, therefore, they developed alternative frameworks.

The difficulty spotted in the probabilistic framework was related to the fact that it requires to build complete models in which every hypothesis is represented by additive probabilities. Notwithstanding, the then recently revisited subjective interpretation of probability [21], most artificial intelligence researchers were apparently looking at probability from a frequentist perspective and felt that, since frequency data are seldom available in applications of artificial intelligence, probability was not a viable option.

Expert systems is one of the first domains in which a dissatisfaction was manifested. A notable example is MYCIN [23, 3], a system for medical diagnosis, which is possibly the best known expert system. The reason why its designers decided to adopt an alternative framework was precisely that it appeared extremely difficult to make explicit and to model in the form of precise additive probabilities, all the intuitions that drive a physician in the medical diagnosis.

The criticisms moved to probability, and to the Bayesian interpretation in particular, concerned mainly its representational apparatus: it was remarked that the Bayesian framework supposes more knowledge than what agents really possess [32, 3]. For example, the Bayesian framework requires that a belief on a hypothesis be functionally related with the belief on its negation [3], forcing thus to a double assignment that is often unsupported by the available evidence. The maximum entropy principle, which in the Bayesian framework is intended to supply the missing evidence, was perceived as problematic: Rather than "artificially" filling some lack of information, many researchers felt inclined to explicitly represent partial knowledge [22]. In particular, they wished to overcome the inability of the probabilistic framework at representing different grades of ignorance, from partial to complete, and at distinguishing it from equiprobability as well as from contradiction [30]. Another difficulty with the probabilistic framework was that it appeared unable to represent linguistic imprecision, which was interpreted as one of the main sources of uncertainty [35, 37]. Some psychological experiments [33] enforced the opinion that probability should be abandoned. They showed that both laymen and experienced researchers systematically violate the axioms of probability when modelling their personal knowledge.

All these criticisms raised doubts about the normativity of probability and, as a result, part of the artificial intelligence community focused on a number of alternative approaches. In the following, we briefly introduce some of them. Our aim is not to provide a thorough survey, but simply to discuss how they addressed the criticisms raised against probability and to highlight the issues they left open. In particular, we will focus our attention on the belief function model, since the transferable belief model of Philippe Smets is inspired by this approach.

One of the first alternative methods is certainty factors, an *ad hoc* formalism especially conceived for the expert system MYCIN [23, 3]. Even if certainty factors did not have a clear semantics, they were preferred to probability because they are less computationally demanding and because they simply require to provide graded values of certainty and no statistical frequencies are needed. This overcomes one of the main difficulties of the probabilistic framework. Yet, certainty factors do not address all issues with probability. Although non-additive, the certainty factor of a hypothesis and of its negation are not independent and they fail to distinguish ignorance from contradiction.

The theory of possibility [36, 9], which is a development of fuzzy sets [35], was introduced to represent natural-language judgments and their intrinsic vagueness. This framework drew the attention of the artificial intelligence community because it was based on non-additive measures and on simple rules of combination. Yet, it was noted that, although simple, these rules "do not appear to have any compelling justification" [34].

The theory of belief functions, also known under the name of Dempster-Shafer model, is another well known alternative model. The Dempster-Shafer model emerged from some studies started by Dempster in the 60's [6] and it was further developed by Shafer [22]. In the design of the Dempster-Shafer model, Shafer departed from Dempster's assumption that the Bayesian inference remains the basic instrument for dealing with uncertainty, and rejected the idea that the Bayesian framework is normative. His stand against the Bayesian subjective interpretation of probability is indeed the cornerstone of his attack to probability. Shafer started from the consideration that the two notions of *chance* and belief do not have to obey to the same probabilistic rules as it happens in the Bayesian approach. In particular, he rejected the additivity axiom for beliefs, and maintained that if an agent has information that justifies the assignment of a probability p to an hypothesis A, but no information in favor of $\bar{A}$, the hypothesis $\bar{A}$ should receive a support equal to 0 rather than a support equal to $1 - p$. In the latter case, it would be impossible to distinguish between the case in which we have no knowledge at all on $\bar{A}$, and the one in which we have it so that we can assign a probability $1 - p$ to $\bar{A}$. The explicit representation of the lack of knowledge is the essential trait of the Dempster-Shafer model. Yet, it has been pointed out that, despite its representational advantages, Shafer's theory of belief functions "lacks a formal procedure for decision making" [31]. In the following, we will discuss how the transferable belief model, which is a development of the Dempster-Shafer model, constitutes an original and effective answer to this issue.

3 The dichotomical structure of the transferable belief model

Smets' transferable belief model is a development of the Dempster-Shafer model. Its peculiarity is that it is a two-level model in which at the *credal level* beliefs are quantified by belief functions and at the *pignistic level* beliefs are transformed into precise additive probabilities before decisions are made. This dichotomical structure, in which a non-Bayesian knowledge representation co-exists with a Bayesian decision making, represents an original solution to the difficulties of the probabilistic approach. On one hand, the adoption of a two-level structure allows to build a non-additive representational apparatus in which it is possible to represent different degrees of knowledge and contradiction. On the other hand, it allows to exploit the well founded decisional apparatus of probability, which ensures rational decision making.

In Section 3.1 we consider how Smets presents and justifies this dichotomical structure. In Section 3.2 we make explicit the underlying motivations and their epistemological implications.

3.1 The believing/betting dichotomy

In the design of the transferable belief model, Philippe Smets starts from the idea that two mental levels exist: the *credal* level, in which beliefs are represented and updated, and the *pignistic* level, in which beliefs are used to make decisions. At the former, beliefs are quantified by belief functions while at the latter, they are quantified by probability functions.

Philippe Smets presents and justifies this dichotomical structure as follows. He points out that beliefs emerge from an epistemic state of uncertainty and that they are responsible of guiding our practical behavior. He notices that the two mental levels at which beliefs manifest themselves are not distinguished in the probabilistic framework and that probability functions are used to represent beliefs at both levels. He clarifies that this procedure is solely motivated by the argument that this guarantees optimal decision making [18, 21, 4]. Smets accepts this assumption, but in the approach he proposes:

> *probability functions quantify the uncertainty only when decision is really involved*—[27].

He acknowledges that uncertainty *must* be represented by a probability function at the pignistic level and that this function is induced by the beliefs entertained at the credal level, but what he refuses is the assumption that:

> *this probability function used at pignistic level represents the uncertainty at the credal level*—[27].

In other words, he makes a clear distinction between betting on a hypothesis and believing it: In the real world, we often bet on hypotheses on which we have little, or no evidence at all, and the fact to represent beliefs and bets in the same way, as it happens in the probabilistic framework, does not allow to account for this subtle distinction. The point for Smets is that knowledge and action, although interrelated, should be kept distinct and represented with two different mathematical frameworks. This is the only way to represent the actual knowledge on which decisions are based, *before* decisions themselves are made. This allows, for example, to make a distinction between ignorance and equiprobability that, from a decisional viewpoint, are identical. In the probabilistic framework, these two cases are indiscernible since they are considered only on a decision making level. On the contrary, in the transferable belief model their difference is clearly expressed at the credal level where equal support is given in case of equiprobability, and a support equal to 0 in case of ignorance.

The inability of making explicit the "justified specific support" in favor of a given hypothesis is precisely what has been indicated as the major shortcoming of the probabilistic formalism. Through the two-level structure, Philippe Smets manages to represent partial and complete ignorance and to distinguish it from contradiction which, as we have seen in Section 2, other alternative formalisms like certainty factors failed to do.

The identification of believing and betting in the probabilistic framework has historical motivations. Born in the XVII century, probability was conceived for dealing with the sphere of "doxa" (belief in Greek) as opposed to mathematics and logics that were intended to deal with the sphere of "episteme". In the first applications of probability, like games of chance and death-rate tables of insurance companies, the issue was to deal with uncertain reasoning in which only approximate premises are available and from which only approximate conclusions can be drawn. The task was thus to find a procedure for taking decisions in spite of uncertainty. This characterized probability as an approach in which the focus is on the product of thought, that is, the action, and thought is evaluated uniquely on the basis of the action it produces. Thought became so a function of action to be represented in the same mathematical framework. A clear formulation of this idea can be found in Ramsey's claim: "the kind of measurement of belief with which probability is concerned [...] is a measurement of belief *qua* basis for action" [18]. In this statement, he summarized the main assumption on which the Bayesian approach is based: personal beliefs can be rationally evaluated uniquely on the basis of the actions they determine.

Philippe Smets, among others, maintained that this approach to uncertainty does not meet the needs of artificial intelligence, since the goal of the latter is to reproduce human-like reasoning processes on machines. To this aim, it is necessary to explicitly represent the knowledge on which these reasoning processes are based and, since this is inherently uncertain, such uncertainty should be expressed. The very fact that probability is focused on decision making has been perceived as an obstacle to a fine-grained representation of uncertain knowledge. What the transferable belief model allows to do with its dichotomical structure is precisely to provide, at the credal level, an explicit representation of the knowledge actually available, so that, when decisions are performed, it is known with precision the extent to which the available knowledge is reliable.

Although Smets insists on the fact that the transferable belief model aims at representing the actual knowledge out of any decision context, he attaches a great importance to its practical use. Smets points out that the adoption of a dichotomical structure is not an academic exercise, but can have relevant consequences on decisions. Through his favorite example of "Peter, Paul and Mary" [30], Smets shows that a Bayesian approach can lead to charge a person of a murder with no conclusive evidence whatsoever, and he indicates how this can be avoided by using the transferable belief model.

The concern of Philippe Smets for the practical use of the transferable belief model explains why he did not conceive the credal and the pignistic levels as completely separated as Plato did with the world of ideas and the one of physical objects. For Smets, the world of knowledge is tightly connected to the one of action and this is why he says about his transferable belief model:

> *It is obvious that such a model, whatever its beauty and elegance, would be useless if it could not be used when decision must be made*—[27].

He insists:

> *Decision under uncertainty, as we consider here, is pervasive and is of course related to the belief held by the decision maker. So we have to show how the decision maker should decide when his/her opinion about which situation will prevail is represented by a belief function. For that purpose we develop the so-called pignistic transformation*—[27].

It is what he calls the *pignistic transformation* that allows to connect the world of beliefs to the world of actions and more precisely to transform non-additive beliefs into precise probabilities functions that obey the axioms of probability. Smets makes clear that this transformation should happen only when decision are made in order to keep track, as long as possible, of the uncertainty affecting the available knowledge. Until decision, the transferable belief model focuses only on the representation of beliefs and on their updating so that any change in the belief state is recorded. Smets states that this would be impossible in the Bayesian approach because its laws

> *[. . .] always center on forced decisions (or preferences) but not of belief itself. They relate to observable behaviors that reflect an underlying credal state, not the credal state itself.*—[30].

3.2 The pragmatic concern

The dual structure of the transferable belief model constitutes an original and interesting solution to the two main requirements that emerged from the debate on uncertainty: a representational structure for dealing with uncertainty as it emerges in artificial intelligence, and a well founded decision making procedure. The ability to meet both requirements is not a trivial achievement and most alternative approaches provided a rather satisfactory solution to the first issue but are typically more problematic on the second one.

At the beginning of the century, the problems concerning the use probability were mainly semantical: they were concerned with the issue of the interpretations of the concept of probability. In artificial intelligence, the problem becomes syntactical and concerns the issue of the normativity of probability, that is, whether alternative approaches can be defined that yield coherent decision making. The original idea of Philippe Smets of having a *pignistic* level that obeys to the Bayesian rules allows his approach to avoid this problem. In particular, by preserving a probabilistic decision making apparatus, Smets prevents the Dutch Book argument to be advocated against the transferable belief model [26]. Ensuring coherent decision making is one of the main concern of Philippe Smets, since this means to maximize the expected utility. In practical applications, this is a key issue.

In artificial intelligence, the effectiveness of a model is measured both by its ability to address a given problem and by the computational overhead it entails. Alternative techniques were preferred to probability because they more suitable to represent different grades of knowledge and offered some computational advantages [3]. Further reasons favored the choice of alternative methods. In particular, researchers in artificial intelligence were mainly computer scientists with a strong background in logic and a weaker one in probability and statistics. Alternative methods, like for example fuzzy sets, appeared more friendly and with a less awkward mathematical structure.

Although Philippe Smets, as an expert in medical statistics, had a strong formal training in statistics and probability theory, he felt nonetheless the need to depart from the probabilistic approach. His original motivation for studying uncertain reasoning was the modelling of medical diagnosis [25, 24, 28, 29] and it was within this domain of application that he conceived the transferable belief model. The reason for proposing this model was not mathematical simplicity. Rather, similarly to Shortliffe and Buchanan [23], he was motivated by the observation that it is difficult to determine a complete model in a context like that of medical diagnosis, in which reliable statistical data are often unavailable. The transferable belief model is, in this respect, a more convenient framework that avoids the assessments of complete probability distributions. Moreover, as it has been recently shown, the transferable belief model turns out to be computationally lighter than the probabilistic approach [5].

The reason behind the introduction of the transferable belief model is eventually practical efficiency which is a concern that drove all Smets' work. His theoretical studies on uncertain reasoning were always connected to practical applications as testified by the great number of problems to which the transferable belief model has been applied. Beside medical diagnosis, Philippe Smets adopted the transferable belief model for the solution of practical problems emerging in different engineering applications such as classification [10], sensor fusion [11], data association [2, 20], and target identification [5, 19].

The importance Philippe Smets gave to practical aspects of the scientific work is just one of the elements that draw a connection between his views and the epistemological school of *pragmatism*. Smets meets the two characterizing principles of this school: i) beliefs are personal dispositions that induce actions, and ii) the effectiveness is the criterion for evaluating a conceptual framework [16, 13]. Notwithstanding his insistence on the distinction between the credal and the pignistic level, Philippe Smets clearly states that the probability functions living at the pignistic level are determined by the beliefs that are entertained at the credal level. Besides, although the transferable belief model represents beliefs outside any decision context, decision is a product of belief and its practical result is the test of the validity of the beliefs from which the decision has been derived. In this sense, the pignistic transformation, which is the link between beliefs and actions, can be considered as an implementation of the pragmatist principles. In particular, the dynamical interaction Smets establishes between knowledge and action reminds the Deweyan conception according to which knowing is an evolving process that transforms reality, rather than simply interprets it [7, 8].

Dating back to the stoics maxim that "the end of man is action" and to the Greek tragedy principle that "action is more important than actors" [1], pragmatism is one of the oldest epistemological views. It becomes influential at the beginning of the last century when, with the refutation of classical mechanics, scientists faced the ultimate conclusion that empirical science cannot provide any certain knowledge, but only conjectures to be kept as valid as far as they resist to the test of experience. Henry Poincaré, authoritative protagonists of the post-Newtonian physics, embodies the pragmatist viewpoint. His answer to the dramatic discovery of the provisional character of physical laws, is that the aim of science is to produce effective solutions to practical problems rather than conclusive truths and that it is precisely in the practice that science finds its justification [17]. In this way, Poincaré revisits the 200 years old Humean view [12] according to which, although uncertainty is an essential character of our knowledge, the inferences we draw from it are justified in practice on the basis of the reliability of the solutions they provide.

The issue of the link between knowledge and practice and the strictly related problem of uncertainty is the crucial problem that every empirical science has to tackle and artificial intelligence is no exception. Indeed, the practical commitment of artificial intelligence to design models for representing uncertain knowledge and to implement techniques for using it in order to solve problems, is nothing but the issue that empirical science, as well as epistemology, dealt with for centuries. In this sense, the solutions given by the transferable belief model, and more in general by artificial intelligence, to this problem represent an interesting example for realizing how far science went in this direction and of how much remains to be done. The dichotomical structure of the transferable belief model provides stimulating hints for understanding the relation between knowledge and practice, and the role that this relation plays in science. The emphasis that the transferable belief model places on this relation and the practical solutions it gave to it are interesting for the scientific enterprise in general: they provide a measure of the achievements as well of the limits of our control and understanding of the external world.

4 Conclusions

This paper focused on the solutions that the transferable belief model provided to the issue of uncertainty in artificial intelligence. The problem of uncertainty is a major problem in artificial intelligence and, for about three decades, artificial intelligence has been concerned with a debate about the adequacy of probability for dealing with this problem. In particular, the representational apparatus of probability has been criticized and alternative methods were developed with the aim of providing a better representation of uncertainty. Yet, these alternative methods opened the issue of the incoherent decisions making.

The transferable belief model of Philippe Smets represents an original and interesting solution. Thanks to a dichotomical structure in which a non-Bayesian knowledge representation co-exists with a Bayesian decision making, Philippe Smets managed on the one hand to explicitly represent uncertainty, and on the other hand to respond to the pragmatic concern of ensuring optimal decision making.

The way in which Philippe Smets manages to connect a non-Bayesian knowledge representation level to a Bayesian decision making level has important epistemological implications. Indeed, Smets touches the more general issue of the link between knowledge and practice, which is one of the most puzzling problem that both science and philosophy faced for centuries. He realizes this link through the *pignistic transformation*, which suggests a *pragmatist* conception of science. The intuition of the pignistic transformation conveys the idea that although uncertainty pervades our knowledge, we can nonetheless address practical issues. The very fact that uncertain knowledge is converted into actions is what allows to evaluate how reliable this knowledge is in order to solve the problem at hand.

Coherently with this pragmatist perspective, Smets conceived his approach as one among many possible approaches for dealing with uncertainty, the adequacy of which is relative to the specific uncertainty problem is applied to. As he puts it:

> *Uncertainty is a polymorphous phenomenon [...] No single model fits all cases. The real problem is to recognize its nature and to select the appropriate model. The Bayesian model is only one of them. The*

transferable belief model is also one of them. Each has its own field of applicability—[30].

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